GLOBAL INSTABILITIES OF A GALAXY MODEL

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A simple two-component fluid model of a galaxy is analyzed numerically. For this equilibrium configuration a large number of unstable spiral modes is found. It is of particular interest that some of these modes are well described by the asymptotic theory developed for tightly wound trailing spirals, while others are best understood in terms of the swing formalism which includes both leading and trailing waves.

The model consists of a Toomre disk of order 5 and length scale 12, plus a Plummer sphere with length scale 2 containing half as much mass. Only the disk is dynamically active. The sphere, regarded as frozen, affects only the total gravitational potential. This combination produces the rotation curve shown in Figure 1. To complete the description I specify the stability function as

$$Q(r) = 1 + \exp\left(\frac{-r^2}{2}\right) \; . \qquad (1)$$

In other words, I assume that the innermost portion of the disk (residing more or less within the sphere) is quite "hot", whereas its exterior is just warm enough to avoid Jeans instability. These two conditions were adopted purposely to favor - and thereby test - the refraction and amplification of the short and long trailing waves involved in the asymptotic theory of Lau, Lin and Mark (1976).

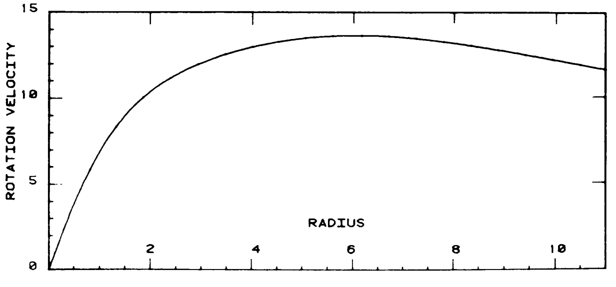

Figure 1. Rotation curve for the adopted equilibrium model.

E. Athanassoula (ed.), Internal Kinematics and Dynamics of Galaxies, 121–124.

The dynamics are assumed to be governed by the linearized equations of motion for the system. I concentrate on the unstable spiral modes using methods similar to those developed by Pannatoni (1979), and find that there are plenty of them. Figure 2 reports the modal "spectrum" for the basic model in a form suggested by Toomre. It obviously offers a lot of information. For example, the third two-armed mode (2C) has a growth rate 0.512 and corotation radius 5.91 corresponding (with the help of Fig. 1) to a pattern speed 2.305. That this basic equilibrium is quite unstable makes it all the better to illustrate two distinct sources of instability.

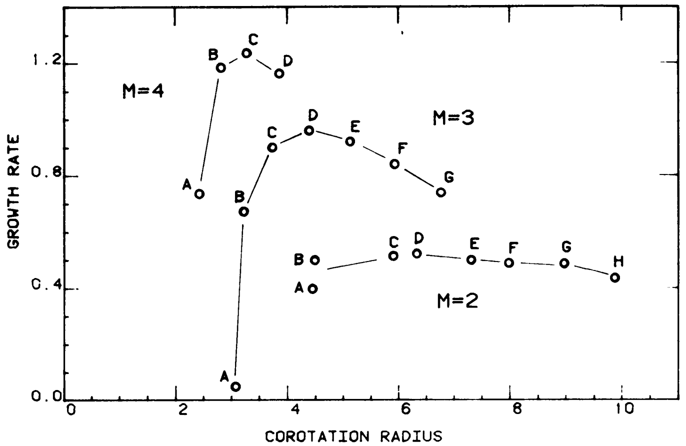

Figure 2. The spectrum of the various modes for m = 2,3 and 4.

One feature seen in the spectrum is the nearly identical pattern speed and growth rate for the first two m=2 modes. This "degeneracy" is now known to be one of the signs that the model (for these modes) is approaching the asymptotic regime where the instability can be traced to a short-long trailing wave feedback cycle first discussed by Lau, Lin and Mark (1976). As evidence for the correctness of this description, I have computed the mode based on the asymptotic second order equation and found agreement in pattern speed and growth rate to within 5% and 15% respectively. Further, the eigenfunctions produced by the different methods are nearly indistinguishable. (See Fig. 3.)

The spectrum in Figure 2 also cautions, however, that these all-trailing modes are not the whole story even in this favorable setting. Notice that several of the m=3 modes grow about twice as rapidly as the m=2 modes we have just been discussing. And although the first two m=3 modes are close in pattern speed, their structures and growth rates are not. These modes owe their instability to a feedback loop quite different from the aforementioned cycle. This possibility was first recognized by Bardeen (1976) in his own gas-disk calculations. He realized that the regularly spaced interference patterns typical of these modes (see especially mode 3D in Fig. 3) signify a superposition of trailing and leading waves of similar length and

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amplitude. This intuition is now supported by a solid theory. Given the pattern speed and model, Toomre's (1981) method for calculating the growth rate from group transport and swing amplification yields agreement with the "exact" values to within 10% for modes 3D, 3E, etc. Again the essence of the modes seems to have been grasped.

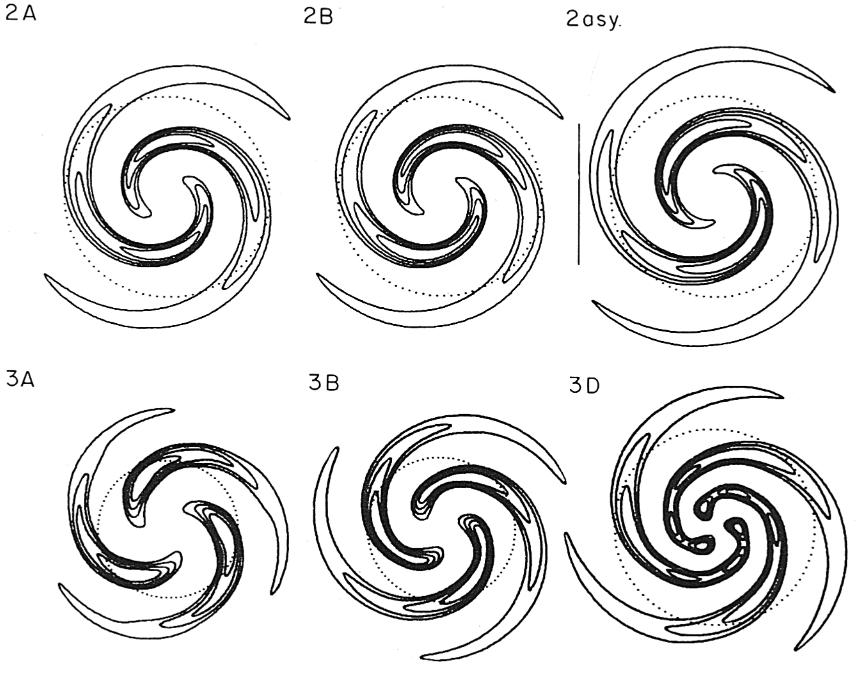

Figure 3. The perturbation density for modes 2A, 2B and 2-asymptotic is shown in the top row. Modes 3A, 3B and 3D are along the bottom.

Alas, most galaxies or their modes are not so simple as either of these two pure cases. But surely an understanding of these building blocks is a prerequisite for the analysis of the muddier situations where both cycles may be operable. Parameter variations, other modes, fluid effects and resonances have not been mentioned. (See Haass, 1982.) A detailed description of the numerical method and some variations of the model is in preparation.

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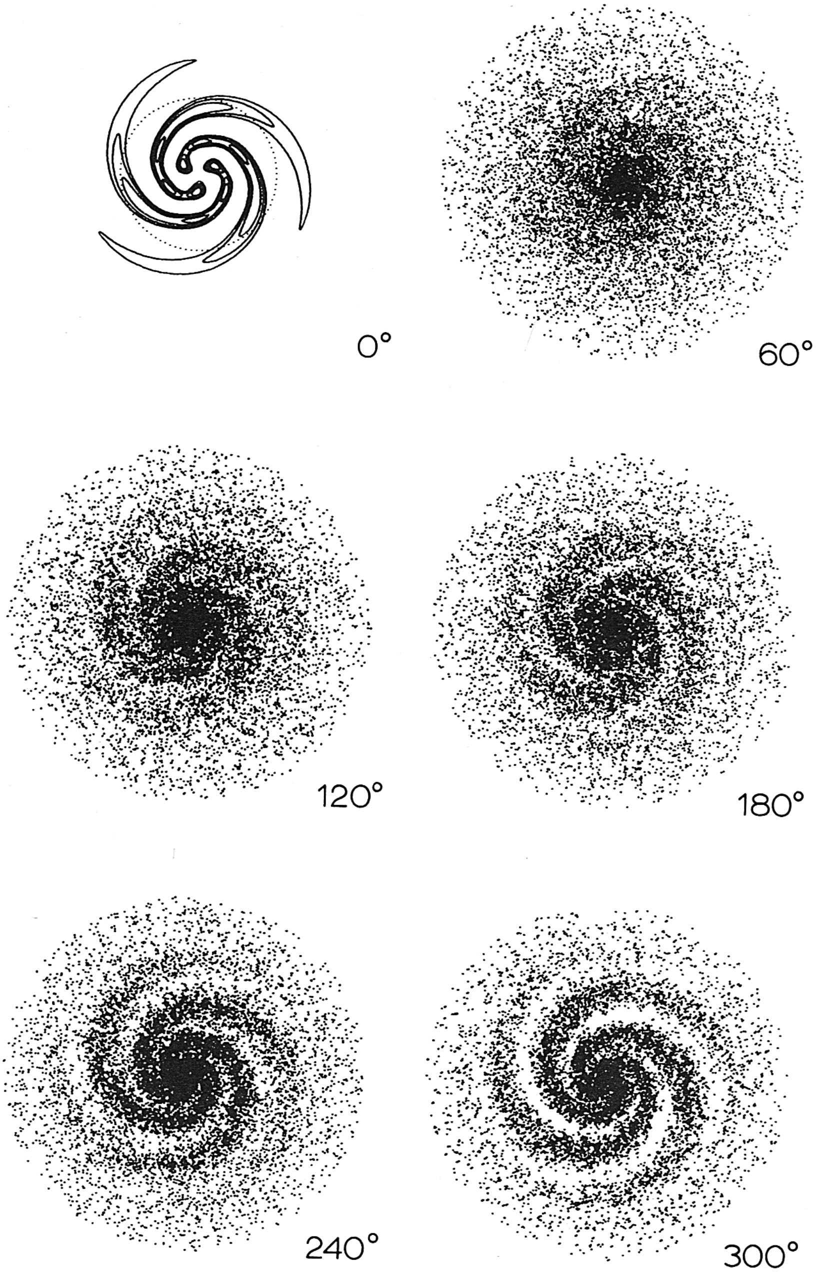

PS from Haass: 10,000-dot visualization of the rapidly-growing mode 3D from Figure 3, shown here at 60° intervals of pattern rotation.

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