Index

#P-hard, 45
adiabatic state preparation, *see* quantum adiabatic algorithm
AES (Advanced Encryption Standard), 108
amplitude amplification, 205, 235–238, *see also* Grover search
  applications of, 64, 104, 108, 165, 175, 265, 288, 302
  fixed-point, 209, 237, 245, 254
  oblivious, 204, 219, 222, 237
  on top of phase estimation, 9, 36, 59, 73, 171, 229
  on top of QAOA, 71
  variable-time, 273
  with Gibbs sampling, 80, 244
amplitude estimation, 205, 233, 238–242, 279, *see also* overlap estimation
  applications of, 19, 59, 80, 118, 131, 143, 150, 158, 165, 175, 307
  nondestructive, 241
  unbiased, 241
analog quantum simulator, 14, 23, 44, 49, 56
approximate counting, *see* amplitude estimation

backlog problem, 318, 321
backtracking algorithms, 68
barren plateau, 96, 184, 283
Bernstein–Vazirani algorithm, ix, 277
Betti number, 173–179
block-encoding, 189–198
  for Hamiltonian access, 7, 18, 34, 59, 211, 244
  of an LCU, 7, 19, 26, 34, 156, 192, 197, 223, 264
  of classical data, 84, 135, 152, 155, 158, 191, 193, 248, 266–269, 273, 295
  of Gram matrices, 156, 191, 267
  of observables, 10, 38
  of sparse matrices, 85, 153, 160, 172, 191, 269, 273, 297
Boltzmann machine (BM), 161
  quantum (QBM), 164, 249
  restricted (RBM), 162
Born–Oppenheimer approximation, 30, 45
BQP, 342
BQP-complete, 20, 43, 56, 168, 210, 343
BQP-hard, 125, 172, 183, 307, 343
branch-and-bound method, 68, 75, 134, 136

Carleman linearization, 122, 124
cheap gradient principle, 92, 96, 278
coherent arithmetic, 34, 54, 92, 101, 108, 143, 144, 234, 263, 265
collateralized debt obligations (CDOs), 130
color code, 324, 329
computational fluid dynamics (CFD), 111, 114, 124
configuration interaction (CI) methods, 42, 43
conjugate gradient method, 118, 126, 139, 275, 293, 297
constraint satisfaction problems, 65, 253, *see also* satisfiability problem
contrastive divergence, 163, 167
convex optimization, 90–93, 134, 154, 279, 291, 299
coupled cluster (CC) methods, 13, 42, 43, 60, *see also* unitary coupled cluster (UCC) ansatz
credit valuation adjustments (CVAs), 130, 142
cryptocurrency, 107, 110
cryptography, 98, 169
  elliptic curve, *see* ECC
  post-quantum, 104, 106, 107, 131

public-key, 99
symmetric-key, 99, 108
curse of dimensionality, 123, 146
cutting-plane methods, 89

decoding (in QEC), 65, 316, 318, 321–324, 330
density functional theory (DFT), 42
dequantization, *see* quantum-inspired algorithms
derivative pricing, *see* option pricing
differential equations, 111–129
nonlinear, 114, 150
ordinary (ODEs), 113
partial (PDEs), 113
stochastic (SDEs), 122, 142
Diffie–Hellman, 99
Dijkstra's algorithm, 67
discrete logarithm problem, 102, 183, 226
elliptic curve, *see* ECDLP
distance (of a QEC code), 316
formula for, 330
of the surface code, 320
DMRG (density matrix renormalization group), 13, 42, *see also* tensor networks
DQC1-complete, 308
DQC1-hard, 177
duality (in optimization), 79, 84, 182, 183, 292, 302
dynamic programming, 67, 68, 259

Eastin–Knill theorem, 317, 329
ECC (elliptic curve cryptography), 99, 103, 106
ECDLP (elliptic curve discrete logarithm problem), 100, 105
effective field theory (EFT), 58
eigenstate filtering, 8, 18, 35, 233, 234, 272
electronic structure problem, 29–44, 61, 204, 248
element distinctness problem, 68
elliptic curve cryptography, *see* ECC
energy-based models, 160–169, *see also* Boltzmann machine
exact diagonalization, 13, 27, 42, 247

factoring, 100, 103, 227, 233, 285
Fermi–Hubbard model, 5–15, 328
finite difference methods (FDMs), 33, 112, 142, 277, 283
finite element method (FEM), 112, 121, 123, 126
finite volume method (FVM), 112
Fokker–Planck equation, 123

Gaussian elimination, 89, 118, 126, 138, 155, 157, 274, 293
Gaussian process regression, 151–154, 159
Gibbs sampling, 205, 228, 233, 243–249
applications of, 8, 18, 25, 37, 47, 86, 166, 302
dynamic, 80, 86, 304
of classical systems, 80, 82, 164, *see also* Monte Carlo methods, Markov chain
gradient descent, 93, 94, 97, 163, 164, 181, 279, 283
Greeks (in finance), 130, 141, 145, 280
Grover search, 62–68, 71, 86, 175, 259, 294, 300, *see also* amplitude amplification

Hamiltonian simulation, 202, 205, 209–224, 228, 264
applications of, 8, 18, 26, 37, 47, 53, 59, 95, 117, 122, 171, 192
as access model, 7, 34, 36, 59, 247
of adiabatic evolution, 72, 251, 294
Hamming weight phasing, 11, 20
Hartree–Fock, 42
heat equation, 112, 123
Heisenberg model, 17, 19, 21
hidden subgroup problem, 102, 227
high-performance computing (HPC), 60, 75, 109, 111, *see also* parallelism in classical implementations

initial value problem, 114
integer factorization problem, *see* factoring
integer program (IP), 134
mixed- (MIP), 68, 134, 136
interior point methods (IPMs), 81, 88, 138, 291, 301, *see also* quantum interior point methods
software implementation, 88
Ising model, 16, 21, 70, 161, 164, 252
iterative refinement, 274, 288

Jordan–Wigner transformation, 7, 25, 34

Kaczmarz method, 89, 139, 274, 297
Kaiser window, 37, 232
kernel methods, 151, 174, 181, *see also* quantum kernel methods
Kikuchi method, 171, 173

LABS (low autocorrelation binary sequence) problem, 72
Lagrange multipliers, 133, 135, 155, 292
lattice Boltzmann equation, 115, 122, 124
lattice gauge theories (LGTs), 53–56

lattice Schwinger model, 54, 56
lattice surgery, 263, 326
  resource estimate, 329–330
linear combination of unitaries (LCU), 195–197, *see also* block-encoding of an LCU
  applications of, 117, 245, 273
  method of Hamiltonian simulation, 218–221, 264
linear program (LP), 78, 83, 292, 299, 303
low-density parity check codes, 317, 329

magic state distillation, *see* state distillation
matrix product state (MPS), 7, 14, 22, 23, 35, *see also* tensor network methods
  tomography, 287
MAX-CUT, 70, 298
Maxwell's equations, 112, 114, 123
Metropolis–Hastings algorithm, 163, 243, 245
Monte Carlo methods
  estimating expectation values, 141, 241
  Markov chain (MCMC), 161, 163, 167, 243
  multilevel, 146
  quantum, *see* quantum Monte Carlo
  quantum-accelerated, 122, 131, 143–146, 165, 265
  quasi–, 146
  style of Gibbs sampling, 8, 20, 25, 90, 166, 233, 245–247
multiplicative weights update (MWU) method, 248, 299–304
  applications of, 79, 86, 136

Navier–Stokes equation, 111, 114, 122
neural networks, 122, 125, 150, 161, 174, 179, 323, *see also* quantum neural networks
nonconvex optimization, 94–97, 134, 136, 298
NP, 62, 66, 342
NP-complete, 20, 343
NP-hard, 68, 134, 166, 177, 247, 283, 307, 343
nuclear magnetic resonance (NMR), 16, 17, 188, 198

option pricing, 130, 140–147, 241
order finding, 100
overlap estimation, 241
  applications of, 26, 38, 118, 153, 156

parallelism
  in classical implementations, 104, 126, *see also* high-performance computing
  in decoding, 322
  in QRAM, 66, 256
  of classical methods, 74, 109, 110, 126, 146, 167, 258
  of quantum methods, 75, 95, 102, 103, 137, 242, 262, 268, 338
  of state distillation, 257, 328
parameterized quantum circuit, *see* variational quantum algorithms
partition function, 162, 244, 248, 309
phase estimation, *see* quantum phase estimation
Poisson equation, 113, 115
portfolio optimization, 68, 87, 131–140, 269, 297, 303
power method, 171, 176, 178
preconditioning (linear system), 119, 124, 126, 275
principal component analysis (PCA), 149, 169, 234, *see also* tensor PCA
product formulas, 211–216, 230, 252
  applications of, 7, 18, 26, 38, 47
  resource estimate for, 11, 12, 21, 40, 41, 54, 60, 74
projected entangled pair state (PEPS), 22, *see also* tensor network methods

QAOA (quantum approximate optimization algorithm), 69, 71, 75, 139, 173, 253, 284
qDRIFT, 216–218
QMA, 342
QMA-complete, 20, 22, 343
QMA-hard, 12, 13, 39, 177, 247, 343
QRAM (quantum random access memory), 65, 85, 150, 255–259, 303
  applications of, 67, 68, 80, 81, 86, 92, 117, 135, 136, 149, 155, 165, 193, 261, 273, 295
QROM (quantum read-only memory), 258
QSVT (quantum singular value transformation), 205–209, 229, 236, 242
  applications of, 73, 175, 221, 237, 245, 265, 271, *see also* eigenstate filtering
  method of Hamiltonian simulation, 222–224
quadratic program, 133, 154
quantum adiabatic algorithm (QAA), 69, 72, 76, 233, 250–254, 284
  applications of, 6, 18, 26, 36, 47, 72, 164, 165, 265, 272, 294
quantum annealing, 76, 134, 139, 165–168, 249, 252
quantum arithmetic, *see* coherent arithmetic
quantum eigenvalue transformation, 204, *see also* QSVT

quantum field theories, 51–57, 210, 214, 227
quantum Fourier transform (QFT), 225–227
  applications of, 101, 231, 277
  approximate, 102, 226
  generalized, 102, 227
  operator, 245
quantum gradient estimation, 241, 276–280, 288
  applications of, 10, 38, 86, 91, 95, 145, 166, 294
quantum interior point methods (QIPMs), 290–298
  applications of, 84, 135, 155
quantum kernel methods, 181–183
quantum key distribution, 98, 131
quantum linear system solver (QLSS), 205, 209, 228, 234, 253, 270–275, 284
  applications of, 84, 117, 135, 149, 152, 155, 293
quantum minimum finding, 63, 80, 86, 237
quantum Monte Carlo (QMC), 13, 20, 42, 44, 52, 55, 58, 75, 284
quantum neural networks, 180–181, 285
quantum phase estimation (QPE), 203, 205, 218, 227–234, 253
  applications of, 9, 18, 26, 36, 47, 59, 73, 101, 165, 171, 175, 239, 244
  consistent, 232
  unbiased, 232, 288
quantum random access memory, *see* QRAM
quantum signal processing (QSP), 198–201, 229, 236, *see also* QSVT
  applications of, 144, 233
  method of Hamiltonian simulation, 8, 18, 26, 38, 222–224, 230
  resource estimate for, 12, 21, 145
quantum simulated annealing, 165
quantum singular value transformation, *see* QSVT
quantum tomography, 148, 226, 241, 260, 274, 279, 286–290, *see also* shadow tomography
  applications of, 84, 118, 135, 291
quantum walks, 67–68, 211, 237
quantum-inspired algorithms, 48, 150, 153, 157, 158, 275, 305
qubitization, 201–205
  for Hamiltonian simulation, 12, 26, 38, 54, 222
  for QPE, 8, 9, 37, 231
  resource estimate for, with QPE, 11, 40, 41
QUBO (quadratic unconstrained binary optimization), 16, 70, 134, 140

rate (of a QEC code), 316
reaction-diffusion equations, 122
recommendation systems, 149, 234, 258
rejection sampling, 36, 165, 175, 233, 245, 246
reversible arithmetic, *see* coherent arithmetic
RSA (Rivest–Shamir–Adleman), 99, 103, 106

SAT, *see* satisfiability problem
satisfiability problem, 64, 69, 70, 74, 342
Schöning's algorithm, 64, 67
Schrödinger equation, 30, 45, 58, 95, 114, 123, 210, 251, 335
second-order cone program (SOCP), 83, 135, 154, 294, 297
semidefinite program (SDP), 83, 248, 288, 294, 297, 299, 303
shadow tomography, 289, 290
Sherrington–Kirkpatrick model, 68, 70, 74, 162, *see also* spin glass
Shor's algorithm, ix, 99–103, 228, 233
short-path algorithm, 73, 233
sign problem, 52, 56, 75
Simon's algorithm, ix, 102, 344
simplex method, 291
simulated annealing, 163, 167, *see also* quantum simulated annealing
single-shot QEC, 322, 329
spectral methods, 112, 115, 116, 123, 127
spin glass, 71, 162, 169
spin models, 15–23, 70, 161, 224
stabilizer code, 320
state distillation, 103, 257, 263, 327–330
state preparation
  adiabatic, *see* quantum adiabatic algorithm
  from classical data, 7, 88, 152, 155, 156, 158, 197, 259–265
  garbage-free, 263
  of ground states, 6, 12, 17, 26, 30, 53, 58, 164, 178, 229, 250
  of thermal states, *see* Gibbs sampling
  of trial/ansatz states, 7, 26, 35, 272
stoquastic, 75
supercomputer, *see* high-performance computing
superconductor, 5, 6
support vector machine, 154–157, 181, 297, 300
surface code, 103, 147, 263, 320–331
  resource estimate for, 65, 103, 329–330

SYK model, 24–28
tensor network methods, 13, 14, 20, 42, 56, 147, 158, 168, 226, 285, 305–309, 321
tensor PCA, 76, 169–173
threshold (in QEC), 316–318, 322
tomography, *see* quantum tomography
topological data analysis (TDA), 173–179, 205, 259
traveling salesperson problem, 64, 67, 68, 259
Trotterization, *see* product formula
turbulence (fluids), 122, 124
unitary coupled cluster (UCC) ansatz, 284
value at risk (VaR), 131, 142
  conditional (CVaR), 131
variational quantum algorithms (VQAs), 63, 96, 180, 252, 279, 281–285
  fault-tolerant applications of, 7, 35, 47, 71, 144, 164
  NISQ applications of, 14, 23, 44, 49, 57, 61, 105, 110, 128, 139
variational quantum eigensolver (VQE), 14, 44, 178, 283
Vlasov equation, 112
wave equation, 111
zero-sum game, 78–82, 300, 303