

CLASSIFICATION OF MAGNETOATMOSPHERIC MODES IN SUNSPOT UMBRAE

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ABSTRACT. We examine the wave modes in a sunspot umbra. Assuming a stratification, based on a model atmosphere in a sunspot, the normal mode spectrum is determined. The modes are classified using a scheme based on a Helmholtz decomposition of the displacements into l (longitudinal) and t (transverse) components. In certain cases these can be related to the usual fast and slow MHD waves. We compute the theoretical eigenfrequencies and note the existence of umbral oscillations with periods in the range 2-3 min, which are interpreted as slow and mixed modes. The frequencies of the Alfvén waves are also calculated. It is suggested that these modes might also have been observed.

1. Introduction

Oscillations in the umbrae of sunspots have been widely reported (e.g. Beckers and Schulz,1972; Balthasar and Wiehr,1984; Lites and Thomas,1985; Lites,1986; Abdelatif *et al.*,1986;Thomas *et al.* ,1987 and Gurman,1987). The periods of these oscillations typically lie in the range 2-3 min, although larger periods of some 300-400 s (Bhatnagar *et al.*,1972; Soltau *et al.*,1976 and Balthasar *et al.*,1987) or smaller ones around 100 s (Schröter and Soltau,1976) cannot be ruled out.

A number of theoretical investigations have been carried out to examine the nature of wave propagation in sunspot umbrae (e.g. Uchida and Sakurai,1975; Scheuer and Thomas,1981; Thomas and Scheuer,1982 and Leroy and Schwartz,1982). The interpretation of the observed modes is still not established. Scheuer and Thomas (1981) suggested that umbral oscillations are essentially fast waves trapped in a photopheric cavity. On the other hand, Zhugzhda *et al.* (1983) and Gurman and Leibacher (1984) have argued in favour of a slow mode, trapped in a chromospheric cavity above the temperature minimum. The aim of the present analysis is to look carefully into this question. Our plan is first to calculate the normal modes of a model sunspot atmosphere with a uniform vertical magnetic field. A classification of these modes is then attempted, based upon a technique developed earlier by us (Hasan and Sobouti,1987; henceforth HS). We also discuss the connection between the modes, classified according to our scheme, and the conventional MHD modes of an unstratified medium.

2. Equations and Method

2.1 EQUILIBRIUM UMBRAL MODEL

In order to compute the wave modes in an umbra, we require an equilibrium model which mimics a real atmosphere in a sunspot. We selected the core umbral model M of Maltby *et al.* (1985) (kindly provided to us by T. Abdelatif). Beneath the photosphere, the atmosphere is matched to a convection zone model. Figure 1 shows the temperature variation with z (depth) in a sunspot. The convention used is that z increases into the Sun and $z=0$ corresponds to optical depth unity.

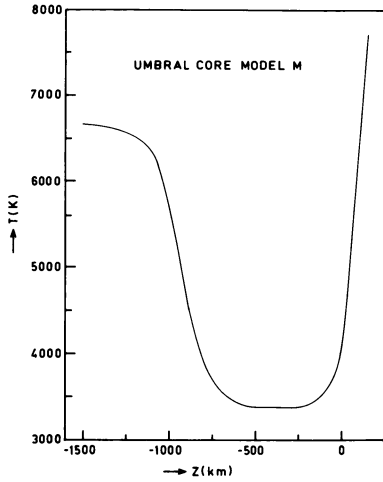


Figure 1. Temperature as a function of z in a sunspot umbra (based on the core umbral model M of Maltby *et al.*).

2.2 LINEARIZED EQUATIONS

We assume that a uniform vertical magnetic field is embedded in the model atmosphere. The normal modes can be determined by solving the wave equation

$$\rho \frac{\partial^2 \xi}{\partial t^2} = -\nabla \delta p + \mathbf{g} \delta \rho + \frac{1}{4\pi} (\nabla \times \delta \mathbf{B}) \times \mathbf{B} \tag{1}$$

where $\delta \rho, \delta p, \delta \mathbf{B}$ denote perturbations in density, pressure and magnetic field respectively and ξ is a small displacement of a fluid element. Equation (1) can be recast into an energy integral, which lends itself to a variational treatment (for details see HS). Assuming a time dependence of the form $e^{i\omega t}$, the normal frequencies can be calculated using a Rayleigh Ritz variational method.

2.3 MODE CLASSIFICATION

In order to classify the modes, we decompose ζ , where ζ is a linear displacement, using a modified form of Helmholtz' theorem (Sobouti 1981), as follows

$$\zeta = \zeta_l + \zeta_t + \zeta_a \tag{2}$$

where the various components can be expressed in terms of scalar functions χ as

$$\zeta_l = -\nabla \chi_l \quad , \quad \zeta_t = \frac{1}{\rho} \nabla \times \nabla \times (\hat{z} \chi_t) \quad \text{and} \quad \zeta_a = \nabla \times \nabla \times \nabla \times (\hat{z} \chi_a)$$

The irrotational vector ζ_l is associated with longitudinal motions, whereas the remaining two are essentially solenoidal transverse displacements. It turns out that ζ_a can always be identified with Alfvén waves. We expand ξ as a linear combination of ζ , so that

$$\xi = \sum_i Z_i \zeta_i$$

where Z_i are proportionality constants and $i = l, t$ or a . For a uniform field, it is found that the Alfvénic motions get decoupled from the rest and can be treated separately.

3. Results

Equation (1) was solved using a Rayleigh Ritz method (see HS for details). The boundary conditions used were $\xi_z = d\xi_x/dz = 0$ at $z = 1000$ km and $\xi_x = d\xi_z/dz = 0$ at $z = -90$ km. A constant vertical field $B = 2000$ G, corresponding to $\beta = 0.84$ at $z = 0$ was used. We first consider the solutions for the l and t modes.

3.1 l AND t MODES

Figure 2 shows the $\omega - k$ (diagnostic) diagram for umbral oscillations, where k is the horizontal wave number. For a fixed value of k , a number of solutions exist satisfying the boundary conditions. These correspond to the normal mode frequencies or harmonics, which form a discrete spectrum. The curves depict the variation of ω with k for fixed order. We also classify the modes by comparing the magnitudes of the l and t components of ξ . Thus, in a l mode $|\xi_l| \gg |\xi_t|$. Open and closed circles correspond to t and l modes respectively, whereas the squares denote mixed modes. In the latter, the first letter denotes the components which has larger magnitude. Physically, the l and t modes can be crudely related to the conventional MHD modes. Above the photosphere, the Alfvén speed is much greater than the sound speed. In this case, the fast mode is essentially a transverse mode (t type) and the slow mode is mainly longitudinal (l type) for parallel propagation to the field. For perpendicular propagation, however, the fast and slow modes are of the l and t types respectively.

3.2 ALFVÉN MODES

The frequencies of the Alfvén modes were also calculated. Since these are independent of the horizontal wave number, we present the results in tabular form, for different mode orders only. Table 1 gives the frequencies and corresponding periods for the lowest five modes.

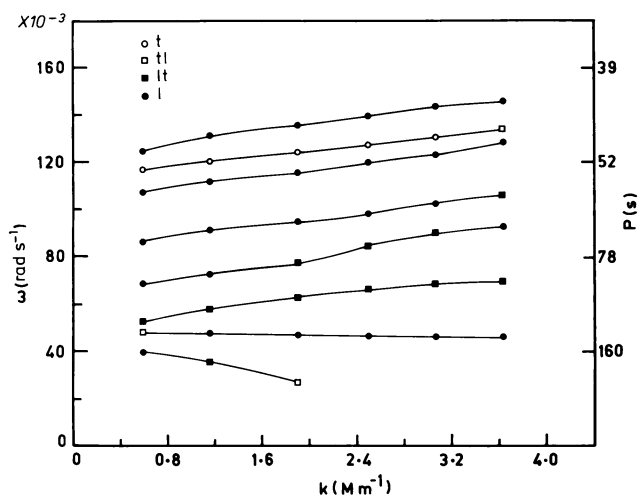


Figure 2. Variation of frequency ω with k in a sunspot umbra, assuming a uniform vertical magnetic field of 2000G. Open and filled circles denote t and l modes respectively, whereas open and filled squares correspond to mixed modes tl and lt respectively.

TABLE 1. Alfvén modes in a sunspot umbra

n	$\omega(s^{-1})$	$P(s)$
1	0.025	250
2	0.098	64
3	0.160	38
4	0.230	27
5	0.310	2

4. Discussion and Conclusions

We now consider the interpretation of oscillations with periods around 180 s. Assuming an umbral radius of some 3000 km, we find $k=1.3 \text{ Mm}^{-1}$ (i.e., for the lowest order mode which has a vanishing radial displacement at the umbral boundary). The corresponding frequencies are 180 s and 130 s for the lowest two modes, and are of type lt and l respectively. For parallel propagation above the photosphere, the 130 s mode resembles a slow wave, whereas the 180 s mode is a mixed mode, which has a dominant slow component. It should, however, be noted that the nature of the modes changes with k (although the period is comparatively insensitive to k). We also find that oscillations in the 2-3 min range are unlikely to be of the Alfvénic kind. It is conceivable that the large period (around 300 s) oscillations observed for example by Balthasar *et al.* (1987), might possibly be Alfvén waves in the photosphere. Owing to the rapid increase of the Alfvén speed with height, these waves are unlikely to reach chromospheric levels, since they suffer strong reflection as they propagate upwards.

In conclusion, we have theoretically calculated the spectrum of umbral oscillations. We suggest that the observed modes in the 2-3 min range are of the slow or mixed type. The higher period oscillations may possibly be of the Alfvén type.

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