The main table gives the sines, cosines, tangents and cotangents from 0° to 45° at intervals of 10″. This of course covers the quadrant conveniently. First differences are provided, and the proportional parts of the differences on a page are given in a margin to the table on that page. Auxiliary tables provide the cotangent for the first three degrees for every second, and the sines and tangents for the first degree at intervals of 10″, as in the main table, but in this case six significant figures are given, irrespective of the number of zeros following the decimal point.

Dr. Comrie has supplied a list of eight errors, not all serious, and this has been inserted ; so short a list from so severe a critic indicates a high standard of reliability.

The tables are convenient for use. Aesthetically they fall short of elegance by what seems to me to be too frequent a use of " rules ", giving the page a crowded, jig-saw appearance ; but this will not be considered a fault by all table-users, and matters little since the fount is of clear, old-style figures.

T. A. A. B.

CORRESPONDENCE.

To the Editor of the *Mathematical Gazette.*

Dear Sir,—A personal reference by Professor G. H. Livens, on p. 10 of the February *Gazette* affords me an opportunity of paying a tribute to the late Mr. William Welsh of Jesus College ; an opportunity which I cannot resist. Mr. Welsh was a very great teacher, and I have always considered myself fortunate to have been one of his pupils. He succeeded Mr. William Walton (eighth wrangler, 1836) as lecturer at Magdalene, in my first term of residence. As we were not then combined with any other College for lectures, he had the difficult task of covering the schedule for the tripos (the old Part I) by giving the men of each year three lectures weekly, each lecture lasting only 45 minutes. His method, which worked well with a small class, was to sit at a table surrounded by his class and write out his lecture (using no notes) in an easily legible hand, at the same time dictating what he wrote, and at the end giving what he had written to one of the class. Though he covered a vast amount of ground, there was not time for everything ; and I have no doubt that Professor Livens got more from him at Jesus College, some twenty years later, including the interesting application of the method of energy to problems of changing mass, which I had not seen until I read the February *Gazette*. Yours faithfully, A. S. Ramsey.

THE INTEGRAL DEFINITION OF THE LOGARITHM.

To the Editor of the *Mathematical Gazette.*

Dear Mr. Editor,—Will you allow us to comment on Mr. Tuckey's note in the *Gazette* of February, 1945?

No doubt it is true that Hardy's *Pure Mathematics* (1st edition, 1908) has been a strong influence in favour of the integral definition of the natural logarithm. But Prof. Hardy would be the last to claim either priority or a monopoly in this matter. T. J. I'A. Bromwich, in the preface to his *Infinite Series* (1907), writes : " It will be noticed that from Art. 11 onwards free use is made of the equation $\frac{d}{dx}\log x = \frac{1}{x}$, although the limit of $\left(1+\frac{1}{n}\right)^n$ from

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which this equation is commonly deduced is not obtained until Art. 57. To avoid the appearance of reasoning in a circle, I have given in Appendix II a treatment of the theory of the logarithm of a real number starting from the equation $\log a = \int_1^a dx/x$. The use of this definition of a logarithm goes back to Napier, but in modern teaching its advantages have been overlooked until comparatively recently."

Bromwich adds references to a paper by Bradshaw dated 1903 and to Osgood's *Lehrbuch der Funktionentheorie*. Reference may be made here also to the English translation of Felix Klein's *Elementary Mathematics from the Advanced Standpoint* (analysis volume, p. 155).

We mention this as a matter of historic interest. It does not of course affect Mr. Tuckey's argument.

The teacher who accepts the recommendation to use the integral definition, without conviction, merely from deference to the authorities who have made it, and does not otherwise alter his teaching arrangements, will probably find himself in the ridiculous position suggested by Mr. Tuckey of saying to his pupils : Come, now, let's pretend that we cannot differentiate $\log x$. This will be because he has proceeded too far in his course of differential calculus before starting integration. The serious objection to Mr. Tuckey's differentiation of a^x is not at all that it requires more intuitive ideas about the gradient of a curve, but that a^x is being differentiated simply because the teacher is scratching about for something to differentiate: not because he wants to differentiate it in connexion with some problem that arises naturally in a logical or a practical course of mathematics.

When Mr. Tuckey says that differentiation comes before integration, he cannot mean that *all* differentiation comes before *any* integration, although that may have been approximately true in his school-days and ours. But nowadays the differentiation that precedes applications of integrals to areas and volumes is generally limited to powers of x. It need not even include the differentiation of $\sin x$, although the trigonometrical functions are more likely to arise in elementary work than logarithmic or exponential functions. Thus the pupil is brought, at an early stage, up against the missing link in the formula

$$\int x^n \, dx = x^{n+1}/(n+1), \quad n \neq -1,$$

and it is then easy and can be exciting to investigate $\int_1^a dx/x$.

Another disadvantage of proceeding too far with differentiation is that it tempts the pupil, when he eventually reaches integrals, to regard integration as a purely tentative process to be carried out by guessing when such and such an answer was obtained to a differentiation. Although it is true that there is a tentative side to integration, it is far more important for the pupil to realise that there is a systematic side. Prof. Hardy's influence has also been in this direction, notably in his tract on *Integration* in the Cambridge series of tracts.

It must be admitted that the integral approach to logarithms loses its interest if the pupil knows the answer beforehand. But he ought not to know it. The ideas on which the method is based are not difficult, and should be introduced when the integral of $1/x$ or the natural logarithm first appears.

Nobody can describe the method which Mr. Tuckey advocates as exciting, and at this stage it cannot be given any purely theoretical support. On the other hand, the integral method, apart from its interest, also serves as an introduction to an important form of mathematical procedure, and so it appears to us to possess just that "outlook value" that should be a dominating consideration in the choice of subject-matter for mathematical sixth forms. Yours, etc., C. V. DURELL and A. ROBSON.

POETS' CORNER.

To the Editor of the *Mathematical Gazette.*

SIR,—For some time I have collected specimens of verse (mostly light) on mathematical and astronomical themes. I should be glad if you could spare a little space to enable me to ask members of the Association to help in retrieving scattered masterpieces. It will be enough to give references to the books or journals in which they can be found, and the smallest contribution will be gratefully acknowledged. Yours faithfully, A. P. ROLLETT,

4 Oak Lane, Sevenoaks, Kent.

TEXTBOOKS AS TEACHERS.

To the Editor of the *Mathematical Gazette.*

SIR,—Would that they were! I speak of university student textbooks. Roughly, these are either class-books, designed as aids to the training of examinands in classes; or they are lectures without a lecturer.

In no case that I know of does any one of them, modern or not, solicitously regard and properly provide for the needs of the learner when he is alone with his books. Yet during that part of his learning time the learner has no help within reach except his books. And it is then that there occurs, for very many learners, an appalling waste of their time.

Responsibility for this wrong done to them divides between authors and publishers. A textbook, like a teacher, is under obligation to answer every question which a learner legitimately puts to it. Further, he must not be needlessly delayed in getting his answer.

I have not space to set out separately here the multiple unnecessary compulsions to waste of time enforced upon a learner in solitude by every one of these textbooks. Each separate unnecessary one of these is in itself a wrong. The accumulated total of them piles up to a great evil. This evil is the more cruel in that the injuries which it inflicts are not merely according to weakness, but become harsher in steeply rising proportion to it. The nature and the hurt of each of these injuries I know by experience. The feelings of the weak concerning them I know from within, and I will voice them.

I am ready to justify my indictment by as many instances as may be desired from well-known current textbooks. The worst of the injuries arise from gross neglect of the mere mechanism of presentation, especially from almost unbelievable deficiency in: (1) references back to the book itself; (2) indexes; (3) reference to ancillary books. The multitude of other injuries I must leave unclassified and uncounted.

Finally, and it would be my best contribution, I believe that I can point out a remedy that would be efficacious, unobjectionable to authors, and yet *cheap* to publishers. Yours, etc., OWEN MADDEN.

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