19

Temporal Correlations

19.1 Introduction

Our aim in this chapter is to extend the Bell inequality discussion in Chapter 17 to its temporal analogue, known as the *Leggett–Garg (LG) inequality*.

Bell inequalities involve spatial nonlocality, that is, signal observations distributed over space. It was shown by Leggett and Garg that analogous inequalities involving temporal nonlocality can be formulated (Leggett and Garg, 1985). We shall discuss one of these, known as the *LG inequality*.

We saw in Chapter 17 that Bell inequalities are based on certain classical mechanics (CM) assumptions about the nature of reality. Likewise, the LG inequality is based on two CM principles that are not incorporated into quantum mechanics (QM).

Definition 19.1 The principle of *macrorealism* asserts that if a macroscopic (that is, large-scale) system under observation (SUO) can be observed to be in one of two or more macroscopically distinct states, then it will always be in one or another of those states at any given time, not in a quantum superposition of those states, even when it is not being observed.

This principle is a foundational principle in CM but is incompatible with QM in at least two ways. First, it is vacuous, as it asserts the truth of a proposition in the absence of empirical validation: how can we define the concept of *always*, without introducing counterfactuality? Second, it is violated in quantum theory, for instance, in path integral calculations.

This principle in embedded in the nexus of issues explored in this book. The quantized detector network (QDN) version of it takes the form "If any SUO can be observed in any number of possible states, then it will be observed in one of them in any run." This is a near tautology. The classical version says the same thing but makes an additional assertion about something going on in the absence of observation, which is the vacuous element that QM cannot accept.

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Definition 19.2 The principle of *noninvasive measurability* asserts that the actual state an SUO is in can always be determined cost-free, that is, without having any effect on that state or on its subsequent dynamical evolution.

This principle is subtle. We experience its apparent validity all the time in our ordinary lives: we look at objects and they appear not to change by those acts of observation. However, quantum physics tells us otherwise, for all information comes to us via quantum processes, and those involve quanta.[1] Newton's third law, *action and reaction are equal and opposite and act on different bodies*, really does have its quantum counterpart: *no observation leaves the observed completely unchanged*.

The LG inequality involves temporal correlations, which measure and compare changes in observables. We shall focus our attention on the temporal correlations of the signal states of dynamically evolving bits and qubits.

19.2 Classical Bit Temporal Correlations

Without loss of content or generality, we shall restrict the discussion in this section to an SUO consisting of N interacting classical bits. In such discussions, it is not necessary to say what these bits represent physically. What matters is that if the observer chose to look at any one of these bits, that bit would be observed to be in precisely one of two possible states. We will refer to these states as *up* and *down*. A useful mental image is that the *up* state represents a raised flag denoting a signal, while the *down* state represents a lowered flag denoting an absence of a signal.

Consider an experiment where at initial stage Σ_0, all N of these bits are definitely in the *up* state. We then allow the state of the SUO to evolve until stage Σ_1 and then we look at the state of the SUO at that stage.

Suppose that at stage Σ_1 we find a total of N^{up} bits in the *up* state and $N^{down} \equiv N - N^{up}$ bits in the *down* state. A natural question is: *how different is the state of the SUO at stage* Σ_1 *compared with its state at* Σ_0?

There are many ways to answer this question, that is, to define temporal change. Before we can do that, we need to clarify a metaphysical point, concerning the notions of *permanence* and *identity*.

Change Is Contextual

Up to now, our exposition of QDN has emphasized that *everything changes*. But on reflection, that is a vacuous inconsistency, for change can be measured

[1] We have not been much concerned with Planck's constant $\hbar$ so far in this book, but it comes in here. Changes occurring to states of SUO when they are observed are quantified by that unit of action, which is relatively negligible on our ordinary real-life (emergent) scales of measurement.

only by observers. But those observers themselves change, according to their own subjective experience. Therefore, any change in an SUO has to be carefully distinguished from the changes going on naturally in the laboratory.

Leggett and Garg chose *temporal correlations*. We discussed these for single qubits in the previous chapter.

Definition 19.3 If a classical bit is in the same state at stage Σ_n as it was at an earlier stage Σ_m, then the states are said to be *perfectly correlated*, with a correlation $C_{n,m} = +1$. If, conversely, a classical bit is in a different state at stage Σ_n compared with its state at stage Σ_m, then the two states are said to be *perfectly anticorrelated*, with a correlation $C_{n,m} = -1$.

Our interest will be in the average correlation $\overline{C}_{1,0}$. From the information given above, we find

$$\overline{C}_{1,0} = \frac{N^{up}}{N} - \frac{N^{down}}{N} = \frac{2N^{up}}{N} - 1. \tag{19.1}$$

By inspection, it is easy to see that the average correlation satisfies the constraints $-1 \leqslant \overline{C}_{1,0} \leqslant 1$.

19.3 The Classical Leggett–Garg Inequality

The LG inequality is based on a three-stage experiment, as follows. An SUO consisting of r classical bits, each in its *up* state at stage Σ_0, evolves to stage Σ_1, and the average correlation $\overline{C}_{1,0}$ is measured. The state is then allowed to evolve to stage Σ_2, where two new average correlations are measured. One of these is $\overline{C}_{2,1}$ and the other is $\overline{C}_{2,0}$.

We note that (1) each bit is followed from stage Σ_0 to stage Σ_1 and then finally to stage Σ_2 and its *up* or *down* state is observed noninvasively and recorded at each stage, and (2) according to the CM principles of macrorealism and noninvasive measurability, the observer will have all the information needed to calculate these three ensemble average quantities precisely and without error. Because the dynamics is assumed deterministic, only one run is required. If on the other hand, the dynamics is classical stochastic, or the initial state is random, then the argument can adjusted to take probabilities into account, with exactly the same result.

Given these three correlations, we define the LG correlation $\overline{K}$ by

$$\overline{K} \equiv \overline{C}_{1,0} + \overline{C}_{2,1} - \overline{C}_{2,0}. \tag{19.2}$$

We derive the LG inequality $-3 \leqslant \overline{K} \leqslant 1$ as follows.

Given an SUO of r bits, noninvasive measurability allows us to track any or all of the bits in the SUO over the three stages, without interfering in the dynamics. Suppose we tracked the ith bit in the SUO over the three stages Σ_0, Σ_1, and Σ_2.

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Table 19.1 *Calculation of the possible values of the LG correlation K^i for the ith bit*

signal status at Σ_0	up	up	up	up
signal status at Σ_1	up	up	down	down
signal status at Σ_2	up	down	up	down
$C^i_{1,0}$	1	1	−1	−1
$C^i_{2,1}$	1	−1	−1	1
$C^i_{2,0}$	1	−1	1	−1
K^i	1	1	−3	1

Table 19.1 shows the possible *up* or down states of that bit at each stage, the correlations associated with those bit states, and the LG correlation for that bit.

By inspection of Table 19.1, we see that the LG correlation $K^i \equiv C^i_{1,0} + C^i_{2,1} - C^1_{2,0}$ for the ith bit lies in the interval $[-3, 1]$. This is perfectly general, being valid for any interaction whatsoever between the bits in that SUO. Since this is true for any bit in the SUO, we conclude that the average $\overline{K}$ satisfies the same condition. Hence we arrive at the LG condition

$$-3 \leqslant \overline{K} \leqslant 1. \tag{19.3}$$

There is no way that the LG inequality could ever be violated classically, given the principles of macrorealism and noninvasive measurability.

19.4 Qubit Temporal Correlations

Extending the above classical bit discussion to the quantum case involves significant differences. Specifically, each bit in the ensemble is replaced by a qubit, and then all of the qubits are tensored together to create a quantum register of rank r.

As will be appreciated by now, such registers are exceedingly complicated structures. We will assume in the first instance that the qubits in the quantum register do not interact with each other, but can interact with other elements of their environment. This assumption means that we can meaningfully discuss the evolution of a single qubit. We shall follow the evolution of a typical single qubit in a noninteracting ensemble from initial stage Σ_0 to intermediate stage Σ_1 and then to final stage Σ_2.

We shall discuss the correlation $C_{1,0}$ of two signal observations, at stage Σ_0 and stage Σ_1. Assuming that the semi-unitary operator acting on the chosen qubit is in the standard form

$$U_{1,0} = \begin{bmatrix} a_1 & -b_1^* \\ b_1 & a_1^* \end{bmatrix}, \tag{19.4}$$

where $|a_1|^2 + |b_1|^2 = 1$ and we ignore an overall phase, we use the results of the previous chapter to find

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$$C_{0,1} = |a_1|^2 - |b_1|^2 = 2|a_1|^2 - 1. \tag{19.5}$$

As noted previously, this result is independent of the initial state of the qubit.

Now consider a further evolution from stage Σ_1 to stage Σ_2 with evolution operator

$$U_{2,1} = \begin{bmatrix} a_2 & -b_2^* \\ b_2 & a_2^* \end{bmatrix}, \tag{19.6}$$

where $|a_2|^2 + |b_2|^2 = 1$. Then $C_{2,1} = 2|a_2|^2 - 1$.

According to QM principles, evolution from stage Σ_0 to stage Σ_2 is given by the evolution operator $U_{2,0} = U_{2,1}U_{0,1}$. We find

$$U_{2,0} = \begin{bmatrix} a_2 & -b_2^* \\ b_2 & a_2^* \end{bmatrix} \begin{bmatrix} a_1 & -b_1^* \\ b_1 & a_1^* \end{bmatrix} = \begin{bmatrix} a_1 a_2 - b_1 b_2^* & -a_1^* b_2^* - b_1^* a_2 \\ a_1 b_2 + b_1 a_2^* & a_1^* a_2^* - b_1^* b_2 \end{bmatrix}, \tag{19.7}$$

from which we deduce

$$C_{2,0} = 2|a_1 a_2 - b_1 b_2^*|^2 - 1. \tag{19.8}$$

19.5 QDN Spin Correlation

In this section we show how QDN deals with spin correlation. The first thing to note in the above calculation is that according to QDN principles, a single persistent qubit representing a *calibrated* detector would show no changes in its signal status, by definition. Because we wish to discuss an actual spin, such as that of a proton, which would show changes in its signal status, bitification tells us that we need *two* detector qubits, one for proton spin *up* and the other for proton spin *down*. QDN qubits do not model angular momentum or spin *per se* but *yes/no* logic.

Suppose then that we want to investigate temporal spin correlation for a single proton. Following the discussion in previous sections, we will have a three-stage process, represented by Figure 19.1.

The initial labstate $\mathbf{\Psi}_0$ is taken to be given by

$$\mathbf{\Psi}_0 \equiv (\alpha \widehat{\mathbb{A}}_0^1 + \beta \widehat{\mathbb{A}}_0^2)\mathbf{0}_0, \tag{19.9}$$

where 1_0 detects spin *up* and 2_0 detects spin *down*.

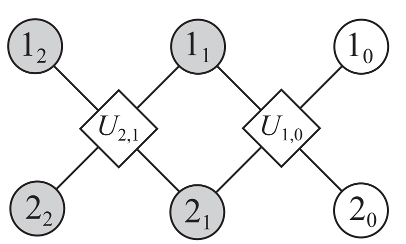

Figure 19.1. Stage diagram for a proton temporal spin correlation.

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We encounter here an example of a contextual subspace. Our rank-two quantum register has four basis states, but only two of these are needed to describe the spin dynamics. For example, semi-unitary evolution from stage Σ_0 to stage Σ_1 is given by

$$\begin{aligned}\mathbb{U}_{1,0}\widehat{\mathbb{A}}_0^1\mathbf{0}_0 &= a_1\widehat{\mathbb{A}}_1^1\mathbf{0}_1 - b_1^*\widehat{\mathbb{A}}_1^2\mathbf{0}_1, \\ \mathbb{U}_{1,0}\widehat{\mathbb{A}}_0^2\mathbf{0}_0 &= b_1\widehat{\mathbb{A}}_1^1\mathbf{0}_1 + a_1^*\widehat{\mathbb{A}}_1^2\mathbf{0}_1,\end{aligned} \tag{19.10}$$

and similarly for evolution to stage Σ_2.

It is not hard to see that in this case, the required temporal correlation coefficient $C_{1,0}$ is given by $C_{1,0} = 2|a_1|^2 - 1$, exactly as above. Our conclusion is that, as elsewhere, QDN reproduces the standard QM results without any difficulty. Note that the bitification process appears to make QDN quantum registers unduly larger than the standard QM counterparts. That is not seen as anything other than an advantage, because it permits the modeling of more situations, such as labstates of signality greater than just one.

19.6 The Leggett–Garg Correlation

From the above correlations, using Definition 19.2 we find

$$K = 2|a_1|^2 + 2|a_2|^2 - 2|a_1a_2 - b_1b_2^*|^2 - 1. \tag{19.11}$$

Now the parameters of an experiment, such as $a_1, a_2, \dots$ are under the control of the experimentalist. It is reasonable therefore to assume that the parameters a_1, b_1, a_2, and b_2 can be chosen to be whatever we wish, subject to the unitarity constraints $|a_1|^2 + |b_1|^2 = |a_2|^2 + |b_2|^2 = 1$.

Consider the reparametrization

$$\begin{aligned}a_1 &= \cos\theta_1, b_1 = \sin\theta_1 e^{i\phi_1}, \\ a_2 &= \cos\theta_2, b_2 = \sin\theta_2 e^{i\phi_2},\end{aligned} \tag{19.12}$$

where θ_1, θ_2, ϕ_1, and ϕ_2 are real. Now take $\phi_1 = \phi_2$. Plotting K as θ_1 and θ_2 range from $-\pi$ to π each gives Figure 19.2.

It is clear that there are values of the parameters where K exceeds the classical limit $+1$. In fact, the maximum value of K over the region shown is 1.5, in clear violation of the Leggett–Garg upper bound of one. This is an entirely nonclassical result that is the temporal analogue of the violation of Bell inequalities in experiments such as those of Aspect (Aspect et al., 1982) and others.

19.7 Understanding the Leggett–Garg Prediction

The predicted QM violation of the Leggett–Garg inequality is another demonstration of the thesis running throughout this book: that empirical truth is contextual and that QM is best regarded as a statement of the laws of entitlement, of what we can legitimately say about SUOs, rather than an explicit description of those SUOs.

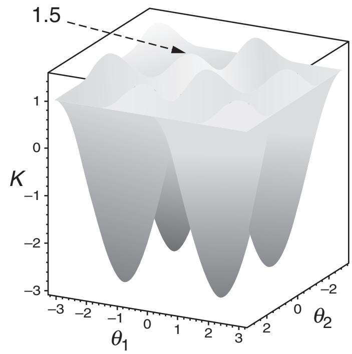

Figure 19.2. Values of K versus angle parameters θ_1, θ_2.

In view of the great predictive success of QM, it would be unwise to believe that we can replace it. At best, we can enhance it. But that does not answer the question, what is the origin of the discrepancy between the classical LG inequality and the predicted QM violation of it?

Our view is that there must be hidden assumptions somewhere that, if identified, would explain the discrepancy. Since QM has never yet been proved empirically incorrect while classical mechanics has, there must be something that we have missed in the way we think classically. It is not QM that is wrong, but our preconditioned classical way of thinking about physical reality that is not quite right.

The answer is given clearly by Wheeler's participatory principle that *if we have not actually done something, we should not make any undue assumption about it.*

With this in mind, consider the LG correlation K discussed above. It is calculated from three separate correlations that classically could be calculated from a single run. This is because of the assumed principle of noninvasive measurability that is one of the standard assumptions of classical mechanics.

But, according to the requirements of quantum experimentation, the correlation $C_{2,0}$ cannot involve any observation at stage Σ_1. The evolution operator $U_{2,0}$ has to be applied on the strict understanding that no attempt is made to extract information between initial stage Σ_0 and final stage Σ_2.

The essential fact is that the principle of noninvasive measurability cannot be assumed to hold in QM (although as discussed in Chapter 25, there are some experiments that can be described in such terms). Classically, our normal human conditioning is to think automatically that the stage-Σ_1 signals observed in the measurement of the correlation $C_{1,0}$ should play a role in $C_{2,0}$. But how can they? They are not observed when the SUO evolves undisturbed from stage Σ_0 to stage Σ_2.

In order to calculate the LG quantity K, we would have to perform three separate "subexperiments," one for each of the three correlations involved. These

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are separate experiments with separate contexts. Classically, we could believe that we could get away with just one experiment. Quantum mechanically, we have to recognize context is critical and perform three subexperiments.

The explanation then is that in the $C_{2,0}$ subexperiment, there is a lack of which-path information at stage time Σ_1, analogous to the architecture of the double-slit experiment, where an interference pattern on a screen is observed provided no attempt is made to determine through which slit the particle had gone.

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