This equation asserts that the rate of diminution of the rate of change of temperature with depth is proportional to the rate of change itself. In other words, the rate of change diminishes in geometrical progression as the depth increases in arithmetical progression. And its rate of diminution is $\sqrt{c}/\sqrt{k\mathrm{T}}$, k being assumed to be constant. But, since the rate of diminution of the rate of alteration of the range is proportional to the rate of alteration itself, it follows that the rate of alteration bears the same ratio to the range. Hence *the range diminishes in geometrical progression as the depth increases in arithmetical progression*, the rate of diminution being directly as the square root of the thermal capacity, and inversely as the square roots of the conductivity and the periodic time conjointly.]

The above examples will serve to illustrate the extreme ease with which the consideration of dimensional equations leads to the solutions of problems which are usually attacked by the aid of recondite methods alone.

Some relations between the orthic and the median triangles.

By A. J. Pressland, M.A.

Figure 15.

Let ABC be the triangle, X, Y, Z the feet of the altitudes, H the orthocentre, A′,B′,C′ the mid points of the sides.

Let ZY meet A′B′ in D, A′C′ in D′, B′C′ in R :
ZX meet B′C′ in E, A′B′ in E′, A′C′ in S :
XY meet C′A′ in F, B′C′ in F′, A′B′ in T.

§ 1. The following triangles are similar to ABC ; AYZ, XBZ, XYC.

B′YD, C′D′Z are similar to AYZ and have parallel sides.
EC′Z, XA′E′ are similar to XBZ and have parallel sides.
XFA′, F′YB′ are similar to XYC and have parallel sides.

Y	is the	internal	centre	of similitude	of the	circles	AYZ, B′YD.
Z	,,	,,	,,	,,	,,	,,	AYZ, C′D′Z.
Z	,,	external	,,	,	,,	,,	XBZ, EC′Z.
X	,,	internal	,,	,,	,,	,,	XBZ, XA′E′.
X	,,	external	,,	,,	,,	,,	XYC, XFA′.
Y	,,	,,	,,	,,	,,	,,	XYC, F′YB′.

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Hence the circle	AYZ	touches	the circles	B′YD,	C′D′Z,
and the circle	XBZ	,,	,,	EC′Z,	XA′E′,
,, ,,	XYC	,,	,,	XFA′,	F′YB′.

§ 2. U, V, W the mid points of AH, BH, CH are the centres of the circles AYZ, XBZ, XYC. Now A′U is a diameter of the nine point circle; therefore A′Y is a common tangent to the circles AYZ, B′YD, and A′Z is a common tangent to the circles AYZ, C′D′Z.

Hence A′ is the radical centre of the circles

AYZ, B′YD, C′D′Z.

Similarly B′ and C′ are radical centres of triads of circles.

Now AC is the radical axis of B′YD, F′YB′;

and AB is the ,, ,, ,, C′D′Z, EC′Z;

and

$$AB'.AY = AC'.AZ.$$

Therefore A is the radical centre of B′YD, F′YB′, C′D′Z, EC′Z.
Similarly B and C are radical axes of tetrads of circles.

Hence AA′ is the radical axis of the circles B′YD, C′D′Z; and similarly for BB′ and CC′.

§ 3.

R	is the external	centre of similitude of the circles	B′YD, C′D′Z,
S	,, internal	,, ,, ,, ,,	EC′Z, XA′E′,
T	,, external	,, ,, ,, ,,	XFA′, F′YB′.

It may be shown that

B′C′	is a common tangent to the circles	B′YD, C′D′Z.
C′A′	,, ,, ,, ,, ,,	EC′Z, XA′E′,
A′B′	,, ,, ,, ,, ,,	XFA′, F′YB′.

Since AA′ bisects B′C′, and A is the radical centre of B′YD, C′D′Z, another proof can be deduced that AA′ is the radical axis of B′YD, C′D′Z.

§ 4. The angle YDB′ = ∠ C.
Therefore the four points A′, Y, D, C are concyclic.
Similarly the following tetrads are concyclic

A′ZD′B; B′EZA; B′XE′C; C′FXB; C′F′YA.

As $$AY \cdot AC = AZ \cdot AB,$$

A must be on the radical axis of A′YDC and A′ZD′B. Hence AA′ is the radical axis of these circles.

§ 5. Since $\angle B'C'X = \angle B'YX = \angle B$,

B′C′ touches the circle BXC′.

Therefore the circle C′D′Z touches the circle BXC′F.

Similarly the circle XA′E′ touches the circle A′YDC, and the circle F′YB′ touches AZB′E.

§ 6. Since $\angle ZXB = \angle ZYA' = \angle XCD$,

ZX is parallel to DC.

Similarly BF is parallel to YZ and AE to XY.

If PA′Q be the tangent at A′ to the nine-point circle, and cut XY in P, and AC in Q, then since $\angle B'A'Q' = \angle C = \angle B'DY$, PA′Q is parallel to YZ.

X is the centre of similitude of the quadrilaterals BC′FX, PXA′L. But B,C′,F,X are concyclic.

Therefore P,X,A′,L are concyclic ;

and since X is the centre of similitude, the two circles BC′FX and PXA′L touch

The angle $B'QA' = \angle DYB' = \angle B$,

and $\angle A'XL = \angle XA'L = \angle B$.

Therefore $\angle XLA' = 180° - 2B$.

But $\angle XLA' = \angle XPA'$.

Therefore $\angle XPA' + \angle A'QB' = 180° - B = \angle XA'B'$.

Therefore the circle XA′L touches the circle B′A′Q.

Since B′ is the centre of similitude of B′A′Q and B′YD, the circumcircles of these triangles touch.

Hence from AYZ has been derived the following cycle of circles six in number,

B′YD, AYZ, ZC′D, C′BXF, PXA′L, A′B′Q, B′YD, AYZ,

each of which touches the two adjoining circles.

Other cycles could be obtained from the triangles XBZ, XYC.

As of the four orthic points A,B,C,H any three may be consi-

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dered as forming the original triangle, it follows that four triangles can be obtained each containing three cycles of six circles.

§ 7. Since ZC′S and DB′C are in perspective,

	RS	passes	through	C.
Similarly	ST	,,	,,	A,
and	TR	,,	,,	B.

Since ZC′E and DA′C are in perspective,

	D′E	passes	through	C.
Similarly	F′D	,,	,,	B,
and	E′F	,,	,,	A.

Since ZC is perpendicular to AB, it is bisected perpendicularly by A′B′, and as CD is parallel to ZX, the figure CDZE′ is a rhombus, as are BD′YF and AEXF′.

D′ESR forms a complete quadrilateral two of whose diagonals ZC′ and ZC bisect their corresponding angles and are perpendicular to each other.

Sixth Meeting, April 10, 1891.

J. S. Mackay, Esq., M.A., LL.D., ex-President, in the Chair.

On some properties of a triangle of given shape inscribed in a given triangle.

By R. E. Allardice, M.A.

It is well known that in a given triangle a one-fold infinity of triangles may be inscribed similar to a given triangle This becomes at once obvious on consideration of the converse problem; for we may circumscribe about a given triangle (A), a triangle similar to a second triangle (B), and having its sides parallel to the sides of (B).

We may also show in the following manner that, in a given triangle, one triangle and only one can in general be inscribed having its sides parallel to given directions.

Let D (fig. 16) be a point in the side BC of a triangle ABC; and let DE, EF, FD′, be parallel to the given directions.

Now D and D′ trace out projective ranges on BC; and hence to get the inscribed triangle corresponding to the given directions, we