RELATIVISTIC REDUCTION OF ASTROMETRIC OBSERVATIONS AT POINTS LEVEL OF ACCURACY

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ABSTRACT. The framework of general relativity theory (GRT) is applied to the problem of reduction of high precision astrometric observations of the order of one microarcsecond. The equations of geometric optics for the non-stationary gravitational field of the Solar system have been deduced. Integration of the equations of geometric optics results in the isotropic geodesic line connecting the source of emission (a star, a quasar) and an observer. This permits to calculate the effects of relativistic aberration of light due to monopole and quadrupole components of the gravitational field of the Sun and the planets taking into account their motions and rotation. Transformations between the reference systems are used to calculate the light aberration occurring when passing from the satellite system to the geocentric system and from the geocentric system to the barycentric system. The barycentric components of the observed position vector reduced to the flat space-time are corrected, if necessary, for parallax and proper motion of a celestial object using the classical techniques of Euclidean geometry.

1. Introduction

At present, the discussion of high precision measurements of light deflection and time delay of radio signals in the Solar system gravitational field has confirmed the effects of the post-Newtonian approximation of GRT within the accuracy of 1.5% and 0.1% respectively (Will, 1986). Meanwhile, the precision of measurement technique applied in astrometric observations is still fast increasing. In this respect, some specific programs are now in elaboration aimed to determine the nonlinear effects of the post-post-Newtonian approximation of GRT. A particular attention is paid to the project POINTS of a space interferometer to be placed on an Earth artificial satellite (Reasenberg and Shapiro, 1986; Reasenberg et al., 1988; Chandler and Reasenberg, 1990). Preliminary estimation enable one to conclude that the instrumental precision of measurement of angular distances between

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celestial objects may be of the order of one microarcsecond. This is comparable with the magnitude of light deflection due to the post-post-Newtonian solar gravitational field as well as to the post-Newtonian corrections for oblateness, motion and rotation of the Sun and the planets. Beside these effects, the reduction of observations of space interferometer should involve the relativistic corrections for light aberration occurring in converting the observed positions of stars to the fixed barycentric reference system of the Solar system. Corrections for parallax and proper motion of observed objects should be also taken into account. Some relativistic effects for light deflection in the gravitational field of one fixed gravitating body have been determined in (Epstein and Shapiro, 1980; Fischbach and Freeman, 1980; Richter and Matzner, 1982, 1983; Sarmiento, 1982; Cowling, 1984; Brumberg, 1987). But these papers do not consider the relativistic effects due to the motions of the gravitating body and an observer with respect to the barycentric reference system and present no algorithm for taking into account parallax and proper motion of observed objects. The aim of the present paper is to develop the consistent relativistic approach for reduction of observations of a space interferometer with due regard to all effects of the order of one microarcsecond.

2. Reference systems

Neglecting the influence of the gravitational field of the Galaxy on the light propagation and motion of the gravitating bodies one may consider the Solar system as isolated. In GRT the characteristics of a reference (coordinate) system introduced in some space-time domain and the potentials of the gravitational field are described by a single object, i.e. the metric tensor $g_{\alpha\beta}$. This tensor is determined by solving the Einstein field equations with appropriately chosen boundary and (or) initial values. It is well known that one may impose on $g_{\alpha\beta}$ four arbitrary complementary conditions. We adopt harmonic conditions $\left(\sqrt{-g}\, g^{\alpha\beta}\right)_{,\beta}=0$. In all formulae used below the greek indices run through values 0,1,2,3; the small latin indices take values 1,2,3; each pair of repeated indices means summation; comma denotes ordinary partial derivative; raising and lowering of the latin indices are performed with the aid of the unit matrix.

The gravitating bodies of the Solar system are considered here as spheroids with constant spin vectors. Numerating the bodies by capital latin letters let us characterize body A by the quantities as follows: mass M_A, mean radius L_A, spin vector $\underline{S}_A$, and the oblateness parameter J_A. Let us introduce also barycentric velocity v_A of the body A, velocity v_T of the body's matter relative to its centre of mass, and distance D_A between body A and the nearest body. The independent small parameters of the problem at hand include J_A, $\varepsilon_A=v_A/c$, $\varepsilon_T=v_T/c$,

$\eta_A=GM_A/c^2L_A$, and $\delta_A=L_A/D_A$. G and c are the gravitational constant and the light velocity respectively. Besides, there exists a small parameter α characterizing the ratio of the mass of the planets and their satellites to the mass of the Sun.

Barycentric reference system (BRS) of the Solar system is used to describe the light propagation from the observed celestial object to an observer and to study the motion of the bodies inside the Solar system. BRS serves as a global reference system. Its metric tensor is resulted from the Einstein field equations by using the post-Minkovskian approximation technique in parameter η_A (Damour, 1983; Blanchet and Damour, 1986,1987). The BRS origin coincides with the Solar system barycentre. Its spatial axes are dynamically non-rotating (Brumberg and Kopejkin, 1989a).

Geocentric reference system (GRS) is used to study the motion of the Earth satellite involved in space interferometry observations and to derive the transformation (reduction) formulae of the position vector components of the observed celestial object from GRS to BRS. GRS is constructed in the space domain restricted by the orbit of the Moon. In solving the Einstein field equations one applies therewith the post-Newtonian approximation technique in parameters ε_A and η_A (Kopejkin, 1988,1989a; Brumberg and Kopejkin, 1989a,b). The GRS origin coincide with the centre of mass of the Earth. Its spatial axes are dynamically non-rotating but they rotate kinematically relative to BRS with the velocity of relativistic precession.

Satellite reference system (SRS) represents a coordinate system with an observer (a space interferometer) at its origin. It is designed for the specific description of observed quantities and for the derivation of the transformation formulae for the position vector components of the observed object from the SRS (instrumental) axes to the GRS axes. SRS is constructed in the space domain restricted by the terrestrial surface. Construction of SRS is performed by solving the Einstein vacuum field equations using the post-Newtonian approximation technique (Brumberg and Kopejkin, 1989b; Kopejkin, 1989a). The world line of the satellite is assumed to be geodesic. Therefore, at the SRS origin the metric tensor reduces to the Minkovsky tensor and its first derivative vanish identically. The SRS spatial axes are dynamically non-rotating but are subjected to kinematical rotation with respect to GRS.

Let us denote the BRS, GRS and SRS coordinates by $x^\alpha=(ct,x^i)$, $w^\alpha=(cu,w^i)$, $\xi^\alpha=(c\tau,\xi^i)$ respectively. At the SRS origin the time τ represents the proper time of an observer and the spatial axes ξ^i realize the instrumental triad of the observer's equipment (an interferometer). Up to constant factors the time scales u and t are equal to TT and TB respectively (Brumberg and Kopejkin,1989c). Transformations between BRS, GRS and SRS are given explicitly in (Kopejkin, 1988,1989a; Brumberg and Kopejkin, 1989a,b).

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3. Geometric optics in the Solar system gravitational field.

In optical region the length of the electromagnetic waves is less than the Solar system space-time curvature by many orders. For this reason (Misner et al., 1973) the light propagation is governed by the geometric optics laws implying the motion of photons in null (isotropic) geodesics of the space-time.

The space-time is split up into three regions with the origin at the Solar system barycentre: 1) external region $R > R_0$, R_0 being the radius of the orbit of Pluto, 2) internal region $R < R_1$ and 3) buffer region $R_1 < R < R_0$. Let the light be emitted at the moment t_1 by a source far outside the Solar system and be received by the observer at the moment t_2. Let x^i_1 and x^i_2 be the coordinates of the source and the observer at the moments t_1 and t_2 respectively. Equation of light propagation is solved separately in the external and internal regions with subsequent matching of both solutions in the buffer region leading to the intermediate solution (Kopejkin, 1990).

The external region is dominated by the monopole component of the total gravitational field of the Solar system. Therefore, the equation of the null geodesics may be presented in the form

$$\ddot{x}^i = -\frac{GM}{R^3}x^i + c^{-2}\frac{GM}{R^3}\left(-\dot{\underline{x}}^2 x^i + 4(\underline{x}\,\dot{\underline{x}})\,\dot{x}^i\right) + O\left(\frac{GM}{R^2}\frac{L^2}{R^2}\right) \quad (1)$$

The remainder terms are due to the quadrupole component of the Solar system total gravitational field. To solve Eq. (6) one substitutes into its right-hand member the unperturbed solution $x^i_N(t) = x^i_1 + ck^i(t-t_1)$ with $|\underline{k}| = (k^i k^i)^{1/2} = 1$. The resulting ordinary differential equation is solved under initial conditions: 1) $x^i(t_1) = x^i_1$ and 2) $\lim_{t\to-\infty} c^{-1}\dot{x}^i(t) = k^i$. These conditions mean physically that the light trajectory passes through the point of emission at moment t_1 and the BRS coordinate velocity of a photon at the infinite isotropic past is equal to the light velocity locally measured in SI units. The specific form of the external solution $x^i_E(t)$ is given, for example, in (Brumberg, 1972). The remainder terms of the obtained solution are proportional to $c^{-2}GM\,L^2/R^2$ with $L = R_0$ and $M = \sum_A M_A$.

The internal region is characterized by the gravitational fields of the individual attracting masses moving much slowly than a photon. Due to this and taking into account the smallness of the time interval of the light propagation through the internal region one may present the equations of null geodesics in the form

$$\ddot{x}^i = F^i_1 + F^i_2 + F^i_3 + F^i_4 + O(\eta_A\varepsilon^2_A) + O(\eta^2_A\,\alpha) + O(\eta^2_A\varepsilon_A) \quad (2)$$

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with

$$F_1^i = \sum_A \frac{GM_A}{R_A^3} \left(-R_A^i + c^{-2}\left(-\dot{x}^2 R_A^i + 4 (\dot{\underline{x}} \, \underline{R}_A) \dot{x}^i \right) + \right.$$
$$\frac{4}{c} (\underline{v}_A \dot{\underline{x}}) R_A^i - \frac{4}{c} (\underline{R}_A \dot{\underline{x}}) v_A^i - \frac{2}{c} (\underline{v}_A \underline{R}_A) \dot{\underline{x}} - \tag{3}$$
$$\left. c^{-4}\, 4 (\dot{\underline{x}} \, \underline{R}_A) (\underline{v}_A \dot{\underline{x}}) \dot{x}^i \right),$$

$$F_2^i = \frac{2}{c^4} \frac{G^2 M_S^2}{R_S^6} (\dot{\underline{x}} \, \underline{R}_S)^2 R_S^i - \frac{2}{c^4} \frac{G^2 M_S^2}{R_S^4} (\dot{\underline{x}} \, \underline{R}_S) \dot{x}^i, \tag{4}$$

$$F_3^i = \frac{4}{c^2} \sum_A \frac{G}{R^3} \left((\underline{S}_A \wedge \dot{\underline{x}})^i - \frac{3}{2} R_A^{-2} (\dot{\underline{x}} \, \underline{R}_A) (\underline{S}_A \wedge \underline{R}_A)^i - \right.$$
$$\left. \frac{3}{2} c^{-2} R_A^{-2} (\underline{R}_A (\underline{S}_A \wedge \dot{\underline{x}})) (\dot{\underline{x}} \wedge (\underline{R}_A \wedge \dot{\underline{x}}))^i \right) \tag{5}$$

$$F_4^i = 6 \sum_A \frac{G}{R_A^5} I_A^{pq} R_A^q (\delta^{ip} - 2 \dot{x}^i \dot{x}^p) -$$
$$15 \sum_A \frac{G}{R_A^7} I_A^{pq} R_A^q R_A^p (R_A^i - 2 (\dot{\underline{x}} \, \underline{R}_A) \dot{x}^i) \tag{6}$$

with $R_A^i = x^i - x_A^i$, $x_A^i(t)$ and $v_A^i = dx_A^i/dt$ are respectively the BRS coordinates and velocity components of the centre of mass of body A. I_A^{ij} is the trace-free quadrupole moment of body A. The terms F_1^i and F_2^i entering into Eq. (7) describe respectively the post-Newtonian and post-post-Newtonian effects of the monopole components of the gravitational fields of the attracting bodies taking into account their motion. In calculating the post-post-Newtonian perturbations it is sufficient to consider only the terms depending on the mass of the Sun M_S. The terms F_3^i and F_4^i are due to the rotation and the quadrupole components respectively of the solar and planetary gravitational fields.

The internal solution $x_1^i(t)$ of Eq. (7) is looked up in the form

$$x_1^i(t) = x_N^i(t) + c^{-2} \left(B^i(t) + C^i(t) + D^i(t) \right) \tag{7}$$

with unperturbed solution $x_N^i(t) = x_2^i + c\sigma^i(t-t_2)$. Functions B^i, C^i, D^i satisfy the equations

$$\ddot{B}^{i} = F^{i}_{1} + F^{i}_{2}\,, \qquad \ddot{C}^{i} = F^{i}_{3}\,, \qquad \ddot{D}^{i} = F^{i}_{4} \tag{8}$$

At the moment t_2 the trajectory of the internal solution should pass through the point of observation implying $x^i_1(t_2) = x^i_2$. Vector σ^i being the arbitrary constant of integration is determined later in matching the internal and external solutions. In solving the first of Eqs. (13) the motion of the attracting bodies is assumed to be uniform and rectilinear, i.e.

$$x^i_A(t) = x^i_A(t_A) + v^i_A(t-t_A) + O(a_A\ (t-t_A)^2) \tag{9}$$

with $a^i_A = dv^i_A/dt$ being the acceleration of the centre of mass of the body A and t_A being some fixed moment of time. In solving remaining two Eqs. (13) the centre of mass of any body A may be regarded as being at rest at moment t_A. The remainder terms in Eqs. (7) and (14) are responsible for the errors of the internal solution. By the suitable fixing t_A regarded as the parameter of the solution one can minimize the magnitude of the errors of the internal solution. It turns out that t_A corresponds to the moments of the closest approach with body A provided the latter is located between the light emitter and the observer. For this case the magnitude of the residuals is proportional to $c^{-4}GM_A a_A R_A\, ln(R_A r/d_A^{-2})$ where r is the distance between the observer and body A and d_A is the impact parameter of the light trajectory with respect to body A. If the observer is located between the light source and body A then t_A is to be coincident with the moment of observation t_2 and the magnitude of the residual errors of the internal solution is proportional therewith to $c^{-4}GM_A a_A R_A\, ln(R_A/r)$.

Solution of Eq. (13) is partly given in (Brumberg, 1987; Klioner, 1989). In the complete form it should be published in our future paper. From the methodological point of view it is of interest to estimate the magnitude of the light deflection due to various factors in passing through the Solar system (see TABLE 1). These estimates demonstrate that within the microarcsecond accuracy one has to take into account the whole Solar system. Indeed, we have to consider under certain conditions the influences of three largest asteroids Ceres, Pallas, Vesta, the Galilean satellites of Jupiter, Titan, Triton and perhaps some other satellites of Saturn and Uranus whose physical properties are not well known yet. The values of δ_1 due to these bodies varies from 0.5 μas (Pallas) to 33 μas (Titan).

Matching of the external and internal solutions is performed in the buffer region at some moment t_* provided that coordinates $x^i_E(t_*)$

and $x^{i}_{i}(t_*)$ coincide and the difference between tangent vectors $\dot{x}^{i}_{E}(t_*)$ and $\dot{x}^{i}_{i}(t_*)$ is minimal. These conditions enable to find the matching radius R_* for any body A. More specifically, R_* is determined by either of two equations

$$M_A a_A \ln(R_* \, r \, d_A^{-2}) = c^2 M L^2 R_*^{-3} \;, \quad M_A a_A \ln(R_*/r) = c^2 M L^2 R_*^{-3} \;.$$

For any attracting body the matching radius exceeds the radius R_0 of external region and may be chosen for all bodies in common enabling the difference between the tangent vectors of the external and internal solutions to be less than one microarcsecond.

The matching procedure implies that $\sigma^{i}=k^{i}+O(c^{-2}GML^{2}R_*^{-3})$. This procedure results in some intermediate solution coinciding with the internal solution for $R < R_*$ and identical to the external solution for $R > R_*$. Formally, one may regard the intermediate solution as coinciding with the internal solution in the whole space-time since outside the Solar system the internal solution differs from the external solution only by the terms $O(c^{-2}GML^{2}R^{-2})$.

TABLE 1. Estimates of relativistic effects due to Solar system bodies.

Body	δ_1	δ_2	δ_3	δ_4	δ_5	ψ_{max}
Sun	$1.75\cdot10^{6}$	2	0.1	0.8	11	180°
Mercury	83	0.06	0.02	-	-	9′
Venus	493	0.002	0.06	-	-	4.5°
Earth	574	0.6	0.06	-	-	180°
Moon	26	0.002	0.003	-	-	8°
Mars	116	0.2	0.01	-	-	25′
Jupiter	16300	240	0.8	0.2	0.001	90°
Saturn	5800	95	0.2	0.04	-	16.5°
Uranus	2100	25	0.05	0.007	-	1.2°
Neptune	2600	10	0.05	0.006	-	51′
Pluto	≃20	-	-	-	-	0.5″

The first five columns of the Table present the maximal values in μas of relativistic effects under study. These values have been estimated as follows: the post-Newtonian light deflection due to monopole field of the body $\delta_1=4GM/c^2L$; the correction due to quadrupole field $\delta_2=\delta_1\cdot J_2$; the influence of the motion of the body $\delta_3=\delta_1\cdot v/c$; the effect of the rotation $\delta_4=4GS/c^3L^2$; the post-post-Newtonian deflection due to monopole field $\delta_5=15\pi/4\cdot G^2M^2/c^4L^2$. The absent values are less than 10^{-3}. The last column contains the maximal angular distance between a source and the body at which the influence of the body on the apparent source position is still to be taken into account.

4. Aberration of light

Let us denote the SRS, GRS and BRS spatial components of the tangent vector to the null geodesic by $s^i = c^{-1} d\xi^i/d\tau$, $q^i = c^{-1} dw^i/du$ and $p^i = c^{-1} dx^i/dt$ respectively. Directly measurable quantities are the components of vector $-s^i$ directed oppositely to vector s^i.

Aberration of the light is caused by the transformations from one reference system to another system at the point of observation. Aberration relations between vectors s^i, q^i and p^i have the form

$$s^i = (K^i_0 + K^i_j q^j)/(K^0_0 + K^0_j q^j), \quad q^i = (\Lambda^i_0 + \Lambda^i_j p^j)/(\Lambda^0_0 + \Lambda^0_j p^j) \qquad (10)$$

$K^\alpha_\beta = \partial\xi^\alpha/\partial w^\beta$ and $\Lambda^\alpha_\beta = \partial w^\alpha/\partial x^\beta$ being the transformation matrices of the coordinate bases at the point of observation. The length of vector s^i is equal to unity since at the SRS origin the metric is flat. Using this condition one may calculate from (15) the lengths of vectors q^i and p^i. Aberration relations between the unit vectors s^i, $m^i = q^i/q$ and $n^i = p^i/p$ derived from (15) have the form

$$\begin{aligned} s^i = {} & m^i + c^{-1}(\underline{m}\wedge(\underline{m}\wedge\underline{v}_T))^i + c^{-2}\Big(\tfrac{1}{2}(\underline{m}\,\underline{v}_T)(\underline{m}\wedge(\underline{m}\wedge\underline{v}_T))^i - \\ & \tfrac{1}{2}(\underline{m}\wedge\underline{v}_T)^2 m^i + R^{ij} m^j\Big) + c^{-3}\Big(\tfrac{1}{2}(\underline{m}\,\underline{v}_T)^2(\underline{m}\wedge(\underline{m}\wedge\underline{v}_T))^i - \\ & \tfrac{1}{2}(\underline{m}\,\underline{v}_T)(\underline{m}\wedge\underline{v}_T)^2 m^i + 2\,U_E(\underline{w}_T)(\underline{m}\wedge(\underline{m}\wedge\underline{v}_T))^i + \\ & R^{ij}(\underline{m}\wedge(\underline{m}\wedge\underline{v}_T))^j + O(c^{-4})\Big) \end{aligned} \qquad (11)$$

$$\begin{aligned} m^i = {} & n^i + c^{-1}(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i + c^{-2}\Big(\tfrac{1}{2}(\underline{n}\,\underline{v}_E)(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i - \\ & \tfrac{1}{2}(\underline{n}\wedge\underline{v}_E)^2 n^i + (\underline{n}\wedge(\underline{R}_E\wedge\underline{a}_E))^i + F^{ij}n^j\Big) + \\ & c^{-3}\Big(\tfrac{1}{2}(\underline{n}\,\underline{v}_E)^2(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i - \tfrac{1}{2}(\underline{n}\,\underline{v}_E)(\underline{n}\wedge\underline{v}_E)^2 n^i + \\ & 2\,U_E(\underline{w}_T)(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i + 2\,\bar{U}(\underline{x}_E)(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i + \\ & (\underline{n}\,\underline{v}_E)(\underline{n}\wedge(\underline{R}_E\wedge\underline{a}_E))^i + (\underline{a}_E\,\underline{R}_E)(\underline{n}\wedge(\underline{n}\wedge\underline{v}_E))^i - \end{aligned} \qquad (12)$$

$$((\underline{R}_E \wedge \underline{a}_E)\ (\underline{n} \wedge \underline{v}_E))\ n^i + F^{ij}\ (\underline{n} \wedge (\underline{n} \wedge \underline{v}_E))^j + O(\ c^{-4})$$

Here U_E is the Earth potential, $\bar{U}$ is the potential of all Solar system bodies excluding the Earth, functions $F^{ij} = -F^{ji}$ characterize kinematical rotation of GRS with respect to BRS and functions $R^{ij} = -R^{ji}$ characterize kinematical rotation of SRS with respect to GRS (Brumberg and Kopejkin, 1989a; Kopejkin, 1989b).

Vector n^i is related with the unit vector k^i characterizing the direction of propagation from the source to the observer by means of expression resulted from the internal solution (12)

$$n^i = k^i + c^{-3}\ (\ (\underline{k} \wedge (\dot{\underline{B}} \wedge \underline{k}))^i + (\underline{k} \wedge (\dot{\underline{C}} \wedge \underline{k}))^i + (\underline{k} \wedge (\dot{\underline{D}} \wedge \underline{k}))^i) - c^{-6}(\ \frac{1}{2}\ (\underline{k} \wedge \dot{\underline{B}})^2\ k^i + (\underline{k}\ \dot{\underline{B}})(\underline{k} \wedge (\dot{\underline{B}} \wedge \underline{k}))^i\ ) + O\ (c^{-4}\ \delta_A) + O\ (c^{-4}\ J_A) + O\ (c^{-4}\ \alpha) \qquad (13)$$

with dot denoting the differentiation with respect to time t.

5. Parallax and proper motion

For the sake of convenience let us re-designate the moment of emission by T, the moment of reception by t, the coordinates of the emitter at the moment T by R^i and the coordinates of the observer at the moment t by x^i. Vector k^i is expressed in terms of the coordinates of the emitter and the observer as follows (Brumberg, 1972,1987)

$$k^i = (\ R^i(T) - x^i(t))/\ |\underline{R}(T) - \underline{x}(t)| + O(c^{-2}) \qquad (14)$$

For a limited time interval the coordinates of the emitter may be presented in the form

$$R^i(T) = R^i_0(T_0) + V^i(T_0)\ \Delta T + \frac{1}{2}\ \dot{V}^i(T_0)\ \Delta T^2 + O(\ \Delta T^3) \qquad (15)$$

with $\Delta T = T - T_0$, T_0 being the initial epoch of emission. $V^i = dR^i/dT$ and $\dot{V}^i = d^2R^i/dT^2$ are BRS velocity and acceleration of the emitter respectively.

The parallax may be taken into account by expanding the right-hand member of Eq. (19) in powers of the parallactic ratio $\pi \simeq |\underline{x}|/R$

$$k^1 = R^{-1} R^1 - R^{-3}(\underline{R} \wedge (\underline{x} \wedge \underline{R}))^1 - \frac{1}{2} R^{-5}(\underline{R} \wedge \underline{x})^2 R^1 - R^{-5}(\underline{R}\ \underline{x})\ (\underline{R} \wedge (\underline{x} \wedge \underline{R}))^1 + O(\pi^3) \qquad (16)$$

The proper motion of the light source is taken into account by expanding the right-hand member of (21) in powers of ΔT and using (20). Finally, one gets

$$k^1 = k_0^1 \left(1 + (\underline{\pi}\ \underline{\mu})\ \Delta T - \frac{1}{2} \underline{\pi}^2 \right) + \mu^1 \Delta T \left(1 + \frac{1}{R} (\underline{k}_0\ \underline{x}) \right) - \qquad (17)$$
$$\pi^1 \left(1 + \frac{1}{R_0}(\underline{k}_0\ \underline{x}) - \frac{1}{R_0}(\underline{k}_0\ \underline{V})\Delta T \right) + \frac{1}{2} \dot{\mu}^1 \Delta T^2 + O(\pi^3) + O(\Delta T^3)$$

with $k_0^1 = R_0^1/R_0$, $\dot{k}_0^1 = R_0^{-1}(V^1 - (\underline{k}_0 \underline{V}) k_0^1)$. $\mu^1 = (\underline{k}_0 \wedge (\dot{\underline{k}}_0 \wedge \underline{k}_0))^1$ is the vector of proper motion and $\pi^1 = R_0^{-1}(\underline{k}_0 \wedge (\underline{x} \wedge \underline{k}_0))^1$ is the vector of parallax.

BRS time interval ΔT is not directly measurable quantity. It should be expressed in terms of the BRS time interval $\Delta t = t - t_0$ at the point of observation (t_0 being the initial epoch of observation) by means of relation

$$\Delta T = \left(1 + c^{-2}(\underline{k}_0 \underline{V}) \right)^{-1} \left(\Delta t + c^{-1}(\underline{k}_0 \underline{x}) \right) + O(c^{-1} R_0\ \pi^2) + O(c^{-1} |\underline{x}|\ \mu\ \Delta t) \qquad (18)$$

The term $c^{-1}(\underline{k}_0 \underline{x})$ in (23) is of the sinusoidal form with the maximal amplitude of the order of 500 seconds and the period of one year. This term should be taken into account for stars with large proper motion. For example, for the Barnard star with $\mu \simeq 10''$ per year such term results to the change of the star coordinate by the order of 200 microarcseconds per year. Within the accuracy of the one microarcsecond this term may be easily detected.

6. Conclusion

This paper presents an algorithm of reduction of astrometric observations to be performed on an Earth satellite with the precision of one microarcsecond. To be short, this algorithm is reduced to Expressions (16)-(18) and (22),(23) for the transformation from the observed vector $-s^1$ to the BRS unit vector k_0^1. Two independent components of the latter yield the position of the source on the

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celestial sphere at epoch t_0. A set of vectors k_0^i referred to one and the same initial epoch t_0 for sufficiently large number of sources determines an inertial reference system on the sky.

It may be noted that the relativistic precessions F^{ij} and R^{ij} need not to be known for reduction of observations performed on a satellite insofar one deals here with relative observations. The relativistic precession is wanted only to provide absoluteness to the inertial reference system constructed with the aid of satellite board observations.

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Discussion

HUGHES: This paper, with its complicated considerations, gives us a perfect example of the necessity of very carefully setting up a considered nomenclature for dealing with space/time. It becomes naive in the extreme to speak simply of some "coordinate system" or such, without a very careful specification of the underlying theory, approximations etc. in the "reference system" or whatever we finally call such a thing.

BASTIAN: Is POINTS able to detect low-frequency gravitational waves, of which the universe may be filled?

KOPEJKIN: A level of 0.1 mas would be expected, according to a study by Braginsky.

BASTIAN: So at least *some* challenge remains.

TURYSHEV: How do you define the region of matching of coordinate systems?

KOPEJKIN: The region of matching is initially bounded by the distance to the nearest attracting body. For example, for the matching of barycentric and geocentric reference frames, the region of matching is bounded initially by the distance from the Earth to the Moon. After the determination of the functions incorporated in the coordinate transformations, the region of matching can be extended to a greater distance, namely to the point in space where the determinant of the coordinate transformations is equal to zero.