On this basis the atomic abundances turn out to be:

$\log T(\mathrm{H}) = 23{\cdot}97$	$T(\mathrm{H}) = 10{,}000$
$\log T(\mathrm{C}) = 21{\cdot}03$	$T(\mathrm{C}) = 11{\cdot}5$
$\log T(\mathrm{N}) = 20{\cdot}99$	$T(\mathrm{N}) = 10{\cdot}5$
$\log T(\mathrm{O}) = 21{\cdot}19$	$T(\mathrm{O}) = 17$

Since these values depend in a complicated fashion on the f- and D-values, it is impossible to estimate the accuracy of the calculated figures. If the values of the dissociation energies given by Herzberg (6) [$D(\mathrm{CN}) = 5{\cdot}95$ e.V.; $D(\mathrm{C_2}) = 3{\cdot}6$ e.V.] were adopted, much higher values of the abundances of C, N and O relative to H would be obtained: this is due to the essential role of $D(\mathrm{CN})$ in the calculations.

Slight changes in the f-values would give the abundance ratio of C and N obtained by Bethe.

Conclusions

1. The determination of molecular excitation temperatures gives consistent results. We have to admit low temperatures for the photospheric region of effective absorption of the faint molecular lines.

2. The estimated atomic abundances are very uncertain, owing mainly to the uncertainties of the f- and D-values. Experimental and theoretical work in this field is highly desirable.

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3. THE CHEMICAL COMPOSITION OF STELLAR ATMOSPHERES

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The quantitative analysis of stellar atmospheres requires—if we may put it somewhat paradoxically—already considerable knowledge of their composition and structure. In practice it is therefore necessary to begin with the simplest problems and even there to proceed in successive approximations.

Let us first consider *normal atmospheres* which are completely determined by stating their chemical composition, effective temperature T_e and surface gravitation g. We divide them best into three groups:

I. Early spectral types from O to A5: completely ionized. Continuous absorption by hydrogen and helium atoms. Scattering by free electrons.

II. Medium spectral types from A5 to K5: hydrogen neutral. $\mathrm{H^-}$ continuum.

III. Late types from K5 on: formation of molecules; molecular spectra.

We shall deal here mainly with early spectral types, the Sun as chief representative of group II being treated by Prof. Minnaert. Then I shall say something about *peculiar spectra* and finally add a few considerations on the *methods* which seem to be important for a successful development of our research.

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I. Normal Stellar Atmospheres

(1) *Interpretation of Fraunhofer Lines.* To begin with, the following picture may be recommended. We imagine the atmosphere having—within a spectral line—an effective optical thickness x_ν which determines the 'depth' in the line $R=\frac{F_0-F}{F_0}$ according to the equation:

$$\frac{1}{R}=\frac{1}{R_c}+\frac{1}{x_\nu}. \tag{1}$$

The limiting depth R_c—corresponding to the black-body radiation for the boundary temperature T_0 of the stellar atmosphere—will be determined empirically. Then equation (1) can be well adapted to any reasonable type of radiative equilibrium. The effective depth of the atmosphere can be considered to be practically constant for $\lambda >$ resp. < 3650 A. The ratio of the depths for the two spectral ranges can be estimated.

By applying the well-known theory of the curve of growth—the damping constant being determined empirically or better avoided by using weak lines—we thus obtain the $\log N_{r,s} H.f$, where $N_{r,s} H$ is the number of r-times ionized atoms in the quantum state s above 1 cm.² of the stellar surface, while f means the oscillator strength of the line.

(2) *Computation and measurement of oscillator strengths f.* The chief problem—more important, for instance, than minute details of radiative equilibrium—is now the determination of f-values.

Relative f-values within multiplets, super-multiplets and transition arrays can be computed for (LS)-coupling according to quantum theory—most easily by means of the tables by Russell, Goldberg *et al.*—or measured in the electric arc. For heavy elements calculation of other modes of coupling is occasionally possible. It might be very desirable to have suitable measuring methods for spectra like O II and III, Ne II ....

Absolute f-values can be calculated for simpler atoms according to quantum mechanics. Recently Biermann in Göttingen has made extensive computations, using Hartree's method.

It has not been sufficiently noticed until now that—especially for long wave-lengths and in spark spectra—there are many important transitions between hydrogen-like orbits, whose f-values can be easily got by means of the well-known hydrogen matrices. E.g. Mg II 3^2D–4^2F $\lambda 4481$ corresponds to the same transition in He II 4686. In other cases the f-sum-rule is useful. Its refined form, discovered by Wigner and Kirkwood, gives for the

$$\begin{array}{llll} \text{Series:} & s\text{–}p & p\text{–}d & d\text{–}f \\ \Sigma f= & 1.00 & 1.11 & 1.40 \end{array} \tag{2}$$

Since the intensity-decrement in series with $\Delta l = +1$ decreases with increasing l, we may roughly state the rule: 'The first line of a series with $\Delta l = +1$ has $f \approx 1$ (limits about 0.5 to 2).'

Then the *measurements*. The range of the classical methods of anomalous dispersion, magneto-rotation, etc., is naturally rather limited. On the other hand, the importance of the absorption measurements by R. B. and A. S. King can hardly be overestimated. For higher excitation potentials the emission-work of the Utrecht school should also be quite useful.

Having—by means of the oscillator strengths f—transformed the $N_{r,s} H.f$, read from the curve of growth into numbers of atoms $N_{r,s} H$ we should take into account excitation and ionization using the Boltzmann and Saha formulae and thus determine the chemical composition of stellar atmospheres.

(3) *Average values of pressure and temperature in early-type atmospheres.* Within our limits of accuracy ($\Delta \log N = \pm 0.3$) it should be allowed first to use suitably chosen average values of electron pressure P_e [Bar] and temperature T.

In early-type atmospheres P_e can be determined using the Stark-effect broadening of hydrogen lines. Either we start with the number of observable Balmer lines or we determine the number $N_{r,s}H$ of absorbing atoms by means of the higher members of the series or of the intensity drop near $\lambda 3650$ A. (in optically thin layer) and then make use of the broadening of H_γ and H_δ (absorbed in optically thick layer). Application of Saha's equation to elements which are present in several stages of ionization finally gives us the average temperature T. For the Oe5 stars λ Ori b and 10 Lac I obtained $T = 31{,}000 \pm 1000°$ K., for τ Sco (B0) $T = 28{,}150 \pm 750°$ K. The electron pressures decrease along the main sequence from $\log P_e = 2{\cdot}9$ for the two O stars somewhat towards the later types. An important by-product of such measurements is the absolute spectroscopic determination of the effective surface gravitation $g_{\text{eff.}}$ by connecting pressure and thickness of layer through the fundamental equation of hydrostatics. For the main sequence stars our $g_{\text{eff.}}$-values agree with the g's computed from the mass-luminosity relation; for the c stars $g_{\text{eff.}} < g$, perhaps the pressure of radiation comes into play there.

(4) *Excitation and ionization in medium spectral types.* For the Sun as a representative of medium spectral types it is—the only difference—no longer possible to get the pressure accurately from observations of the hydrogen lines. Here it appeared to be most suitable to determine simultaneously average values of P_e and T by considering the chemical equilibrium between any energy levels of two consecutive stages of ionization of the same element.

Using the excitation of the lowest energy levels of neutral atoms or molecules, numerous authors have obtained temperatures between 4500° and 5000° K. The difference against our $T = 5675°$ K. is almost certainly real. We shall come back to this point and show why we prefer to use the higher temperature as a general average value.

(5) *Abundance of the elements.* We deal now with our chief item, the abundance distribution of elements. In Table I we summarize some results for the lighter elements. The agreement between the solar values by B. Strömgren and myself is excellent. The agreement with the meteorite analyses by V. Goldschmidt and Noddack is better than in H. N. Russell's classical investigation—if we discard (reasonably) the lighter elements (up to Ne). We are surprised to see the complete agreement between the Sun and τ Sco, although these two stars are quite different cosmogonically. τ Sco is a member of the well-known moving cluster and should be of more recent origin than most of our galactic system. Even for c stars of the types O to B3 the ratio of H : He turns out to be about the same as for corresponding main-sequence stars within an uncertainty factor of about two—a fact of great cosmogonical significance.

II. Peculiar Stellar Spectra

Are there now any stars whose spectrum indicates that their composition differs essentially from the 'normal' ones? Among the light elements up to oxygen such differences might be connected with the energy-generation by nuclear transformations.

Especially hydrogen should in course of time undergo transmutation into helium. For the elements heavier than oxygen, however, we can see at present no possibility of transmutation, except perhaps in super-novae.

(1) *Helium in cA stars.* Mrs Payne-Gaposchkin was probably first to point out the unexpected strength of He lines compared with the Balmer lines in *A super-giants*. Recently more detailed papers on υ Sgr and other stars have been published by Greenstein and Popper. If the analysis follows the usual methods, one obtains a small abundance-ratio of H : He. However, even if a different explanation cannot yet be proved, I feel rather sceptical about the proposed one. The spectrum of eruptive solar prominences exhibits in emission exactly the same features, which the cA stars show in absorption although the composition of the prominences certainly is the same as that of the Sun. Probably we are dealing with a phenomenon of excitation and ionization not yet understood in detail.

(2) *Carbon and nitrogen in Wolf-Rayet stars.* We need not deal here with the well-known subdivision of the Wolf-Rayet stars into WC and WN. Reference to the Bethe-v. Weizsäcker process of energy generation is too obvious here. On the other hand, however,

we should point out also that the interpretation of Wolf-Rayet spectra in terms of transparent expanding envelopes has become very doubtful by recent studies of the eclipsing binary V 444 Cyg and other arguments.

(3) *Carbon and oxygen in late-type spectra*. H. N. Russell has, as is well known, attempted to explain the division of the Harvard sequence in the late types by the assumption that in S stars the abundance ratio $\frac{\text{oxygen}}{\text{carbon}} \sim 30$, in R and N stars only $\sim 1/30$. (For ⊙ and τ Sco we found ~ 5.) However, in such complicated dissociation equilibria it might be well possible that some quantitatively important compound has escaped our attention. It need not show up at all in the accessible part of the spectrum! Furthermore, the curves of growth for molecular spectra and the influence of turbulence upon them urgently require a closer study. Van de Hulst's beautiful work on the telluric O_2-bands may indicate what is possible in this direction.

(4) *'Metallic line' stars*. Then we have the 'metallic line' stars which, according to Morgan, form a branch of their own in the H.R. diagram. From the viewpoint of nuclear physics it appears rather senseless to operate here with differences of abundance. Perhaps a recent paper by Greenstein on τ UMa and other F stars as well as the work of O. Struve may indicate that in the atmospheres of such stars turbulence which varies with height changes the curve of growth, the pressure-stratification and the excitation compared with normal stars. One might imagine that in such a way certain elements are considerably 'enhanced'. The problem may also be connected with that of the 'anomalous curves of growth' (K. O. Wright).

III. Some Questions of Principle

The difficulties which we have met emphasize the need of a critical examination and improvement of the applied methods. We shall note only in loose connection a few points which, in my opinion, have not always received sufficient attention.

(1) *Deviation of the temperature stratification from the limiting case of the 'grey' atmosphere*. In calculating the temperature T as a function of the optical depth $\bar{\tau}$ we must chiefly take into account the dependence of the absorption coefficient on wave-length and depth, since the 'grey' approximation is too inaccurate. A layer which gives strong absorption in a limited range of λ's dams, I should say, the flux of radiation. This results in heating its inner and cooling its outer parts. Evaluating for the Sun the temperature-stratification from the centre-limb contrast one should be aware that for $\cos\theta < 0{\cdot}3$ the observations are of rather doubtful accuracy. But just for $\tau < 0{\cdot}3$ we should expect steepening of the temperature gradient by the absorption within the Fraunhofer lines. Only in that way can we probably reconcile the low excitation temperatures measured spectroscopically with the results of Chalonge and Barbier.

Then it will be necessary to pay more attention to *zones of convection*. I have recently constructed an entropy-temperature-pressure diagram for stellar matter which enables us to overlook the constitution of any convective zone by just drawing one straight line $S = \text{const}$.

Besides closer study of convection and turbulence one should pay attention to possible effects of the inflow of cosmical matter into the stellar atmospheres. A paper by Biermann and ten Bruggencate tries to explain in that way the high temperature of the solar corona; investigations at the Mt Wilson and Yerkes Observatories on variables in dark nebulae point towards similar ideas.

(2) *Theory of Fraunhofer lines*. Recent papers by Houtgast, Spitzer *et al.* indicate—we cannot enter here into theoretical details—that within most weak lines and in the wings of strong lines it is legitimate to proceed according to Schwarzschild's schema of *true absorption* or local thermodynamic equilibrium. The difficulties concerning the centre-limb variation of Fraunhofer lines which first arose in this connection could be removed by paying more attention to the effects of stratification.

For computing line contours we have, beside the classical models of Schwarzschild, Milne, Eddington *et al.*: (*a*) *The method of weight functions*, which recently could still be generalized. (*b*) The 'centre of gravity' method by B. Strömgren, which now can be simplified considerably. (*c*) The '$\tau_\nu = 2/3$' method which, following Eddington and Barbier, was applied by the present author successfully, e.g. to the above-mentioned problem of centre-limb variation. (*d*) On the other hand, one should not forget that for true absorption also the *exact calculation of line contours* by simple numerical quadratures (cf. A. Rosa's paper on the hydrogen lines in the solar spectrum) is rather easy, in any case simpler than many of the usual calculations of models.

Looking finally into the future development of our branch we should already in the near future expect progress on the one hand from quantitative studies of peculiar spectra. Theoretical work will probably beforehand be centred mainly on the development of methods, first for normal and later for peculiar spectra. Besides this it appears possible, using high-precision measurements, to achieve already in some cases (e.g. the Sun) through a second approximation an accuracy of the 'astrochemical' analysis which should not yield to that of geochemistry.

Cosmic Abundance of Elements (log NH + const.)

(The additive constants have been chosen in such a way that the various columns become comparable.)

El.	τ Scorpii Unsöld	Sun Unsöld	Sun Strömgren	Sun Russell	Meteorites Goldschmidt
1 H	10·00	—	10·38	8·8	—
2 He	9·25	—	—	7·3 ?	—
3 Li	—	—	—	0·3 :	1·74
4 Be	—	—	—	0·1	1·04
5 B	—	—	—	3·3 :	1·12
6 C	6·24	6·62	—	5·8	3·26
7 N	6·58	6·94	—	6·3 :	—
8 O	6·99	7·06	—	7·3	6·28
9 F	—	—	—	4·3 :	2·06
10 Ne	7·05	—	—	—	—
11 Na	—	4·61	4·32	5·5	4·39
12 Mg	5·76	5·84	5·96	5·6	5·68
13 Al	4·56	4·66	—	4·7	4·68
14 Si	5·80	5·62	—	5·8	5·74
15 P	—	—	—	2·3 :	3·50
16 S	—	5·25	—	4·0 :	4·80
17 Cl	—	—	—	—	3·4 (?)
18 A	—	—	—	—	—
19 K	—	3·53	3·68	5·1 :	3·58
20 Ca	—	4·56	4·59	5·0	4·50
21 Sc	—	1·66	—	1·9	0·92
22 Ti	—	3·29	—	3·5	3·41
23 V	—	2·38	—	3·3	1·85
24 Cr	—	3·91	—	4·0	3·79
25 Mn	—	3·79	—	4·2	3·56
26 Fe	—	6·05	—	5·5	5·69
27 Co	—	3·36	—	3·9	3·28
28 Ni	—	4·28	—	4·3	4·40
29 Cu	—	2·56	—	3·3	2·40
30 Zn	—	3·11	—	3·2	2·30
31 Ga	—	—	—	0·3 :	0·66
32 Ge	—	—	—	1·3	2·01
33 As	—	—	—	—	Trace
34 Se	—	—	—	—	0·92
35 Br	—	—	—	—	1·37

Cosmic Abundance of Elements (*continued*)

El.	Sun Unsöld	Sun Russell	Meteorites Goldschmidt	El.	Sun Unsöld	Sun Russell	Meteorites Goldschmidt
36 Kr	—	—	—	61 Il	—	—	—
37 Rb	—	0·0:	0·57	62 Sm	—	−0·2	−0·20
38 Sr	1·68	1·6	1·34	63 Eu	—	−0·3:	−0·81
39 Y	1·54	0·9	0·73	64 Gd	—	−0·6:	−0·04
40 Zr	0·70	0·8	1·88	65 Tb	—	—	−0·54
41 Nb	—	−0·7:	—	66 Dy	—	−0·1:	0·05
42 Mo	0·11	−0·3	0·72	67 Ho	—	—	−0·50
43 —	—	—	—	68 Er	—	−1·6:	−0·05
44 Ru	—	0·0	0·30	69 Tu	—	−1·2:	−0·80
45 Rh	—	−1·2	−0·15	70 Yb	—	−0·7:	−0·08
46 Pd	—	−0·6	0·14	71 Cp	—	−0·7:	−0·58
47 Ag	—	−0·7	0·25	72 Hf	—	−1·3	−0·08
48 Cd	—	0·5:	Trace	73 Ta	—	−1·7:	—
49 In	—	−1·7:	−0·90	74 W	—	−1·5	0·90
50 Sn	—	−0·5?	1·20	75 Re	—	—	−3·00
51 Sb	—	0·9:	Trace	76 Os	—	−1·2:	−0·02
52 Te	—	—	?	77 Ir	—	−1·9?	−0·50
53 J	—	—	−0·13	78 Pt	—	−0·1	0·20
54 X	—	—	—	79 Au	—	—	−0·50
55 Cs	—	?	−1·26	80 Hg	—	—	—
56 Ba	1·28	1·6	0·66	81 Tl	—	—	Trace
57 La	—	0·1	0·06	82 Pb	0·9	−0·5	0·70
58 Ce	—	0·7	0·11?	83 Bi	—	—	Trace
59 Pr	—	−1·1:	−0·28	90 Th	—	—	−0·49
60 Nd	—	0·3	0·26	92 U	—	—	−0·90

4. THE CHEMICAL COMPOSITION OF PLANETARY NEBULAE

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Calculations of abundances of the elements in planetary nebulae depend on knowledge of the following factors: the physical processes involved in the production of any given line, the atomic parameters governing the excitation processes, and the distribution of matter in the shell surrounding the nuclear star. The basic observations are spectrophotometric, preferably reduced to an absolute scale of magnitude per square minute of arc in each significant spectral line.

Ionization and electron capture are generally recognized to be the primary processes responsible for the emissions of hydrogen, helium and ionized helium. Absorption and reabsorption of the emitted line radiation, in the far ultra-violet, lead to conversion of high-energy quanta into visible radiation. The atomic parameters are known accurately for hydrogen and ionized helium. They are known less accurately for neutral helium, but the uncertainties are probably not a major source of error.

We must estimate the abundances of N II, O II, O III, Ne III, Ne V, S II, S III, Cl II, Cl IV, A IV, and A V, from the intensities of the forbidden lines associated with the lowest electronic configuration of the atom. The line excitation results from electron impact. The transition probabilities for the emission are small but finite. The emission is possible because of the non-vanishing matrix components associated with a departure of the atomic energy levels from those of LS coupling.

Calculation of the Einstein A's for the forbidden lines is not a difficult matter, except for the p^3 configuration, where second-order effects control the magnitude of the result.

A recent analysis of the O II problem, by Aller, Van Vleck, and Ufford, has removed an old discrepancy in the relative intensities of the nebular pair 3727 : 3729. We can, therefore, apply the theoretical transition probabilities for spontaneous emission with some degree of confidence for all the forbidden lines.

The target areas for collisional excitation are a different matter. No laboratory measures of these atomic parameters exist or are likely to become available. And, until recently, theoretical calculations were available for only one atom, viz. O III, according to the studies by Hebb and Menzel. The earlier calculations of abundances were, therefore, based on the assumption that the target areas for collisional excitation were the same for all of the atoms. The assumption was certainly not correct, but no other one seemed possible at the time.

A recent study of the O II collision areas, by Lawrence H. Aller, has indicated that the foregoing assumption was very poor, for this atom at least. The parameter enters the nebular problem as the ratio of the Einstein A for spontaneous emission to the target area for collisional excitation. The A-values for O II are very much less than those for O III. Consequently, the aforementioned ratio of A to collisional target area differed greatly for O II and O III, according to our original assumption. The revised calculations, however, make this ratio almost the same for O II as for O III. In the absence of further information, therefore, it seems somewhat safer to assume that the ratio is constant for all atoms.

A constant ratio was to be expected for very fast electrons, which behave very much like light waves. But the electrons responsible for the collisional excitation are relatively slow and the result is somewhat surprising. Doubtless, exact calculations will show some interesting discrepancies, but we cannot make much further progress without these badly needed parameters.

Perhaps the most important factor of all is the distribution of matter in the shell of gas surrounding the nuclear star. Previous calculations have generally been based on the assumption that the material fills a spherical shell, whose inner and outer boundaries roughly conform to the interior and exterior limits of the ring, where such a ring is evident.

Even so, we have recognized the danger of this assumption. Aller and Menzel have pointed out the possibility that the seemingly continuous shell may possess a filamentary constitution. The structure of knots and striae in various nebulae has clearly suggested that some of the nebulae, at least, do not have a continuous shell of matter. Many of the records presented by Curtis (*Lick Obs. Publ.* vol. **13**) display a very complicated internal structure.

Recent unpublished photographs, by W. Baade of Mount Wilson Observatory, of the so-called Helical Planetary in Aquarius, N.G.C. 7293, depict this filamentary structure in considerable detail. In portions of the nebula where the shell is less brilliant, Baade has noted the existence of numerous, comet-like objects. Each possesses a condensed nucleus and the 'tail' generally points away from the central star of the planetary.

These filamentary objects doubtless possess some very important cosmogonic import, but our present study merely depends on the existence of the formations. From measures on photographs supplied by Baade, to whom full credit should go for the discovery of the 'comets', I have estimated that the tails may be some 30,000 astronomical units in length and perhaps 3000 astronomical units in diameter. The dimensions are based on Berman's parallaxes. The Helical nebula is probably the largest of its class and the above dimensions are certainly not typical of the average planetary. Very preliminary photographic estimates indicate that perhaps only 1/300th of the volume is filled with the luminous matter. The figure in the denominator may be even larger, since I have not taken into account the fact that many of the cometary tails possess a sort of 'beaded' appearance.

Dangerous as it is to assume that other planetaries have the same ratio of luminous to non-luminous volume, I feel that the correction is much safer to adopt than the earlier values. Until information on individual nebulae or better measures for the Helical nebula become available, we are forced to adopt the value of about 1/300 for the ratio.

The correction for the volume effect has its greatest influence in determinations of electron density, which come out about $\sqrt{300}$ or about seventeen times greater than before. For a typical group of planetaries the densities range from about 3×10^4 to perhaps 10^6 electrons per $cm.^3$

Estimates of the total mass of the gaseous shell come out about seventeen times smaller than those given by our earlier calculations. This reduction in total mass is in the direction of making a nova outburst a less violent catastrophe in the history of a star. Even so, the total masses are still large. For example, the calculated mass of N.G.C. 7009 seems to be of the order of 10^{31} g.

The increased densities, coupled with the correction for target areas, previously mentioned, give revised values of the electron temperatures. The mean value is reduced from about 10,000 to 7700°. I.C. 4997, which was previously very discrepant with an electron temperature of 27,000°, now is in much better agreement with the other nebulae and a slight increase of the average density would serve to bring about exact agreement. Such an assumption would seem to be justified by the apparent compactness of this object.

The values of the hydrogen-helium ratios are essentially independent of the electron density, because the processes tending to produce the lines are analogous for these atoms. The primary uncertainty comes from a necessary correction for the relative volumes of gas tending to emit the various lines. Hydrogen, for example, is largely ionized throughout the nebula. Helium, however, may be doubly ionized only in the innermost parts of the shell, and singly ionized through most of the remaining region. If we assume that the emitting regions are the same for both sets of atoms we shall underestimate the true abundance of helium. That such a correction is necessary becomes especially evident when we come to the low-excitation objects, like N.G.C. 6826, N.G.C. 6543 and especially I.C. 418, all of which seem to show an abnormally low abundance of helium. The safest estimates for helium come from the observations of He II. We allow for the fact that the 4686 images are appreciably smaller than those of hydrogen, by assuming that the volume of emitting gas involved is less for H I, by a factor of about 1/2. The means for seven nebulae are very concordant and give a mean of 0·2 for the helium/hydrogen ratio.

The intensities of the nebular lines of O III and of the line Hβ constitute our best data for determination of the abundance ratio of oxygen to hydrogen. If we employ the O II nebular lines, we obtain a very high abundance of oxygen. However, slitless spectra clearly show that O II tends to predominate only in the external regions of the nebula. In fact the lines at 3727 give by far the largest nebular images. Presumably the excess ionization of O II arises from the marked difference between the absorption coefficients of O I and H I. The curve for the former falls very slowly. Various helium emission lines may thus cause considerable ionization of oxygen and not affect hydrogen at all. If we are to employ O II lines for the abundance estimates, we must apply a very large and uncertain correction because the volumes of emitting hydrogen and O II are far from being similar.

The determined ratio of O/H for eight planetaries is 0·0020. This ratio depends markedly upon the density, because the excitation processes are different. The ratio, thus determined, is probably a minimum value. We shall have to increase it if any appreciable amount of oxygen exists in stages of ionization higher than O III. Such stages give no important lines in the visible region, but the occurrence of Ne V in some of the high-excitation planetaries leads one to conclude that the extra ionization may be appreciable. Unfortunately, there seems to be no completely unambiguous way of allowing for the effect, quantitatively.

For the abundances of all the remaining atoms, except carbon, we rely upon measures or, in some cases, upon visual estimates of the intensities of the forbidden lines. The abundances, relative to oxygen, are therefore essentially fixed. The group as a whole, however, depends upon the oxygen/hydrogen ratio for its validity. The spectrum of carbon is represented in planetaries only by a few weak permitted lines. The process of capture is similar to that for H and He, but the estimate here given is a distinct lower limit, because additional ionization stages may be represented.

Some recent observations, by Olin Wilson of Mount Wilson Observatory, have shown a remarkable phenomenon that leads us to wonder how far we may trust our assumptions concerning the sizes of emitting volumes for different atoms. In slitless spectra, for N.G.C. 2392, the inner rings of Ne III and Ne V display very similar patterns of illumination. One would suppose that the emitting volumes for these atoms were essentially identical. Slit spectrograms clearly depict the splitting phenomenon, discovered by Campbell and Moore, for the lines of Ne III. The splitting is, equally clearly, absent or vanishingly small for the lines of Ne V.

If we interpret the phenomenon in the usual sense, in terms of an expanding shell, we are forced to conclude that the Ne III nebulosity is moving through a stationary Ne V plasma, of nearly identical appearance. No simple interpretation of this phenomenon exists at the moment. Perhaps shock-wave action, of the type discussed by Oort, is responsible. But the mere existence of the peculiar discrepancy introduces uncertainties in our interpretations of the spectra of planetaries, where a knowledge of the volumes occupied by the emitting atoms is so important a parameter.

The calculated abundances, expressed as the logarithm of the numbers of atoms on an arbitrary scale, appears in Table I.

TABLE I

Abundances from Planetaries

	log N		log N
H	10·0	F	4·8
He	9·3	Ne	6·5
C	6·2	S	5·6
N	7·2	Cl	5·3
O	7·3	A	6·2

I wish to thank Drs Olin Wilson, W. Baade, L. Aller, C. H. Ufford and J. H. Van Vleck, for making available to me the observations and data previously reported upon, in advance of publication.

5. THE ABUNDANCES OF THE ELEMENTS IN INTERSTELLAR SPACE

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The early work on gaseous interstellar matter was confined to the lines of Ca II. Even after the discovery of interstellar Na I lines, the possibility of an interstellar gas consisting of a few special elements remained. By 1938, however, there was observational evidence of the presence of gaseous hydrogen, oxygen, calcium, sodium, potassium, and titanium in interstellar space. The relative abundances of these elements seemed to agree, very roughly at least, with the relative abundances found in stellar atmospheres. Since then interstellar absorption lines of the molecules CH I, CH II, and CN have been identified.

Our present knowledge of the density and chemical composition of the interstellar gas is based on observations of interstellar atomic and molecular absorption lines, and interstellar emission lines.

Methods of analysis of observed strengths of interstellar absorption lines were developed by Unsöld, Struve and Elvey (1930), by Eddington (1934), by Wilson and Merrill (doublet ratio method, 1937), and by Dunham (empirical curve of growth method, 1939). Through the application of these methods to an increasing material of observed strengths of interstellar lines it gradually became clear that the interstellar gas (around the galactic plane) was not uniformly distributed in space. The observations by Beals (1936) of double interstellar Ca II lines in several stars pointed in the same direction.

The work of Adams (1943) on the multiplicity of interstellar H and K absorption lines has shown that the interstellar gas is largely concentrated in separate clouds differing in radial velocity, at least as far as calcium is concerned.

The realization of this fact has necessitated a revision of the methods of analysis of the observed strengths of interstellar absorption lines, in a direction indicated by the work of Wilson and Merrill (1937). First, it is necessary to consider the effect of the radial velocity distribution of the absorbing atoms corresponding to the combination of cloud motions, internal motion in the clouds, and thermal motion. Secondly, the concentration of the absorbing atoms in clouds must be taken into account, when densities are calculated. These problems have recently been considered by Spitzer (1948), and by the author (1948).

The analysis of observed line strengths of interstellar absorption lines yields numbers of absorbing atoms (or ions) in the column between star and observer. In order to derive relative chemical abundances it is necessary to consider the state of ionization of the interstellar gas.

The early work on ionization in interstellar space by Eddington (1926), Gerasimovic and Struve (1929) and Rosseland (1936) was followed by investigations by Struve (1939) and Dunham (1939). These authors considered the role of the free electrons produced when interstellar hydrogen was ionized. The importance of hydrogen as a possible source of free electrons in interstellar space was clear after the observations of interstellar hydrogen emission by Struve and Elvey (1938) had shown that hydrogen has a very high relative abundance in interstellar space. Also, Struve and Elvey's observations with the nebular spectrograph of extended areas of hydrogen emission were direct observational evidence of the existence of vast interstellar galactic regions in which hydrogen is ionized (H II regions), while the areas with no observable hydrogen emission indicated regions of un-ionized hydrogen (H I regions). A method for calculating the extent of the H II region produced by a star as a function of the effective temperature and absolute magnitude of the star, was developed by the author (1939) and applied to the case of a uniform interstellar hydrogen gas, later to an interstellar hydrogen gas consisting of clouds (1948).

The distinction between H I and H II regions is essential in the calculation of chemical abundances in interstellar space from densities of absorbing atoms (or ions), since the number of free electrons per unit volume, for a given gas density, may well be expected to vary by a factor of more than 1000 from H I to H II regions, this being the order of magnitude of the ratio of the hydrogen abundance to the total abundance of the elements contributing free electrons in H I regions (probably chiefly carbon, also magnesium, silicon and iron).

The ionization equilibrium in interstellar space depends upon the local kinetic gas temperature. Spitzer has shown (1947) that in H I regions the inelastic collisions between hydrogen atoms and solid particles lower the kinetic gas temperatures very appreciably. Probably the temperatures are between 100° K. and 1000° K. The velocity distribution of the atomic particles in interstellar space can be assumed Maxwellian (Eddington, 1926; van de Hulst, 1946; Aller and Bohm, 1947).

It has been pointed out by Dunham (1939) and by the author (1939) that in calculating the state of ionization in interstellar space it is necessary to take account of the fact that ionizations occur practically only from the ground state, whereas recombinations occur also (and often largely) to excited states. Corresponding correction factors have to be introduced into the previously used ionization equation. Numerical values have been calculated by Bates and Massey (1941), and by the author (1948) for Ca I, Ca II, Na I, and K 5. For these atoms and ions the correction factors range from about 3 to about 1600.

An analysis of observed strengths of interstellar hydrogen emission lines yields densities of interstellar hydrogen. Work of this kind has been carried out by Struve (1939) and by the author (1939, 1948), following methods developed for planetary nebulae by Cillié (1932, 1936), and Menzel and Baker.

The strengths of interstellar molecular absorption lines have been observed in relatively few cases (Dunham, 1941), these lines being generally faint. The analysis of these observations hinges upon the determination of the dissociation and ionization equilibrium of the molecules in question in interstellar space. This problem has been attacked by Swings (1942), and by Kramers and ter Haar (1946). The results are as yet relatively rough. (The determination of the oscillator strength of the molecular lines in question also as yet presents certain difficulties.) The results derived from the molecular lines are, therefore, at present probably somewhat less accurate than those found from atomic lines.

From the observational material on interstellar absorption and emission the following picture of the interstellar gas, necessarily uncertain in many respects, is derived.

The interstellar gas is largely concentrated in clouds. In most of these clouds hydrogen is un-ionized, i.e. they are H I regions. The density of the average cloud in the neighbourhood of the Sun is about 10 hydrogen atoms cm.$^{-3}$. In between the clouds the density is probably less than 0·1–0·2 hydrogen atoms cm.$^{-3}$. There is considerable scatter in the density of the clouds, with densities as high as 50 hydrogen atoms cm.$^{-3}$ (absorbing cloud in front of χ^2 Orionis), or perhaps even more.

In the case of the dense cloud in front of χ^2 Orionis conditions are relatively favourable to the determination of relative abundances of the elements (cf. Dunham, 1939). Owing to the difficulties mentioned above regarding the molecular lines, however, the results are as yet rather uncertain. The relative abundances derived, taking account of the various circumstances mentioned above, are given below (the abundance of H being put equal to 10^6).

H	Na	Ca	K	Ti
10^6	6	10	0·4	0·2

On the whole, these relative abundances are remarkably close to those found in the stars.

With regard to the derivation of the relative abundances, the following points should be noted. In order to find the required result, it is necessary to know the number of free electrons per unit volume. This can be found from a comparison of the strengths of the Ca I line 4227 and the H and K lines of Ca II (Struve, 1939), and also by comparison of the lines of CH I and CH II (Swings, 1942; Kramers and ter Haar, 1946). However, in the case of χ^2 Orionis the Ca method is not very accurate owing to the approximate saturation of the H and K lines. Also the quantum-mechanical data are as yet very inaccurate in the case of CH II. The available data indicate an electron density around 0·2 in the case of χ^2 Orionis, as a compromise between the upper limit found by the Ca method, and the (considerably higher) rough value found from the preliminary data for CH I and CH II.

A value of the electron density which is probably not far from the truth may be derived from the observed strength of the Ca I line on the assumption that the ratio of the elements furnishing the free electrons (chiefly carbon) to calcium is equal to that found in stellar atmospheres. The important point in this connection is that the derived value of the electron density is proportional to the cube root of the hypothetical ratio. It is further assumed that we have to do with an H I region of kinetic temperature 300° K. The electron density thus found is 0·05 cm.$^{-3}$.

It should be noted that the relative abundances of H, Na, and K are practically independent of the electron density. The same is true of the geometrical mean of the abundances for Ca and Ti when compared with H, Na, and K.

For the average clouds the observational material available is much less complete. The abundance ratio of Ca and Na, however, can be derived. There is some indication that it is considerably smaller than in the dense cloud, but the calculation is as yet affected by many uncertainties, among which may be mentioned the uncertainty of the strength of the ionizing radiation in interstellar space in the wave-length region 1000–2000 A.

The H II regions have a total volume of perhaps about one-tenth of that of the H I regions in the galactic stratum in question. The average emission nebulae observed in H II regions seem to be comparable in density to the denser clouds (about 50 hydrogen ions cm.$^{-3}$), although exceptional regions such as the centre of the Orion nebula have densities as high as 10^3 or 10^4 hydrogen ions cm.$^{-3}$ (Greenstein, 1946). It is yet an open question whether hydrogen in the H II regions is largely concentrated in clouds. This problem can possibly be solved through detailed observations of the structure of the hydrogen emission of these regions, especially if these are combined with observations of interstellar absorption lines on the spectra of stars seen through such regions.

A question that has not as yet been quite satisfactorily answered is whether or not hydrogen—in the normal H I regions—shows the same degree of concentration into clouds as, say, calcium. Increased accuracy in the analysis of the CH I and CH II lines will be important to the solution of this problem.

The investigations of Oort and van de Hulst (1946) on the building up and destruction of solid particles in interstellar space have lead to a picture according to which the interstellar gas and the interstellar solid particles are closely related and should show similar spatial distribution.

The measures of colour excesses by Stebbins, Huffer and Whitford (1940) have demonstrated that the solid particles are unevenly distributed in space and the work of Ambartsumian and Gordeladse (1938, 1940, 1944) indicates that the solid particles are probably concentrated in clouds similar in extent and number to the gas clouds (diameter about 10 parsecs, about 10^{-4} clouds per cubic parsec). It is indeed plausible to assume that the spatial distributions of solid particles and gas are generally similar (Oort and van de Hulst, 1946), although marked exceptions certainly exist (Greenstein and Struve, 1939; Morgan, 1939). (The exceptions may possibly be explained as a consequence of exhaustion of the gas through building up of solid particles.)

Spitzer (1948) has investigated the observed strengths and doublet ratios of the sodium lines for unreddened and reddened stars, respectively. He has found that the gas clouds producing the interstellar sodium lines in unreddened stars are considerably less dense than those giving rise to the lines in reddened stars. Thus the proportion of solid particles should be considerably higher in the denser gas clouds.

Problems regarding the solid particles are closely connected with the problem of the chemical constitution of interstellar matter. For although the amount of interstellar gaseous hydrogen is undoubtedly far greater than the amount of hydrogen in the solid particles, the amounts may well be comparable for other elements.

Further progress in the solution of problems regarding the abundances of the elements in interstellar space may be expected as a consequence of advances along several lines. The following points may be mentioned:

(1) Determination of the strengths of interstellar absorption lines from high-dispersion spectra for a large number of stars (cf. Adams, H. N. Russell Lecture, 1947).

(2) Extension of the high-dispersion observations to lines in the ultra-violet, especially the Na I and Ti II lines, for a not too limited number of stars. Observations of the Na I doublet of 3303 A. are especially valuable for strong Na I line absorption, when the D lines are saturated. Observations of the Ti II are a reliable indication of the total density in the column in question, largely independent of ionization condition.

(3) An extended survey of the interstellar hydrogen emission to determine the extent and properties of H II regions.

(4) Observations and analysis of emission lines of other elements in H II regions.

(5) Revision of the calculations of the ionizing radiation in interstellar space (intensity as a function of wave-length) with special regard to the far ultra-violet radiation and spatial distribution of the B stars.

(6) Improved calculations of the state of ionization and in particular the state of dissociation and ionization of molecules.

(7) Further investigations on the relations between interstellar gas and solid particles.

6. STUDIES OF STELLAR SPECTRA RELATED TO THE ABUNDANCE PROBLEM

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In accordance with the Russell-Vogt theorem, stars which have the same equation of state and the same chemical composition obey the relations

$$L = L\,(M) \quad \text{and} \quad R = R\,(M);$$

that is, the luminosity and the radius are uniquely determined by the mass. If, for a star of a given mass, we find a luminosity differing from the empirical 'mass-luminosity curve', we should then conclude that the chemical composition is abnormal: the brighter the star the smaller must be its hydrogen content and the larger must be its radius.

If we make use of the relation
$$L = 4\pi R^2 \sigma T^4$$
in order to eliminate R, we obtain an alternative expression from the Russell-Vogt theorem:
$$L = L\,(T),$$
and since $T = f\,(Sp)$ we can use this relation to form for each set of abundance factors a theoretical HR diagram
$$L = L\,(Sp).$$
A family of curves with different values of X, the hydrogen abundance, and $(1-X)$, the Russell mixture, was published by B. Strömgren 15 years ago.

If a star falls within the region of the Strömgren diagram occupied by the curves, its H content may be immediately read off. If it falls completely outside the occupied area, its equation of state is not the usual one. This applied, of course, to the white dwarfs.

The empirical HR diagram contains stars in nearly all parts of Strömgren's diagram. It is therefore customary to say that we recognize the existence of stars with hydrogen contents ranging between about $X = 0{\cdot}1$ and $X = 0{\cdot}7$. This follows from Strömgren's own computations, as well as from Kuiper's study of Trumpler's HR diagrams of clusters. It may be noted that the dispersion in the empirical mass-luminosity relation is surprisingly small—though of course physically real. Russell has remarked that the great majority of stars with well-determined values of M and L agree quite closely with the relation derived in his and Miss Moore's work, so that the conclusion suggests itself that nearly all those *binaries* which were used in the derivation of the empirical relation $L = L\,(M)$ happen to have almost identically the same composition.

It is true that Kuiper has found appreciable departures from the $L = L\,(M)$ relation in the case of several visual double stars in the Hyades which lie above the empirical curve, and he, as well as Chandrasekhar, has stressed the fact that ζ Herculis with $M = 0{\cdot}96\,M_\odot$, $R = 2\,R_\odot$ and $L = 4\,L_\odot$ also falls above the curve and requires $X = 0{\cdot}11$ (or $\mu = 1{\cdot}45$), while the Sun requires $X = 0{\cdot}35$ (or $\mu = 1{\cdot}00$). A recent determination of the parallax by G. Hall places ζ Her closer to the main sequence, but still appreciably above it.

Although the recent recognition of the high abundance of He will alter the numerical results, the picture, as a whole, remains essentially the same. We are therefore led to believe that some form of observational selection has resulted in our picking out a sequence of almost uniform X in the construction of the $L = L\,(M)$ diagram, while no such selection was operating in the construction of the $L = L\,(Sp)$ diagram. The fact that we know very little about the masses of the super-giants and of special groups of stars such as those with weak H, or strong Si II, Eu II, etc. may indeed account for this difference.

An obvious approach to the problem of chemical composition is thus offered through the study of (1) departures from the $L = L\,(M)$ relation, and (2) differences in the HR diagrams of closely related groups of stars, such as clusters. Both methods have been employed with notable success by Kuiper, Strömgren, and others. My approach is somewhat different. I have made two investigations:

(1) I have examined several groups of binaries in order to detect departures from the $L = L\,(M)$ relation and have then attempted to answer the following questions: (*a*) Are

there any spectrographic features which support the contention that the H-content is abnormal? (*b*) Are there any other factors involved which may explain the departures without resorting to differences in X?

(2) I have examined the spectra of members of two clusters, the Hyades and the Pleiades, in order to detect any spectroscopic features that might add or detract from the presently accepted view that the former is relatively poor in H, while the latter is richer, though not nearly as rich as the clusters near S and 12 Mon or as the ordinary galactic stars of types B and A of the main sequence.

The results of the first investigation are based mostly upon a large number of observations of spectroscopic binaries at the McDonald Observatory, although I have consulted the results of many other workers. There are, in addition to those departures which had been discussed by previous workers (Russell and Moore, Kuiper, etc.), four groups of binaries which show large departures from the mass-luminosity relation:

I. The mass-ratios of the W Ursae Majoris stars are, on the average, close to $\alpha = m_1/m_2 = 2{\cdot}0$. For some, such as V 502 Oph (Gratton), $\alpha = 2{\cdot}5$. Since in the region of the G-dwarfs $L \sim M^4$, we have $L_1/L_2 = (M_1/M_2)^4 = (2{\cdot}5)^4 = 40$ and $\Delta m = 2{\cdot}5 \log L_1/L_2 = 4$ mag. But both components are easily visible so that in reality

$$\Delta m \sim 0{\cdot}5 \text{ mag.}$$

There is a departure $\delta = 3{\cdot}5$ mag. in at least one of the components, but it is possible that both stars depart. The fainter component is in reality much brighter than can be explained with the $L = L\ (M)$ relation.

It is extremely improbable that the components have widely different X. Perhaps the velocity curves are distorted because of (*a*) the reflection effect, and (*b*) the gravity effect, but it is quite doubtful that the discrepancy can be removed in this way. I have recently proposed (see *Annales d'Astroph.*) that the stars may have a common envelope which tends to equalize the brightnesses of components of very different masses. There is no obvious relation between δ and other parameters of the stars or their orbits.

II. In the normal Algols we often find systems with very small values of K_1 and therefore with small mass-functions $f\,(M)$. In most of these systems K_2 cannot be observed, except as in U Sge during the total eclipse. Therefore α is unknown. But if we recognize that the spectra of the bright components have normal spectra of main-sequence type A or late B, we can assume that their masses are also 'normal' and then determine α from

$$M_1 = \alpha M_2 = \alpha\ (1+\alpha)^2 f\,(M).$$

In this manner we find for XZ Sgr approximately $M_1 = 3M\odot$; $\alpha = 17$; $M_2 = 0{\cdot}16M\odot$. Again, the luminosity of M_2 from the $L = L\ (M)$ relation would be some 10 mags. fainter than it is according to the light curve or according to the fact that we can photograph its spectrum during total eclipse with reasonable exposure times. Other systems of this kind are R CMa, DN Ori, BD Vir, and others.

But the departures δ range among the Algols all the way from 0 to about 10 mags. and I have found no correlation with other parameters. The spectra of at least some fainter components with large values of δ are quite normal sub-giant spectra of types G or K. The H lines are not unusually faint; on the contrary, they are strong in XZ Sgr B!

We cannot be as certain in the case of the Algols, as we were in the case of the W UMa systems, that differences in chemical composition are not involved. But the large range of the values of δ certainly is not favourable to the hypothesis. Perhaps we should first examine the possibility that evolutionary conditions have prevented the fainter components from reaching stability in the sense required by the theory of the interior of a single star. The primaries are, as far as we know, in agreement with the empirical $L = L\ (M)$ relation. The observational basis of the δ's rests upon the smallness of K_1—a quantity which, in these systems, cannot be vitiated by blending, and which is easily and reliably measured, but which conceivably could be distorted by absorption in gaseous streams.

III. Of special interest are several very massive binary systems. The best-determined data give for the empirical $L = L\ (M)$ relation about $M = 40M\odot$ with $L_{\text{bol}} = -10$ or

$L_{vis} = -7$. Plaskett's massive star HD 47129 gave larger masses, of the order of at least $85M\odot$ if the absence of eclipses is taken into account. But I have found that the fainter components in the spectrum show lines which vary erratically in radial velocity and in intensity, so that Plaskett's value of α is not reliable. It remains unknown, so that the individual masses must be larger than about $50M\odot$ each—by how much we do not know. The absolute magnitude is not known. It is probably very great—judging from the appearance of the spectrum.

More perplexing is Pearce's star HD 698 for which $\alpha = 2{\cdot}5$. With $f(M) = 3{\cdot}6M\odot$ this gives $M_1 = 113M\odot$ and $M_2 = 45M\odot$. These values are remarkable, since they exceed all others that are at present known, except perhaps those of the Trumpler stars.

But HD 698 is not a real *c*-star and its absolute visual magnitude cannot be much greater than -3. The H and He I lines (2^3P–n^3D and 2^1P–n^1D) have characteristic wings produced by Stark broadening. The H lines are relatively weak and the spectrum is of type B9s with Hα in emission. The lines of He I and Si II, Fe II, etc., are relatively strong. The star may lie above the main sequence, but it cannot have an absolute magnitude of the order of -8 or -9 on the visual scale such as would correspond to its mass (at B9 the bolometric correction is about 1 mag.).

Perhaps we have here a star of the kind of Maya in the Pleiades which (together with the other bright B-type members of that cluster) is about 2 or 3 mags. more luminous than the main sequence. If this were so, then the large mass determined by Pearce deserves special consideration. Unless for such large masses the $L = L(M)$ relation is altogether different from the one we usually accept, the departure δ would be negative, by something like -6 mags. But the empirical main sequence corresponds to about $X = 0{\cdot}34$. It seems quite doubtful that even $X = 1{\cdot}00$ would bring about so large a departure *below* the $L = L(M)$ relation. We would have to conclude that the equation of state is different—and for that there is at present no very good excuse.

It is, in my opinion, not entirely impossible that Pearce's value of α may be too large. It rests upon the measurements of K_2 and this is intrinsically difficult. My experience in the case of Plaskett's star leads me to be cautious about all values of K_2. Moreover, the second component is very faint. Petrie finds $l = L_2/L_1 = 0{\cdot}06 \pm 0{\cdot}02$ so that $\Delta m = 3$ mags. Observations made recently at McDonald (with M. Rudkjøbing) do not show the fainter component. They suggest that blending with faint lines of Fe II belonging to the primary may have affected Pearce's measures, but they do not disprove the existence of this component in the spectrum. They do justify caution with regard to α. Clearly we have here a most tantalizing case: $f(M)$ is so large for a B9 star, $\sim 3{\cdot}6M\odot$, that even with $\alpha = 1$ the mass of each component would be at least $15M\odot$, and since there are no eclipses the real masses may be considerably larger. But it is still possible that we may find that there is no great departure from the empirical $L = L(M)$ relation.

IV. The Russian astronomers Zverev, Kukarkin, and more recently Krat, have pointed out that UX UMa, with its period of only $4^h\,43^m$ has an absolute magnitude of the order of $L_v = +6{\cdot}5$. This result is very uncertain, being based upon proper motion data, but it does show that the system is intrinsically faint. In order to determine the mass-function I obtained two series of spectrograms covering the entire period. The velocity curve gives $K_1 = 250$ km./sec. and $f(M) = 0{\cdot}3M\odot$; $a_1 \sin i = 7 \times 10^2$ km. $= 1R\odot$. α is unknown, but assuming it to be $\alpha = 1$ we have minimum masses $M_1 = M_2 = 1M\odot$. The spectrum shows very remarkable changes. At maximum radial velocity there are strong absorption lines of H and He I so that Kuiper's classification as B3 is confirmed. But I count the Balmer lines to H15 and this is about normal for the class, so that at this stage there is no indication of sub-dwarf characteristics. About 15 minutes after principal mid-eclipse the absorption lines fade out almost completely. The spectrum is then practically continuous with a broad and faint Hβ emission line and it remains such until about phase $0{\cdot}75P$. The width of this line corresponds to Doppler motions of the order of ± 500 km./sec. The continuous spectrum is very blue, so that we should adopt an effective temperature corresponding to about B3, at all stages of the binary. I have not observed the spectrum of the fainter component.

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This star is, of course, quite abnormal. It departs from the HR diagram in the direction of the white dwarfs and its equation of state must differ materially from that of the normal stars. The departure from the $L=L$ (M) relation is not striking—if we can rely upon the absolute magnitude, and our estimate $\alpha=1$. For $M_1=1M\odot$ we have $L_{bol}=+4\cdot5$. But the spectrum is B3, so that the bolometric correction is $-2\cdot0$ mag. and $L_{vis}=+6\cdot5$, which agrees with the Zverev-Kukarkin result from the proper motions. But if $\alpha>1$ we should have found a larger mass and thus a negative value for δ. In all probability causes other than chemical composition produce this result.

Before abandoning the subject of the δ's I want to call attention to the work of R. M. Petrie on several spectroscopic binaries with composite spectra. Of particular interest is HD 43246 with $P=23\cdot2$ days, $K_1=67$ km./sec., $K_2=31$ km./sec.; $\alpha=0\cdot45$. The brighter component is F5 with a spectroscopic absolute magnitude of $+2\cdot1$, the fainter—visible only through the 'window' of the strong K-line absorption of the F5 component—is B8 with an absolute magnitude of $+3\cdot6$ as determined from the spectroscopic difference Δm. If these data are reliable, they would imply a large negative value of δ because Petrie estimates $M_1=2M\odot$, $M_2=4M\odot$. Systems with underluminous early-type components are, of course, known even among the visual double stars: α Scorpii is an example. But W. P. Bidelman is not quite certain that the F5 components in these composite spectra are really dwarfs. If they were giants the masses and luminosities of the early-type components would be normal, but the mass-ratio α would give too small a mass for the F5 star so that the latter would not fit the $L=L$ (M) relation. In that case the departure δ would be similar to the one which we found in the Algols.

To summarize the results of this phase of the work, there appear to be many departures δ from the $L=L$ (M) relation which greatly exceed the errors of the observation. Most of these are probably not related to the abundance problem, but a few must be investigated further before a conclusion can be reached. The significance of these results lies in the fact that the mass-luminosity relation was, in the first place, derived from double-star observations. It now appears probable that many components of such systems depart greatly from the empirical curve. Hence, we should exercise caution in attributing differences of H content to stars—single or double—which fall above or below the mean curve.

The second procedure was to compare the spectra of the Pleiades with those of the Hyades. I used two sets of plates which I had obtained at McDonald for another purpose. The dispersion of the spectrograms was 40 A./mm. at $\lambda3934$. I had the advantage of being able to use the spectral types as determined by Morgan and Titus for the Hyades (*Ap. J.* **92**, 256, 1940) and by Morgan for the Pleiades (unpubl. material) based partly upon my McDonald plates and partly upon his own observations at Yerkes. The spectral types of the Pleiades range from B6 to K2, but only one member of the cluster was observed at this late spectral type. The great mass of the material ends at spectral type G0. The Hyades begin at class A1, but extend as far as F9 along the main sequence. In addition, the Hyades have the four well-known red giants whose spectral types are between G7 and K0. There are no late-type giants in the Pleiades and of course there are no B-type stars among the Hyades.

If we compare critically the main sequences of the two clusters, we find that at F9 and F8 the spectra are almost identically the same, so that even a fairly detailed study fails to reveal systematic differences between the two groups. At the other end of the main sequence where the two clusters overlap, namely at A1, the difference is most conspicuous and definitely systematic in character. The Hyades are much richer in the metallic lines, including not only Fe I and Fe II but also Ca I, Sr II, and probably Mg II and Si II. At the same time the lines of H and probably those of Ca II are much weaker in the Hyades than in the Pleiades. This effect is somewhat obscured in the case of the Pleiades because of the large rotational velocities which are frequently encountered in that cluster, but even allowing for the rotational broadening it is a most conspicuous phenomenon and one that cannot be attributed to observational effects. It is closely connected with the well-known tendency of the Hyades to possess members which Morgan described as having metallic-line spectra. It is of course also known that these metallic-line spectra are not present in the Pleiades.

I have the impression that although not all the early and middle A's among the Hyades have been recognized as metallic-line stars, there seems to be a tendency of most members in this direction. It is true that there exists among the Hyades a marked range in their characteristics between a behaviour which differs only slightly from that of the Pleiades and that of the typical metallic-line objects. The latter have all been classified by Morgan as A stars and not as F stars. If, following Greenstein, we had described these stars as members of class F we should have found them to differ materially from the corresponding F stars of the Pleiades as well as from those normal F stars which Morgan has found in the Hyades. It is not necessary for us to decide at this stage which classification is the more correct one. The important thing is to recognize that there exists a striking systematic difference between the spectra of the Hyades and those of the Pleiades and that this difference is pronounced among the earlier types and disappears in the later ones.

It is of interest in this connection that most recent observers find that the main sequence of the Hyades falls below that of the Pleiades and even below that of the galactic stars in general. This tendency is most conspicuously shown in the work of Ramberg, who has found that the departure is greatest among the A's, F's, and G's, but disappears not far from K o. This latter point suggests that a serious error in π is not involved. Although the systematic difference in the absorption line spectra to which I have referred is hardly noticeable at F8 and even F5, it is tempting to relate the two phenomena.

The difficulty of classifying the metallic-line stars illustrates the problem we are facing when we encounter conflicting spectroscopic criteria as we always do when we examine stars of different hydrogen content. It is at present difficult and perhaps impossible to decide whether the tendency to develop the characteristic features of the metallic-line spectra may in some way be related to the low hydrogen abundance which has been suggested for this cluster. The criteria, however, are not unreasonable if considered from this point of view. Rather than attempt to predict theoretically what a star's spectrum would look like if the hydrogen abundance were small, we might consider the observed data in the light of information gathered from such a star as Upsilon Sagittarii. Greenstein's work has shown that if we analyse the spectrum by the conventional method we find a small hydrogen abundance and a relatively high abundance of helium and of the Russell mixture. Qualitatively this is the same sort of thing which we have observed in the metallic-line stars. They imitate stars of higher luminosity because they have relatively weak hydrogen lines, strong lines of other elements, spectroscopic characteristics of higher than average luminosity, and appreciable turbulence. It is not surprising that Greenstein's spectrophotometric study of the metallic-line star τ Ursae Majoris brings out precisely these characteristics. A difficulty is encountered when we consider the departures of these stars from the HR diagram. Are they members of the main sequence as previous workers have assumed, or are they several magnitudes brighter? Strömgren's original study of hydrogen abundances led us to expect that the main sequence of the hydrogen-poor clusters would lie higher than the main sequence of a hydrogen-rich cluster, but Kuiper's investigation in 1937 and Ramberg's, more recently, have placed the hot end of the main sequence of the Hyades lower than that of other stars. It seems to me that there is involved a serious difficulty because of the lack of knowledge of the precise relation between spectrum and temperature. Strömgren's curves related L with T. It is obviously not safe to apply to a hydrogen-poor cluster the same relation between T and Sp that was found correct for H-rich stars. This difficulty is illustrated by the uncertainty of placing the metallic-line stars in the HR diagram. If they are A's, then they cause the main sequence of the Hyades to curve downward at the earliest types, as it does in the work of Titus and Morgan. But if we should move these same stars over to the right until they coincide with the middle F's, their absolute magnitudes would be normal or perhaps even suggest a slight upward curvature of the main sequence.

Another difficulty in the interpretation in terms of hydrogen content arises from the great similarity of the spectra of the later subdivisions of class F, the G's and the K's.

If we had observed these same spectra without knowing that they came from different clusters we should certainly have assumed that they belong to the same spectral type and luminosity. We should have then obtained identical compositions from the equivalent widths of the spectral lines. The similarity of these spectra is really quite striking, and it was, to me at least, surprising. We must distinguish between the problem of determining absolute abundances and differences in abundance. The former is difficult and can at the very best be carried out with uncertainties of the order of a factor of 2 or by 100%. But the latter is much easier. If the curves of growth are similar, as they are in these dwarfs, it should be possible to detect differences in abundance of the order of 10%. I have not yet made any accurate measurements, but I believe that even a simple comparison of the spectra would readily reveal differences of the order of 25%.

It would be an unexpected and improbable coincidence if in these spectral classes a change in the hydrogen content would so completely simulate the spectral type and the luminosity of a star in a different part of the main sequence in the HR diagram. Hence I am somewhat doubtful about being able to attribute different compositions to the fainter members of the two clusters. It is much more probable that such a difference does play a role among the earlier spectral types.

In this connection it may be well to recall again that the B-type members of the Pleiades are systematically quite different from the ordinary galactic B stars. They are certainly more luminous than the main sequence, perhaps by as much as 2 or 3 mags. This fact led Trumpler and later Kuiper to draw in the HR diagram a curve steeply pointed upward on the side of higher temperatures than those corresponding to class A0. The similarity of this HR diagram with one of Strömgren's curves led Kuiper to the conclusion that the Pleiades had a smaller hydrogen content than the average B star in the Milky Way. It is again very difficult to decide whether the pronounced spectroscopic features of the brighter Pleiades necessarily indicate low hydrogen content in their atmospheres. The spectrum of Maya is particularly interesting because it is the only star of type B which has practically no rotation. The lines give conflicting results and do not permit us to assign to this star a definite spectral type and luminosity, unless we at the same time recognize the existence of a parameter other than temperature and pressure. On lower dispersion plates Morgan finds that the hydrogen lines of all the brighter Pleiades are fainter than they would be if the spectral type is assigned in accordance with the ratio He I to Mg II. Since this weakness of the hydrogen lines is consistent with higher luminosity he is inclined to attribute slightly later types to these brighter Pleiades than was customary when the hydrogen lines themselves served as a criterion of spectral type. This places Maya and the other bright Pleiades among the stars of types B6 to B8 with luminosities of the order of -3 to -2. There are, however, a few other B8 and B9 stars in the Pleiades which do not have these high absolute magnitudes and whose hydrogen lines are normal.

The weakness of the hydrogen lines in Maya, together with the relatively great strength of Mn II and Fe II and the simultaneous strength of He I, all combine to indicate fairly high luminosity of the order of -2 or $-2\cdot5$. Again, at least qualitatively, the spectroscopic criteria are in accordance with the hypothesis that the hydrogen content of the atmosphere is relatively low.

Since it must be even lower in the Hyades, it is reasonable to suppose that because of the short life times of the B-type stars they have already exhausted their supply of energy provided by the Bethe cycle and have been transformed into other types of stars. This would account for the absence of B stars in the Hyades and other hydrogen-poor clusters.

Again, as in the case of the Hyades, we must conclude that the spectra of the fainter and cooler Pleiades are not appreciably different from those of the great mass of galactic stars.

If present theories may be relied upon, it would perhaps be reasonable to suppose, as Vogt has done in his recent book, that only the original content of a cluster was the same for all its members. In the Hyades the B-type stars may have completely exhausted their supply of hydrogen so that they have disappeared as B stars. The A's and perhaps early

F's are deficient in hydrogen, but the later type members whose evolution proceeds much more slowly have not been greatly affected throughout the history of the cluster. In the Pleiades we must then assume that the B-type stars are in the process of burning up their supply of hydrogen while the A's, F's, G's, etc., still have their original composition. Finally, the great mass of galactic B and O stars are rich in hydrogen because they are relatively young, and we observe them only when they are producing the required amount of energy to give us a spectrum of class B.

In one way this picture differs from that proposed by earlier workers. I assume that the hydrogen content was at first essentially the same in all clusters and in the great majority of the stars of our galaxy. This is much more satisfactory, since we all seem to agree that stars are formed out of interstellar matter, and that in the course of their lives they throw back a part of their substance into the diffuse medium. The composition of the medium and that of young stars is about the same. I therefore suggest that we are concerned more with the evolutionary ages of the clusters, which may not necessarily be identical with ages measured on the ordinary time scale. On this picture then, the Hyades would represent an old cluster, the Pleiades a younger cluster and the greater mass of galactic O and B stars a group of particularly young objects.

I realize that the data at our disposal are entirely insufficient to resolve the many difficulties which this or any other hypothesis would involve. A disturbing thought arises immediately when we consider the wide range in L for a given spectrum, among the late B stars of the Pleiades and among the A stars of the Hyades, or when we examine the two distinct branches in Eggen's recent Colour-Absolute Magnitude diagrams of the Hyades and Coma clusters. It is not at present possible to overcome these difficulties, except by taking refuge in the fact that conflicting criteria prevent us from assigning these stars to their proper temperature classes. One of the most fruitful types of investigation would undoubtedly be the extension of Trumpler's work with the help of spectrograms of high dispersion. This can now be done for at least some of the clusters, and I conclude this report by expressing the hope that in the next few years stellar spectroscopists will make use of the facilities of the great telescopes and powerful spectrographs in order to elucidate some of the points I have raised.

7. THE ABUNDANCES OF THE ELEMENTS IN STELLAR INTERIORS

By F. Hoyle

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Provided it is assumed

(*a*) that the contribution of electron scattering to the opacity is negligible,

(*b*) that the distribution of the chemical elements is uniform throughout the star,

it can be shown that

$$L=\frac{A\ (\mu\beta)^{7\cdot5}\ M^{5\cdot5}\ R^{-0\cdot5}}{\tau\,(1+x)\,(1-x-y)}, \tag{1}$$

$$R=B\ (\mu\beta)^{0\cdot5}\ M^{0\cdot7}, \tag{2}$$

where τ is the guillotine factor, and

$$\mu=4/(6x+y+2),$$

$$(\mu\beta)^4/(1-\beta)=CM^2,$$

define μ, β when M, X, Y are given. A, B, C are known from calculation, and, to within a sufficient approximation for present purposes, may be regarded as constants. When (*a*) is changed to the opposite hypothesis (*c*) that the opacity is wholly due to electron scattering, the results (1), (2) are altered to

$$L=D\ (\mu\beta)^4\ M^3/(1+x), \tag{3}$$

$$R=E\ (\mu\beta)^{0\cdot6}\ M^{0\cdot75}, \tag{4}$$

where again, to sufficient approximation, the known quantities D, E may be regarded as constants.

The calculation of L is not much affected by hypothesis (b), but the formulae for R depend very sensitively on this assumption. It has been demonstrated* that a certain type of distribution, in which μ is appreciably less in the atmosphere than in the core of the star, leads to a large increase of radius (above the result given by the uniform model). Lyttleton has examined in detail a case where the extension in radius is by a factor of more than 40. Since this type of distribution for μ is exactly what we should expect from the conversion of hydrogen to helium, taking place within the core, it is clear that assumption (b) must be eschewed if we are to confine the present investigation to the most trustworthy considerations. Accordingly, we must drop (2), (4) and regard R as given by observation.

Our main purpose is to reach a decision between the hypotheses (a) and (c). As is well known, the observed luminosities of stars with masses $\geqslant 1\cdot5 \odot$, if expressed empirically as a power of M, are best represented by

$$L \propto M^{3\cdot5}.$$

This gives a closer correspondence with (3) than with (1), and therefore suggests that (c) be adopted for stars in this mass range. This conclusion is confirmed by a detailed inquiry. Thus, using data given by Kuiper in his discussion of the empirical mass-luminosity relation, it is found that

(i) hypothesis (a) requires μ to decrease with increasing mass,

(ii) hypothesis (c) requires μ to increase with increasing mass.

The second alternative is much to be preferred, because the fraction of the original hydrogen converted into helium by thermonuclear reactions, must, on the average, increase with increasing mass. Accordingly, since such a conversion has the effect of increasing μ, we have a clear indication that case (c) should be adopted for stars with masses $> 1\cdot5 \odot$.

When we turn to stars with masses $< \odot$ we meet a somewhat different state of affairs. The observational data then fit an empirical relation

$$L \propto M^{4\cdot5},$$

which is midway between (3) and (1). It appears therefore that the opacity changes from being predominantly due to electron scattering for masses $> 1\cdot5 \odot$ to being predominantly of Kramers' form for masses $< \odot$. For the Sun we accordingly expect the two sources of opacity to make roughly equal contributions. This inference is confirmed by direct calculation, which shows that, for the conditions of density and temperature occurring in the central regions of the Sun, the opacity due to free-free electronic transitions in hydrogen (Kramers' form) is closely comparable with the opacity due to scattering by electrons. It is particularly to be noticed that no contribution from heavy elements is required. This means that elements such as Ca, Fe must have an abundance appreciably less than 1% by mass. C, O, on the other hand, can have abundances of several per cent. without affecting the opacity. These requirements are consistent with the heavy elements abundances found for the reversing layer of the Sun. Accordingly, there is no reason to believe that the composition of the interior of the Sun differs markedly from the constitution of the atmosphere, except in the helium to hydrogen ratio which may be expected to take its largest value in the energy-generating core.

* F. Hoyle and R. A. Lyttleton, *Mon. Not. Roy. Astr. Soc.* **102**, 177, 1942.

8. ON THE THERMODYNAMICAL THEORY OF THE ABUNDANCE DISTRIBUTION OF CHEMICAL ELEMENTS

By O. KLEIN

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As is well known, the attempts to develop a thermodynamical theory for the abundance distribution of chemical elements, although very promising, have met with great difficulties. Thus, on suitable assumptions about the temperature and the density of the original distribution of matter, it has been possible to give a fairly satisfactory account of the relative abundance of the lighter elements up to mass numbers of about 60, but, on the other hand, these same assumptions lead to impossibly small values for the heavier elements, those of the upper end of the periodic table coming out so rare that there would be a negligible chance of finding a single atom of them in the known parts of the universe (1). I shall review now some attempts to overcome the difficulties mentioned, which, although by no means completed, would seem to strengthen considerably the arguments in favour of the thermodynamical theory.

Thus Beskow and Treffenberg(2) have investigated the distribution within a star in temperature equilibrium embedded in radiation corresponding to the temperature in question taking into account the gravitational field not only of the matter inside the star but also of the radiation.

The relation between the concentrations of the different kinds of atomic nuclei, including protons and neutrons, in a temperature equilibrium are most easily expressed if the chemical potential introduced by Gibbs is used. Thus, let C_τ be the number of particles of the kind τ in unit volume, $\theta = kT$ the temperature modulus (k = Boltzmann's constant), M_τ the mass of the particle in question and E_τ its energy; then, according to classical statistical thermodynamics (neglecting quantum effects), the chemical potential μ_τ is defined by means of the relation

$$C_\tau = g_\tau \left(\frac{2\pi M_\tau \theta}{h^2}\right)^{\frac{3}{2}} \exp\left(\frac{\mu_\tau - E_\tau}{\theta}\right), \qquad (1)$$

where h is Planck's constant, g_τ the multiplicity of the state of the particle in question (equal 2 for a proton or neutron due to spin), and where μ_τ and E_τ are measured from the same arbitrary zero-point. If the particle in question is an atomic nucleus consisting of N_τ neutrons and Z_τ protons, then the chemical equilibrium between the different possible compound systems is expressed by means of the simple relation

$$\mu_\tau = N_\tau \mu + Z_\tau \lambda, \qquad (2)$$

where μ and λ are the chemical potentials of neutrons and protons respectively, and where for E_τ in (1) we may take the binding energy of the nucleus in the state in question. If the binding energies and multiplicities of all possible nuclei, in all possible states of excitation, are known, the distribution will, according to (1) and (2), be completely determined by means of the three parameters λ, μ and θ.

While for atomic nuclei the quantum-theoretical modifications of statistical thermodynamics will in general be negligible, playing a role only for the neutrons at the very highest concentrations occurring, they are of decisive influence in the case of electrons. Here the relation (1) has to be replaced by the following formula

$$C_e = \frac{8\pi}{h^3} \int_0^\infty \frac{p^2 dp}{\exp\left(\frac{E(p) - \mu_e}{\theta}\right) + 1}, \qquad (3)$$

where $E(p) = c\sqrt{(m^2c^2 + p^2)}$ is the relativistic energy of the electron of rest mass m as a function of its momentum p, c being the vacuum velocity of light. Owing to the reaction neutron $\rightleftharpoons$ proton + electron + neutrino, there will be a relation between the chemical

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potentials of the four particles in question which, assuming the potential of the neutrinos to be zero as in empty space, will take the following form

$$\mu_e = \mu - \lambda + \tfrac{1}{2}(M_n - M_p)\, c^2, \tag{4}$$

where M_n and M_p are the masses of neutrons and protons respectively. Thus the concentration of the electrons will also be given as a function of the three parameters λ, μ and θ, and the same will be the case with the positrons, their potential being equal to $-\mu_e$. As to the neutrinos and the electromagnetic temperature radiation they are given as functions of θ alone. We see thus that the electric density as well as the mass density will be a function of the three parameters in question.

At a given point in space the equilibrium is thus determined by the local values of λ, μ and θ and the problem now arises how these three parameters vary from point to point. Neglecting the modifications of general relativity theory, θ will be constant throughout space, while λ and μ according to Gibbs are governed by the following two conditions for the gravitational and electrical equilibrium respectively

$$M_n \phi_g + \mu = \text{const.}, \tag{5a}$$

$$M_p \phi_g + \epsilon\phi_e + \lambda = \text{const.}, \tag{5b}$$

where ϕ_g and ϕ_e are the gravitational and electrical potentials respectively at an arbitrary point of space and ϵ the electric charge of the proton. To these equations we have to add the Poisson relations in differential or integral form between the potentials and the corresponding densities.

The obvious way of procedure is now to replace the equation (5 *b*) by the condition of electrical neutrality, which, owing to the enormous electric forces created even by a small deviation from this state, will be satisfied to a high degree of approximation. The relation between λ and μ following from this condition has been worked out by Beskow and Treffenberg for $\theta = 1$ Mev and by Beskow for $\theta = 0{\cdot}5$ Mev, using for the unknown nuclear binding energies an extrapolation formula given by Mattauch and Flügge, expressing the binding energy of the ground state of an arbitrary nucleus as a function of its charge number Z and mass number $A = N + Z$. Moreover, the influence of excitation, which is not very fundamental at the temperatures chosen, was estimated by means of well-known approximate formulae.

Using the λ-μ-relation for a given temperature the total mass density at an arbitrary point was expressed in terms of the value of μ at the same point. Applying this to a centrally symmetrical star model the following procedure was used. From equation (5 *a*) it follows that

$$\frac{d\mu}{dr} = -M_n \frac{d\phi_g}{dr} = -G\,\frac{M_n M(r)}{r^2}, \tag{6}$$

$M(r)$ being the mass contained in a sphere of radius r around the centre of the star passing through the point in question and G the gravitational constant. If now μ and M are known for a certain value of r, equation (6) together with the equation

$$\frac{dM}{dr} = 4\pi r^2 \rho(\mu) \tag{7}$$

allows the calculation of μ, ρ and M for arbitrary values of r.

In the actual calculation of most of their models Beskow and Treffenberg have made use of the fact that according to the formula mentioned above for the binding energy of an atomic nucleus at a certain μ-value ($\sim 5{\cdot}7$ Mev) condensation into nuclear matter of arbitrary size will take place. Thus they have assumed the inner part of the star to consist of a core of such nuclear matter of a density of $\sim 10^{14}$ g./cm.^{3} starting the numerical integration of equations (6) and (7) at the surface of this core where, the radius given, M as well as μ is known. In some models they have started from the centre of the star with a μ-value $< 5{\cdot}7$ Mev.

While in the inner parts of the star model the material mass density will dominate, in the outer parts the mass density due to radiation (including electrons, positrons and neutrinos) will be predominant, so that, denoting the mass density of the radiation by ρ_s, equation (6) will here take the form

$$\frac{d\mu}{dr} = -GM_n \frac{4\pi}{3} \rho_s r \quad \text{or} \quad \mu = \text{const.} - G\frac{2\pi}{3} M_n \rho_s r^2; \tag{8}$$

from which follows for the material mass density (neutrons and protons being here predominant over other nuclei)

$$\rho \sim \exp\left(-G\frac{2\pi}{3}\frac{M_n \rho_s r^2}{\theta}\right). \tag{9}$$

Owing to this the star will have a finite mass and a comparatively well-defined radius, which in most of the models considered is about 10^{10} cm., the density ρ_s being $\sim 10^6$ g./cm.^{2} at the higher temperature and $\sim 10^5$ g./cm.^{2} at the lower temperature considered.

The results of the work of Beskow and Treffenberg are exemplified in the following table:

Abundances of certain nuclei in models with close-packed cores of radius $r_0 = 5 \cdot 10^5$ cm. at the two temperatures $\theta = 1$ Mev and $\theta = \frac{1}{2}$ Mev.

At 1 Mev the total mass is ~ 40 Sun-masses, which is the case for several models at this temperature, at 0·5 Mev the mass is ~ 5 Sun-masses, while the masses of the other models vary between 1 and 10 M. The abundances are given in powers of ten and related to hydrogen.

Element	H^1+n	He^4	C^{12}	30*	60*	90*	120*	180*	240*
$\theta=1$ Mev	10·0	7·8	3·9	3·7	3·6	2·1	1·1	−0·1	+0·2
$\theta=\frac{1}{2}$ Mev	10·0	8·5	1·8	1·7	5·7	3·8	3·1	2·8	1·3
Empirical	H^1	He^4	C^{12}	Si^{28}	Fe^{56}	Zr^{90}	Sn^{120}	W^{184}	U^{238}
(Goldschmidt)	10·0	8·6	8·0	6·6	6·55	2·5	1·6	1·3	0·0

* Total abundance of nuclei with the given mass number, calculated on basis of Mattauch-Flügge's general expression for the binding energies. For comparison Goldschmidt's values are given for the most abundant of the adjoining elements.

As is seen from this table, both temperatures, and particularly the lower one, which with respect to the total mass would at first seem preferable, give much too small abundances for the common elements in the neighbourhood of oxygen, not only in comparison with hydrogen but also compared with the heavier elements. It is probable that a temperature somewhat above 1 Mev will give better abundances for these elements. Since this would seem to be a more crucial point than the total mass, one would then have to try the assumption that a considerable part of the mass, mainly consisting of hydrogen, is lost during and after the explosion mentioned below, not only from the stars but even from the galactic systems to which they belong. The resulting hydrogen content of intergalactic space would, however, seem to be hardly above the present limit of observability.

The most important result of the investigation reviewed here seems to me to be the fact that there is a range of models *where the abundance of the lightest and the heavier elements is of about the right order of magnitude and that the corresponding stars have masses of the order of magnitude of real stars.*

In connection with this work the question of the equilibrium of the radiation itself and its bearing on the transition to the present state of the universe will be shortly reviewed. Here the starting-point is the fact that a homogeneous universe of the high density of the radiation at the temperature in question (10^5–10^6 g./cm.^{3}) would either have a comparatively very small mass and radius or would have a very short life due to its rapid expansion. Thus, a total mass exceeding considerably 10^5–10^6 Sun-masses would lead to an expansion velocity whereby the density would fall to its half value in a time of the order of magnitude of a second. In order to surround all stars in a quasi-stationary way by an atmosphere of such dense radiation it would thus be necessary to assume an accumulation around each star separately. An estimate of the mass of the radiation belonging to a single star might be obtained in different ways with approximately the same result. We might consider for instance a closed world and require that its density should be such that the velocity of expansion were momentarily equal to zero. This would give for the radius R and the mass M the expressions

$$R=\left(\frac{3c^2}{4\pi G\rho}\right)^{\frac{1}{2}}, \quad M=\left(\frac{27\pi c^6}{16G^3\rho}\right)^{\frac{1}{2}}, \tag{10}$$

giving with $\rho=10^6$ g./cm.^{3}

$$R=5{\cdot}7\times10^{10}\text{ cm.}, \quad M=1{\cdot}8\times10^6\, M_\odot,$$

where $M_\odot$ is the mass of the Sun. We see that the value of R is not very much larger than the radii of the Beskow-Treffenberg star models.

In order to get a somewhat clearer idea of the conditions governing such radiation stars a static solution of the Einstein equations of general relativity corresponding to a centrally symmetrical distribution of radiation has been investigated (3) with results very similar to those of Emden for an ordinary isothermal gas under the influence of its own gravitational field. Thus, at great distances from the centre, the density approaches more and more the singular solution

$$\rho=\frac{3c^2}{16\pi G}\cdot\frac{1}{R^2}, \tag{11}$$

and in a solution with the density 10^6 g./cm.^{3} for $R=0$ it has fallen to half its value for $R\simeq5{\cdot}10^{10}$ cm.

In close connection with the question of the quasi-stability of the radiation at the initial state stands the question of its disappearance in a later state of the universe by means of an accelerated process, the latter stages of which must have been very rapid. The reason for this may be sought in the decrease of the gravitational forces of the radiation itself, which in the beginning have maintained a state of approximate equilibrium. The radius of the star models being of the order of magnitude of a light-second, the latter stage of the disappearance would on this view take only a few seconds. The sudden disappearance of the large part of the gravitational force due to the radiation would lead to a violent explosion of the star and a corresponding rapid decrease of its density. At the same time the disappearance of the neutrinos would initiate a series of nuclear processes, the most important ones being such β-processes whereby the enormous neutron excess is removed. This again would entail a general decrease in reaction velocities, so that the freezing-in of the main features of the original state would not seem excluded.

In connection with the radiation the question of the evolution of the stellar systems should also be mentioned. Thus, it is reasonable to speculate on a primary quasi-statical state not only of each galactic system separately but also of the metagalactic system, the gravitational effect of the radiation contributing to the approximate equilibrium. On such an assumption the removal of the radiation would entail a process of evolution of the stellar systems and may possibly also constitute the beginning state for the expansion of the metagalactic system. This would raise a number of dynamical problems the solution of which may perhaps give a clearer view of the original distribution of the radiation density in the universe.

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9. SUMMARY AND CONCLUSIONS

By OTTO STRUVE

The problem of the abundances of the chemical elements in the universe had its beginning almost 20 years ago when Russell published his fundamental paper on the composition of the reversing layer of the Sun. Since all the later investigations have been based upon this investigation, it will be of interest to you to hear an excerpt from a recent letter by Prof. Russell:

> It may be pardonable to remark that the reconnaissance by my colleagues and me 20 years ago is a rather remarkable example of the results of an approximate theory applied to empirical estimates of low accuracy. The relative abundance of the commonest and the rarest elements was overestimated by a factor of the order of 10 and the absolute abundance of the former by about the same factor. Owing to ignorance of the correct damping factor the values given for the elements lighter than sodium were based on scanty uncalibrated data and extreme extrapolation. Fortune appears to have been remarkably kind to audacity. It is much to be hoped that accurate determinations for all the elements will soon replace this exploratory work. The successful study of the solar atmosphere may be attributed to its being nearly in thermodynamic equilibrium—at least for most radiations. The central parts of H and K and of the Mg II pair near $\lambda 2800$ and still more the He absorption near $\lambda 10{,}830$ would probably tell a different story. We are also fortunate in the wave-length region available for ordinary observation. If we had to work in the region short of $\lambda 2500$ where the molecular absorption appears to distort the solar energy curve enormously, or long of $\lambda 15{,}000$ where telluric absorption is heavy and solar lines feeble, the problem would be far more difficult.

Enormous progress has been made in determining the composition of the Sun's atmosphere and as we have learned from Prof. Minnaert's paper the advance was brought about partly by the creation of a more exact theory of the formation of an absorption line, partly through the determination of theoretical and accurate laboratory intensities of different lines and partly through better observations of the kind we now have in the photometric atlas of the solar spectrum.

Nevertheless, as Minnaert has pointed out, there are relatively large differences between the determinations made by different methods. It should be possible to measure the absorption of a spectral line to at least 10% of its value. Yet, as Minnaert has remarked, an incorrect assumption concerning the temperature may produce a difference in the resulting abundances by factors of the order of several hundred. No wonder then that he and other workers prefer to use methods which avoid these large uncertainties in the Boltzmann factor.

The theory of the formation of an absorption line has entered explicitly or implicitly into the results of almost all the workers in this field. It was referred to not only by Minnaert and Hunaerts, but also by Unsöld and Strömgren. One advance which is now possible results from the application of Chandrasekhar's exact theory of the Milne-Eddington transfer problem. Following Chandrasekhar, Marshal Wrubel has computed a set of curves of growth, all based upon the exact theory, with η, the ratio of the absorption coefficient in the line to the continuous absorption coefficient, considered constant within the atmosphere. Wrubel has used as the parameters the ratios of the coefficients in the linear expansions of the Planck function with optical depth and the damping constant expressed in the usual way in terms of the Doppler broadening b. I am showing on the screen a set of his curves for one specific value of the first parameter and for six values of the damping constant. These curves of growth differ quite appreciably from those which Unsöld computed with the Minnaert approximation making use of the expression $1/R_c$. Chandrasekhar tells me that in some parts of the curves, notably those where $\log \eta_0$ is 0·8, Unsöld's curves run higher, so that the transition is less gradual than in the case of this exact theory. Different sets of curves computed with different values of the first parameter differ very greatly. Wrubel points out that if the observational data are approximated by means of a family of curves using the wrong ratio of B^0/B^1, the result may often be a serious error in the turbulent velocity and in the damping constant. Chandrasekhar is of the opinion that Wrubel's new curves of growth should be used in preference to other theoretical curves.

The seriousness of this problem is emphasized by some comments which I have recently obtained from Lawrence Aller and from K. O. Wright. The former has reduced his observations in several different ways. For γ Pegasi he obtains appreciable differences depending upon whether he uses Unsöld's method or Strömgren's curve of growth. For the ratio of H to O in this main-sequence B2 star he finds a value of the order of 10,000, which disagrees with the results from other stars. For τ Scorpii Unsöld finds 1000. Yet Aller is not prepared to attribute reality to this difference because there are large uncertainties not only from the choice of the theoretical curve of growth but also from possible departures from thermodynamic equilibrium. Similarly, K. O. Wright has experimented with different curves of growth in the case of stars of type F. He finds similar relative abundances in Procyon and the Sun and he also concludes that the giants agree among themselves. He suspects that the giants differ slightly from the dwarfs, with sodium and yttrium more abundant in the giants and with magnesium and silicon less abundant. Despite the importance of these results Wright and the other observers have all felt the need for a more exact set of theoretical curves of growth.

I hope that these details do not obscure the main purpose of this symposium. We have here been concerned with a problem of method. As the previous speakers have pointed out, our purpose is to determine the relative numbers of atoms of different elements within unit volume of stellar substance. We observe only a thin outer fringe and the spectroscopic results apply only to that fringe, but Hoyle, among others, has indicated that the stirring of the stellar substance through convection may be sufficient to bring about a uniform distribution of the elements. If this were absolutely certain, it would greatly simplify the entire problem. I understand, however, that some theoretical astrophysicists are not completely convinced that the stirring actually is adequate. I have only recently learned that Dr Ledoux of Liège has taken the opposite view. I hope that in the discussion at the end of this session he will tell us briefly about his work. Dr G. Gamow also recently wrote me that in his opinion 'the abundance of elements in stellar atmospheres must have very little to do with the reactions in the central regions.... How else could one understand the presence of Li, Be and B in the solar atmosphere in view of the fact that these nuclei would be completely destroyed in a few minutes under the conditions near the sun's centre? How else could one understand the presence of H in the white-dwarf atmospheres when we are sure now, after Kuiper's revision, that there is no H inside these stars?'

Most of the speakers have emphasized the problem of determining what might be called the absolute abundances of the elements. Perhaps the most striking result of the discussions is the remarkable degree of uniformity that has been observed in the most widely different astronomical sources. The sun, the main-sequence stars of type F and the He stars like Tau Scorpii and even the O-type star 10 Lacertae have all approximately the same composition. The observers tell us that the precision with which the abundances are now determined corresponds to a factor of perhaps a little better than 2 or 3. Even more surprising is the fact that Strömgren's result for interstellar matter and Menzel's for the planetary nebulae also indicate a composition that is essentially the same as that of the normal stars. Even the interiors of the stars as investigated by Hoyle and others lead essentially to the same composition. Not many months ago Martin Schwarzschild made use of the mass-luminosity relation and of the energy-generation equation in order to determine the hydrogen and helium content of the Sun. His result lead to a ratio of hydrogen to metals or, as we say, of hydrogen to Russell mixture, that was considered to give an excessive amount to the metals. More recently Mrs Harrison has repeated Schwarzschild's computations through the addition of a separate parameter—the abundance of the elements of the oxygen group. These elements differ appreciably from the metals because at temperatures above about 2×10^7 degrees they rapidly become less efficient absorbers. The metals which are not stripped to the bare nuclei at these temperatures retain their high opacity. In this manner Mrs Harrison was able to adjust the observed mass and luminosity of the Sun by assuming a composition by weight of 59% of hydrogen, 30% of helium, 9% of the oxygen group and 2% of the metals. This is not unreasonable from the observational point of view.

The first conclusion of this symposium is the establishment of a list of what we might call the normal abundances of the elements in the universe. These abundances seem to have the character of a universal law of nature. They strikingly reflect the well-known terrestrial scarcity of lithium, beryllium and boron and they also bring out Harkins's rule concerning the relative abundances of the even and odd atomic numbers. It is with this normal distribution of the elements that Prof. Klein has been concerned in his analysis of the conditions that might have produced the heavier elements.

Our next big problem was to consider possible departures from the normal abundances. We are here encountering some disadvantages but also some decided gains. The stars which we suspect of having unusual abundances are often faint and therefore not within the reach of our most powerful spectrographs. Moreover, few of them are binaries, so that we are lacking the important information provided by the mass and even the luminosities of some of these stars and their temperatures are usually not known with the precision we would require for a complete analysis. However, there are also two important advantages. The first consists in the fact that we do not necessarily have to determine absolute abundances but may sometimes be satisfied with differential measurements against normal stars or against other stars of certain known characteristics. I have used this method in my discussion of the two galactic clusters. The second consists in the conspicuousness of the observational features and the ease with which they are measured.

Some of these important problems have only been touched upon in this symposium. Unsöld has already commented upon the metallic-line stars. They form a large group of objects with conflicting criteria so that if we use the hydrogen lines or the calcium line K we obtain spectral type A, while if we use the lines of the neutral and ionized metals we find a middle F. Greenstein has made a thorough investigation of some of these stars and if his results are interpreted in the conventional manner, there is no escape from the conclusion that certain elements, such as calcium, scandium, zirconium, and magnesium, have abundance deficiencies by a factor of 10, and that, as you know, is a large quantity when it is reflected directly in the equivalent width of an absorption line. Greenstein has struggled with this problem for a long time. Why should it be that these particular elements are deficient and is the deficiency real or only apparent? Unsöld has remarked that differences in turbulence may cause spurious abundances, but Greenstein finds that this alone will not account for the observations. After a long search he found that when he plotted the observed deficiencies against the second ionization potentials of the elements involved a definite regularity became apparent. All elements with second ionization potentials between 11·8 V. and 16 V. are deficient and no others. This range of potentials is close to the ionization potential of hydrogen, namely 13·5 V. However, they were not all on one side of 13·5, being distributed somewhat unsymmetrically about this value, the maximum deficiency occurring for calcium at 11·8 V. and for scandium at 12·8 V. Greenstein correctly concludes that the hydrogen ionization potential must have something to do with the observed distribution. He has advanced two possible explanations, one in terms of Lyman emission lines and another in terms of a quantum-resonance collisional process in which a proton may remove an electron from a singly ionized metal and thereby cause an excessive ionization of those metals whose second ionization potentials are close to 13·5 V.

I have dwelt upon this result because it demonstrates how careful we must be in attributing reality to observed discrepancies in the abundances. At the same time we must not lose sight of the fact that some physical cause must exist that somehow produces the anomaly of the metallic-line stars but fails to produce it among the normal stars of similar effective temperature. I have already suggested in my own contribution to this symposium that the evidence from the Hyades suggests that we are, after all, concerned in the case of the metallic-line stars with a peculiar condition resulting from an abnormally low hydrogen content. If this were so, then we should be observing not so much the direct anomaly of the hydrogen scarcity which, incidentally, would not be very apparent by itself because the continuous absorption coefficient is also due to hydrogen so that the ratio of line absorption to continuous absorption would hardly be affected.

Another important group of stars with anomalous observed line intensities are those which resemble Upsilon Sagittarii. Unsöld has referred to these stars and attempted to explain them as a result of super-excitation. I believe that I must agree with Russell and Greenstein rather than with Unsöld that these stars cannot be simply explained in terms of a factor ϕ which measures the departure from the ordinary Saha ionization. Greenstein's most important result is the weakness of the hydrogen lines compared with the metals as well as to helium. It is this weakness which is also striking in the case of Popper's helium star HD 124448 and of Bidelman's recently discovered star HD 30353. The latter seems to form a transition between Upsilon Sagittarii and R Coronae Borealis. The former extends this unusual sequence towards the higher members. I agree with Greenstein, Popper and Bidelman that it is at present impossible to explain the almost complete absence of the Balmer lines through any known physical process. Because the hydrogen lines are so strong in the shells and even in the nebulae I am inclined to believe that these reversing layers are really deficient in hydrogen. But it would be unreasonable at this time to attribute this deficiency to any particular process such as the carbon cycle. Although I do not wish to draw a close parallel between the two cases, it does strike me forcibly that in the nebula surrounding the B3 sub-dwarf companion of Antares there are no hydrogen emission lines, but only the forbidden lines of Fe II. This case may have some similarity to the deficiency of hydrogen in the planets, satellites, comets and meteors. Perhaps we should not altogether lose sight of the possibility that gaseous masses could have been formed around certain stars through the vaporization of solid particles or chunks of matter which had previously lost their hydrogen content through causes altogether different from those that are involved in the carbon cycle. The possible association of Upsilon Sagittarii and Bidelman's star with R Coronae and the old theories of smoke-screens which produce the light variation of this famous variable lend some degree of respectability to the hypothesis I have suggested.

Among the anomalous A-type stars, those with strong lines of the rare earths deserve particular attention. Why should europium, for example, be unusually abundant in such a star as α Canum Venaticorum or β Coronae Borealis? A partial answer has been given in a paper presented last month by H. W. Babcock at a meeting of the American Astronomical Society. Many of these peculiar stars have strong magnetic fields. In at least one of them, HD 125248, the magnetic field varies between a large positive and a large negative value. When the magnetic field is strong the lines of europium are split into numerous Zeeman components. These components are blended, producing a broadened line. The result is a striking change in the curve of growth. When the magnetic field is strong the europium lines appear greatly enhanced. When the magnetic field is absent they are weak. I am sure that those who have not been previously acquainted with Babcock's work will be as surprised as I was when Babcock announced that the line $\lambda 4205$ of europium II is enhanced by a factor of almost 10 when the field reaches its maximum value of 7800 gauss. This tremendous effect rivals that of turbulence in producing a change in the curve of growth. It almost certainly suffices to eliminate a large number of previously suggested anomalies in composition among the A-type stars.

I must next refer to the remarkable results of McKellar, Herzberg, and others on the abundances of certain isotopes. McKellar has sent us an account of his work on the carbon isotopes C^{12} and C^{13}. On the Earth the ratio is about $C^{12}/C^{13}=90$. In the N-type stars McKellar and Herzberg invariably find a much greater abundance of C^{13}. In the R-type stars, however, McKellar finds for three stars a ratio C^{12}/C^{13} greater than 50 which could well be similar to that of the Earth. But in 15 stars the ratio is between 3 and 4. It seems at present impossible to explain these differences either through the curve of growth effects or through any other simple processes. McKellar points out that two suggestions may be made: Klein, Beskow, and Treffenberg had found in their closely packed cores that values of the ratio between 10 and 50 may be produced in thermal equilibrium at about 1 million eV. This would in effect mean that the unusual abundances were formed in the prestellar state of the galaxy. The other suggestion is one advanced by Fermi. It presupposes that in the original medium the ratio was about 3. Since the capture cross-section of the nuclei of C^{13} for protons is about 70 times larger than that of C^{12}, the ratio would gradually change as a result of the carbon cycle from its original value of about 3 to its final value of around 70. This would, of course, mean that the Bethe cycle must have been operating for a long time, and the question to which as yet we have no answer is whether the effectively complete conversion to a ratio of even more than 70 in the Sun, the Earth and many stars would not at the same time require a pretty far-going degree of conversion of hydrogen into helium? It should be easy to give the answer, but I do not believe that this has been done. McKellar writes me that low C^{12}/C^{13} would indicate youth; high C^{12}/C^{13} would indicate more advanced age. But because of the extremely high temperature coefficient of the nuclear reactions the transition stage would be short. The conversion of H into He would start when the Bethe cycle goes into operation and would continue long afterwards. It would be reasonable to expect that in the stars with small C^{12}/C^{13} the ratio H/He would be as great as or greater than in stars with large C^{12}/C^{13}. There should be some interesting observational tests of Fermi's suggestion, and McKellar is working on some of them. Dr G. Shajn of the U.S.S.R. has also sent me some very interesting new results on the relative strengths of the bands of $C^{12}N^{14}$ and $C^{13}N^{14}$ in the N3 star Y Can Ven.

It is of interest that O. C. Wilson found for interstellar space that the ratio must be larger than about 5. Hence, it is not impossible that the interstellar medium may have a composition that would, on Fermi's hypothesis, correspond to the original condition.

We have not as yet discussed the question of the compositions of Baade's two populations. The observational data are quite scarce. Among the high-velocity giants which belong to population II and therefore represent the kind of stars that may have originated near the centre of the galaxy, the bands of CN are definitely weaker than in stars of population I. These and other spectroscopic results are mostly due to Morgan and Keenan. They also find that among the carbon stars of type R those of population II are marked 'by such tremendous strengthening of the G band that they have been called CH stars'. In both groups the molecules appear to be more sensitive to differences between the two populations than the atoms. The elements that are most affected are H, C, N, and possibly O. Keenan, Morgan, and Munch say that 'it is significant that these are among the abundant light elements the concentration of which appears to be a determining factor in the structure of a star'.

There are also some well-known anomalies among the RR Lyrae stars in which the hydrogen lines are systematically too weak when compared with the metallic lines and among the high-velocity dwarfs or sub-dwarfs which were first designated at Mount Wilson as 'intermediate white dwarfs'. Kuiper has suggested that the general weakness of the lines in these stars may be the result of small hydrogen content so that 'on the whole the hydrogen content increases with decreasing ellipticity of the galactic orbits'. Within the last few days Morgan has found some rather striking indications that even among the high-velocity A-type stars the hydrogen lines are sharper and the metallic lines weaker than in normal stars of the same class, so that it is quite probable that there is a general tendency among all of these objects of population II to be characterized by the same general departure which, if Kuiper is right, may well be due to small hydrogen content. However, I must not conceal the fact that at least the sub-dwarfs most definitely fall below the main sequence and not above it as I had indicated might be the case with the hydrogen-poor stars in the Hyades and the Pleiades.

I am glad that Unsöld has mentioned in his report the great importance of the Wolf-Rayet C and N branches at the one end of the spectral sequence and of the work by Russell on the abundances of oxygen and carbon in the S stars on one side and the R and N stars on the other. Dr Beals of Ottawa and Dr Swings of Liège and Yerkes-McDonald are here and are much better qualified to discuss the Wolf-Rayet stars than I am.

Discussion

J. A. Pearce (Dominion Astrophysical Observatory, Victoria, B.C.): Dr Andrew McKellar of Victoria, B.C., has just completed a survey of 40 N-type stars. He had previously found a very strong line at $\lambda 6707$ in the spectrum of the 7th magnitude N-type star WZ Cas. This is the resonance line of lithium—some 10 A. wide; it was the first identification of lithium in a stellar source. His new survey shows that the N-type stars can be divided into three groups: (*a*) those showing strong lithium lines like WZ Cas, (*b*) those showing weak lines of lithium, and (*c*) those in which there is no evidence of lithium.

E. Schatzman (Institut d'Astrophysique, Paris, France): Les naines blanches se placent bien loin de la séquence principale dans le diagramme masse-luminosité. Ce fait peut s'expliquer par leur structure particulière. Dans le champ de gravitation extrêmement intense des naines blanches un triage des éléments se produit et l'hydrogène flotte à la surface de l'étoile. Ce fait explique l'absence générale de raies métalliques dans le spectre des naines blanches. Les métaux sont tombés à l'intérieur de l'étoile. Par ailleurs ce schéma semble bien s'accorder avec la théorie du débit d'énergie des naines blanches et semble prouver que la réaction proton-proton est permise.

P. Ledoux (Institut d'Astrophysique, Cointe-Sclessin, Belgium): In the Sun, the carbon cycle takes place at an appreciable rate, only in the convective core, very close to the centre of the star. However, owing to the convection currents, the chemical composition of the core remains uniform in the course of time, but it becomes progressively more and more different from the composition of the radiative envelope, hydrogen being continually transformed into helium. There does not seem to be any good reason, at the present time, to assume that a mixing will take place to any appreciable extent between the material of the core and the envelope. We are thus led to the study of configurations with different mean molecular weights μ and μ_2 in the core and envelope.

If one tries to find such a configuration having the characteristics of the Sun ($M\odot$, $L\odot$, $R\odot$) and in which the energy liberated by the carbon cycle is equal to the energy radiated, one reaches the conclusion that this will only be possible if the abundances by weight X_e of hydrogen and Y_e of helium in the envelope are high.

Adopting values similar to those given by Unsöld for τ Scorpii, namely $X_e = 0{\cdot}56$, $Y_e = 0{\cdot}40$ one finds that the equality between energy generated and radiated is realized for a model in which $\mu_i/\mu_e = 1{\cdot}35$, that is, $X_1 = 0{\cdot}26$, $Y_1 = 0{\cdot}70$.

If one supposes that the initial chemical composition of the sun were uniform, it is possible to evaluate the time necessary to bring about the difference found above. One finds that τ is of the order of $5 \cdot 10^9$ years. This can be called the 'age' of the Sun, since during that period, its state was never very different from the present one. There are still many uncertainties in the values of different constants appearing in the theory, but the results must be of the right order of magnitude. The agreement with other determinations of the 'age' of the Sun is certainly arresting as it appears here simply as a by-product of the condition that, at the present time, the energy generated be equal to the energy radiated.

P. LACROUTE (Observatoire, Toulouse, France) signale une possibilité d'obtenir les proportions des éléments lourds dans l'univers, indépendamment de l'analyse chimique et de l'analyse spectrale.

Lorsque les éléments lourds se sont formés, les conditions physiques ont imposé certaines proportions entre les différents types de noyaux et toutes ces proportions étaient liées entre elles; certaines nous sont bien connues, ce sont les proportions des isotopes et ceci constitue une piste sérieuse pour atteindre les autres proportions. Les indications données par les isotopes ne peuvent cependant évidemment pas suffire, il faut leur adjoindre d'autres considérations.

On choisit par tâtonnement les proportions des éléments chimiques de façon:

(1) Que le total des noyaux de même nombre de masse, n_A, varie aussi régulièrement que possible.

(2) Que le rapport $\frac{n_A^2}{n_{A-1} \times n_{A+1}}$ varie aussi régulièrement que possible.

Ces deux règles simples ont été suggérées par certaines considérations sur la formation des éléments lourds (*C.R.* **224**, pp. 1481, 1541). Ces considérations et ces règles ne deviennent valables que pour $A > 80$.

La courbe de fréquence obtenue présente une grande analogie avec la courbe de fréquence obtenue par l'analyse des météorites. Dans l'avenir lorsqu'on comprendra mieux les conditions de formation, l'application d'une méthode de ce genre donnera peut-être les résultats les plus sûrs.

D. H. MENZEL (Harvard College Observatory, Cambridge, Mass.): Nova Pictoris (1925), with its intense sulphur lines, presents an interesting and unique spectroscopic anomaly. Whether the peculiarities arise from unusual abundances or from a special combination of physical conditions and distinctive excitation parameters of S II is not clear.

The use of King's f-values for iron, in the absolute sense, is somewhat dangerous. The details of the calculations have not been published. The f-values are probably much too small; any determination of iron abundance from them leads to a high value. I think that they may be wrong by a factor of 10.

A. UNSÖLD (Astronomical Observatory, Kiel, Germany): It may be that the abundance of iron in the Sun, which I determined using King's f-values, is somewhat too high. Evidence from the analysis of meteorites points in this direction. A factor of more than 2 or 3, however, appears improbable.

D. H. MENZEL: I disagree. When the sum rules are applied, with appropriate factors for equivalent electrons, the discrepancy is about 10. What did you use for Ni?

A. UNSÖLD: For Ni I used the f-sum rule.