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Enhancement of the prediction of geophysical time series by modifying the regularity structure of a signal

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Abstract. The time series of geophysical data are chaotic and, on the other hand, extremely noisy. Thus, though there are a number of advanced methods of chaotic time series prediction, the improvement of geophysical data is crucial to succeed. Mainly, it is connected with low determinism of such time series. The improvement procedure, we are about to represent, does not directly enhance deterministic component, but regularity properties of a signal, and, therefore, causes the increase of a deterministic portion in data. The main advantage consists in the fact that the method preserves the initial structure (information) of a time series, while effectively reduces noises, even knowing nothing about its actual nature.

Nowadays many noise reduction methods are known, see for example Kostelich & Yorke (1998), Grassberger et al. (1993), Parlitz & Bröcker (2001). The most of them assume that the observed signal Y can be presented in the form of composition $Y = F(X, B)$ of a clean signal X and a noise term B. As a rule, the noise term B is assumed to be some, for example, gaussian stochastic process which does not depend on X and X in its turn is considered as a piecewise-smooth function of class $C^n, n \geqslant 0$. Finally, F is a linear or quasi-linear functional dependence. Unfortunately, all these assumptions are too strict for many time series, and for geophysical data especially. Data of such kind are usually the result of interference of large number of uncontrollable processes, for which correct models do not exist at all. Neither the functional relation $F(X, B)$ between the signal and noise, nor the noise nature are known. In this work we apply a more general approach to such data, which has been offered in Lévy Véhel & Lutton (2001). The method idea is based on properties of time series smoothness (Hölder regularity). Measured regularity is increased knowingly by a controllable constant value. After that a new signal with the obtained (prescribed) regularity is constructed. A constructed signal is considered as a clean one. Thus, the noise reduction problem comes to the procedure of reconstructing new signal on the basis of enhanced (prescribed) regularity (Daoudi et al. (1998)).

Let us briefly remind definition of Hölder regularity exponent. A function $f(x)$ is a continuous one at x_0, if $|f(x) - f(x_0)| \to 0$ as $x \to x_0$. A continuity corresponds to the *regularity index* $\alpha = 0$. Similarly, $f(x)$ is differentiable if there exists a linear function (for example, polynomial) P such that $|f(x) - P(x - x_0| \to 0$ faster than $|x - x_0|$ as $x \to x_0$. This case corresponds to a regularity index $\alpha = 1$. In general (Arneodo et al. (1997)), let α be a positive real number and $x_0 \in R$ and a function $f(x) : R \to R$. Then α is called the regularity index of f at x_0, if there are a constant C and a polynomial $P(x)$ of order smaller than α so that, for all x in a neighborhood of x_0: $|f(x) - P(x - x_0| \leqslant C|x - x_0|^\alpha$. *Hölder exponent* $\alpha_f(x_0)$ is a supremum of all α such that previous inequality holds. Since α_f is defined at each point x, we may associate to $f(x)$ a function $x \to \alpha_f(x)$, which measures evolution of its regularity. So, increasing or shifting α_f by a positive constant may be interpreted as a "smoothing" procedure.

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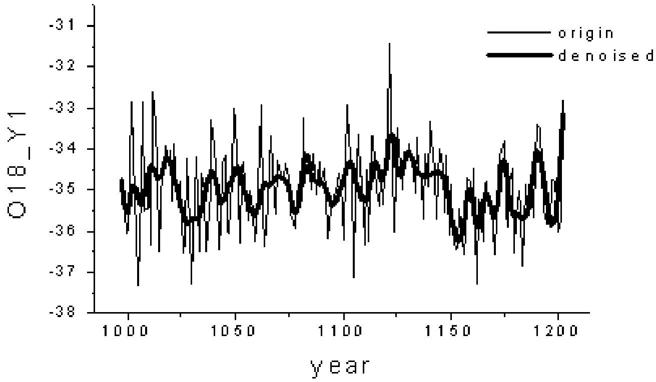

Figure 1. The example of enhancement of the oxygen isotope ratio time series ($\delta = 1.5$)

The numerical estimation of the Hölder exponent is implemented through wavelet decomposition of the original time series: $X(t) = \sum_{j,k} x_{jk}\psi_{jk}(t)$, where x_{jk} are wavelet decomposition coefficients of time series $X(t)$ and $\psi_{jk}(t)$ are wavelet basis functions. So, the Hölder exponent can be estimated using $|x_{j,k}| \leqslant C2^{-j(\alpha_X+\frac{1}{2})}$, where C is a constant, parameters j and k correspond to the values of shifting and scaling of a wavelet function.

Now, the task of signal enhancement can be accomplished in the following way. Let X be an original (true) signal, while Y is an observational time series corrupted by noises. It is necessary to find a signal $\tilde{X}$ of smoothed regularity structure so as to meet the following conditions: 1) $\tilde{X}$ and Y must be close in L^2 metric space, 2) the function of local regularity $\alpha_{\tilde{X}}$ must be prescribed. If the function α_X is known, then we suppose $\alpha_{\tilde{X}} = \alpha_X$. If we do not know α_X, we estimate α_Y. Next we determine $\alpha_{\tilde{X}} = \alpha_Y + \delta$, where a value of $\delta > 0$ is chosen according to practical expediency. Now, we need merely reconstruct $\tilde{X}$ basing on its prescribed regularity. The latter is realized using an inverse wavelet decomposition, i.e. we obtain enhanced signal $\tilde{X} = \sum_{j,k} \tilde{x}_{jk}\psi_{jk}(t)$ from estimation of wavelet coefficients $\tilde{x}_{jk}$, by solving some optimization problem that is a direct formalization of given above conditions (Lévy Véhel & Lutton (2001)).

For numerical experiments, we have used the time series of abundance ratio of oxygen isotope in ice cores of Greenland (figure 1), dated from 8065 BP to 1987 AD (Stuiver et al. (1995)). Since this time series was not measured directly, represents complex behavior and contains information distorted due to influences of a number of another natural processes, we could evenly regard all these negative factors as an impact of some unknown noise and then have applied the enhancement procedure. In general, we have observed the improvement of dynamical properties of the time series and, hence, its predictability.

Acknowledgements

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