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2024 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Francisco, California
Joint Mathematics Meeting
January 3-6, 2024

The 2024 Winter Meeting of the Association for Symbolic Logic was held January 3–6, 2024, in conjunction with the annual Joint Mathematics Meeting. The program committee consisted of Cameron Freer, Juliette Kennedy, Maryanthe Malliaris (chair) and Andrew Marks.

The ASL program included seven plenary speakers, a tutorial, a special session and a contributed paper session. The ASL Special Session *Descriptive Methods in Dynamics, Combinatorics, and Large Scale Geometry* was organized by Forte Shinko and Jenna Zomback. The two-part ASL Tutorial *Large cardinals, determinacy, and inner models* was given by John Steel. The plenary addresses at the meeting are listed below.

Matthew Harrison-Trainor (University of Illinois Chicago), *The complexity of classifying topological spaces.*

Åsa Hirvonen (University of Helsinki), *Games for measuring distance between metric structures.*

François Loeser (Pierre and Marie Curie University), *Model theory and non-archimedean geometry.*

Toby Meadows (University of California Irvine), *A moderate foundational argument for the generic multiverse.*

Dima Sinapova (Rutgers), *Combinatorial principles at successors of singular cardinals.*

Slawomir Solecki (Cornell University), *Descriptive set theory and generic measure preserving transformations.*

Mariana Vicaria (University of California Los Angeles), *Model theory of valued fields.*

Abstracts of the invited talks and the contributed talks by members of the Association for Symbolic Logic follow.

For the Program Committee
MARYANTHE MALLIARIS

Abstract of invited tutorial

► JOHN R. STEEL, *Large cardinals, determinacy, and inner models.*
Department of Mathematics, University of California Berkeley.
E-mail: coremodel@berkeley.edu.

The most fruitful way to strengthen ZFC, the standard set-theoretic foundation for mathematics, is to strengthen its axiom asserting that there are infinite sets. *Large cardinal hypotheses* do this. Much of their strength is due to their consequences concerning the

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existence of winning strategies for infinite games of perfect information. Such *determinacy principles* can be seen as strengthenings of the logical law of the excluded middle.

Large cardinal and determinacy hypotheses fall into hierarchies based on their logical strengths. At the lower levels, where we understand the situation pretty well, the two hierarchies are tightly interconnected. This connection is mediated by the theory of canonical inner models for large cardinal hypotheses.

In the first lecture we shall state the main results leading from large cardinals to determinacy, and outline some of their proofs. Here a regularity property of sets of real numbers that is more fundamental than determinacy, namely, being *homogeneously Suslin*, plays a key role. In the second lecture, we shall focus on the theory of canonical inner models for large cardinal hypotheses. Here again homogeneously Suslin sets play a key role, as "certificates" that a given inner model is canonical. Inner model theory is the site of the main open problems concerning the connection between large cardinals and determinacy at higher levels, and we shall conclude by stating some of them.

We hope to make both lectures accessible to a broad audience of mathematicians.

Abstracts of invited plenary lectures

► MATTHEW HARRISON-TRAINOR, *The complexity of classifying topological spaces.*
Department of Mathematics, Statistics and Computer Science, University of Illinois Chicago, Chicago, Illinois.
E-mail: mht@uic.edu.

Given a topological space, how difficult is it to characterize it up to homeomorphism? The unit interval, for example, is the unique metrizable continuum with exactly two non-cut points. We would like to, first of all, measure the complexity of such a characterization, and second of all, prove that it is best possible. I will talk about these problems both for particular examples and as a more general theory. This is the topological analogue of the theory of Scott sentences and Scott complexity for countable structures.

► ÅSA HIRVONEN, *Games for measuring distances between metric structures.*
Department of Mathematics and Statistics, University of Helsinki, Pietari Kalmin katu 5, 00014 Helsinki, Finland.
E-mail: asa.hirvonen@helsinki.fi.

In classical model theory, Ehrenfeucht-Fraïssé games are used to study similarities between structures. This can, e.g., be applied to prove inexpressibility results (finite games) or for building Scott sentences capturing isomorphism of countable structures (infinite or dynamic games).

Several authors have considered Ehrenfeucht-Fraïssé games – or their cousin, back-and-forth systems – on metric structures. In a metric setting new phenomena arise, related to various forms of approximation, both within structures and between structures. In a game setting these show up as approximate answers to moves, and approximate preservation of formulae. It turns out that elementary equivalence up to a given quantifier depth can be subdivided by the accuracy and steepness of preserved formulae. These notions reconstruct the ability to prove inexpressibility results, but also enable capturing new phenomena, such as given distances between models with respect to various natural pseudometrics.

► FRANÇOIS LOESER, *Model theory and non-archimedean geometry.*
Department of Mathematics, Sorbonne University, Paris, France.
E-mail: francois.loeser@imj-prg.fr.

We shall present an overview of applications of Model Theory of valued fields to Non-Archimedean Geometry. We will start with the Bieri-Groves theorem and then focus on more recent results obtained in collaboration with several coauthors (A. Ducros, E. Hrushovski and J. Ye).

► TOBY MEADOWS, *A modest foundational argument for the generic multiverse.*
Logic & Philosophy of Science, University of California, Irvine, USA.
E-mail: meadadwst@uci.edu.

The generic multiverse is a system of set theoretic universes that is, roughly speaking, closed under the operations of generic extension and its inverse. The underlying idea can be axiomatized and the resultant theory MV might be thought of as a competitor to ZFC. In this talk, I want to make a modest philosophical argument for the value of MV as a foundation for set theory. I'll start by discussing the way in which ZFC provides a satisfying foundation for almost all of mathematics. I'll then argue ZFC does not provide a similarly satisfying foundation for contemporary set theory. Finally, I'll argue that MV can fulfill this role in a very natural manner.

► DIMA SINAPOVA, *Combinatorial principles at successors of singular cardinals.*
Department of Mathematics, Rutgers University, New Brunswick, New Jersey.
E-mail: ds2005@math.rutgers.edu.

Given a singular cardinal κ, mutual stationarity asserts that sequences of stationary subsets of regular cardinals with limit κ have a "simultaneous witness" for their stationarity. This was first defined by Foreman and Magidor in 2001, who showed it holds when restricted to points of cofinality ω. The case for higher cofinalities remained open until a few years ago Ben Neria showed its consistency from large cardinals. In Ben Neria's model SCH naturally holds at $\aleph_\omega$.

We show that we can obtain mutual stationarity at $\langle \aleph_n \mid n < \omega \rangle$ for any fixed cofinality together with the failure of SCH at $\aleph_\omega$ (joint with Will Adkisson). Then we will discuss what this means for various combinatorial principles at $\aleph_{\omega+1}$.

► SLAWOMIR SOLECKI, *Descriptive set theory and generic measure preserving transformations.*
Department of Mathematics, Cornell University, Ithaca, NY.
E-mail: ss3777@cornell.edu.

One of the areas of interest of Descriptive Set Theory is dynamics of Polish groups, that is, groups carrying a group topology that is separable and completely metrizable. Such groups are not, in general, locally compact. Therefore, in studying their dynamics, classical methods relying on Haar measure are not available. These methods can sometimes be replaced by descriptive set theoretic tools.

I will describe how the descriptive set theoretic point of view led to a recent answer to an old question in Ergodic Theory. The question lies within a long-established theme, going back to the work of Halmos and Rokhlin, of investigating generic measure preserving transformations. The answer to the question rests on an analysis of unitary representations of a certain non-locally compact Polish group that can be viewed as an infinite dimensional torus.

► MARIANA VICARÍA, *Model theory of valued fields.*
Department of Mathematics, University of California, Los Angeles.
E-mail: mariana_vicaria@berkeley.edu.

Model theory is a branch of mathematical logic that studies *structures* (that is sets equipped with relations, functions and constants) and their *definable sets*, that is the subsets of various cartesian powers that can be defined in terms of these distinguished constants, relations and functions via the logical connectives and quantifiers. There is a more general class of

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subsets that one could study, called the *interpretable sets*, obtained by taking the quotient of a definable set by a definable equivalence relation. A natural question is: given a structure can one classify the interpretable sets in that structure?

A valued field is a field K equipped with a distinguished subset $\mathcal{O}$, a valuation ring[1]. Examples of valued fields are the p-adic field $\mathbb{Q}_p$ or the Laurent series over the complex numbers $\mathbb{C}((t))$. Given $\mathcal{O}$ a valuation ring of a field and $\mathcal{M}$ its maximal ideal, we commonly refer to the additive quotient $\mathcal{O}/\mathcal{M}$ as the *residue field*, while the multiplicative quotient $K^{\times}/\mathcal{O}^{\times}$ is an ordered abelian group and it is called the *value group*.

One of the most striking results in the model theory of valued fields is the *Ax-Kochen/Ershov theorem* which roughly states that the first order theory of an unramified henselian valued field is completely determined by the first order theory of its residue field and its value group. A principle follows from this theorem: the model theory of valued field is controlled by its residue field and its value group.

In this talk I will make a brief description of valued fields and their model theory. I'll present how the problem of classifying interpretable sets in henselian valued fields can be approached in an Ax-Kochen style: What obstructions come from the residue field? and from the value group? I will conclude presenting the classification of the interpretable sets in valued fields obtained in joint work with Rideau-Kikuchi, building on [1] and [2].

[1] M. Hils and S. Rideau-Kikuchi, *Un principe D'Ax-Kochen-Ershov imaginaire*, preprint, arXiv:2109.12189.

[2] M. Vicaría, *Elimination of Imaginaries in* $\mathbb{C}((t))$, Journal of the London Mathematical Society, Vol. 108, 2, (2023), 482-544

Abstracts of contributed talks

► FRED HALPERN, *Preservation theorems via Smullyan clashing tableau.*
Royal Path to Math, 6131 Melody Lane, Dallas, TX 75231 USA.
E-mail: fredhalp@gmail.com.

We adopt Smullyan's Clashing Tableaux to prove preservation theorems in a unified manner. An Algebraic Description D is a set of structure names and a set of relation names along with a set of "conditions".

Description D is simple if it describes two structures (A and B) and a relation μ between them along with possible conditions: μ is domain-onto, μ is range-onto, and μ preserves the formulas Φ. We associate $\exists$ with domain-onto and $\forall$ with range-onto. D_Q denotes the quantifiers associated with the onto conditions of D.

Theorem 1. *The formulas preserved under simple D (relative to Σ) are the closure of Φ under D_Q, $\vee$, and $\wedge$.*

The method parallels Natural Deduction. Just as we systematically search for a contradictory tableau to show a sentence is unsatisfiable, we search for a clashing tableau system C which shows non-preservation is unsatisfiable. We recursively compute from C the preserved formula. A key observation is that a tableau for Σ yields a weakening sentence σ such that $\Sigma \vdash \sigma$. The weakening has Craig interpolation theorem traits.

► SHAY ALLEN LOGAN, *Varieties of variable sharing or: how I stopped worrying and learned to love nonuniform substitutions.*
Department of Philosophy, Kansas State University, Manhattan KS, USA.
E-mail: salogan@ksu.edu.

[1] Let K be a field, a subring $A \subseteq K$ is said to be a valuation ring of K if for any element $x \in K\backslash\{0\}$ either $x \in A$ or $x^{-1} \in A$.

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Relevance and variable sharing have gone hand-in-hand since the very start. The relationship in fact predates the seminal texts of the movement, as one can see by comparing the publication dates of [1] and [2].

But both both parties to this marriage have changed over the decades, and the marriage looks quite different now than it did when it first began. In particular, variable sharing results no longer play at being quite so hard to get. This is the result of a novel way to prove strong variable sharing results. The key bit (introduced in [3]; [4]) is the use of nonuniform substitutions. It turns out that this key bit is key in more than one way: not only does it unlock easy proofs of strong variable sharing results, it also opens a door behind which hide a plethora of novel and quite unanticipated forms of variable sharing as well. For each of these forms of variable sharing, a proof that is not interestingly different from the proof of the main result in [3] shows that weak-enough logics exhibit that form of variable sharing.

Given all this, the goal of this talk is twofold. First, I'll survey the state of the art in order to show you how to use nonuniform substitutions to achieve profit and fame. After that, I'll try to convince you that you shouldn't feel bad about doing so.

[1] Nuel D. Belnap, *Entailment and relevance*, ***Journal of Symbolic Logic***, vol. 25 (1960), no. 2, pp. 144–146.

[2] Alan Ross Anderson and Nuel D. Belnap, *Entailment: The Logic of Relevance and Neccessity, Vol. I*, ***Princeton, Princeton University Press (1975)***.

[3] Shay Allen Logan, *Depth Relevance and Hyperformalism*, ***Journal of Philosophical Logic***, vol. 51 (2022), no. 4, pp. 721–737.

[4] ———, *Correction to: Depth Relevance and Hyperformalism*, ***Journal of Philosophical Logic***, vol. 52 (2023), no. 4, p. 1235.

► NOAH SCHWEBER, *The Harrison order as an ultraproduct.*
Proof School, 973 Mission St., San Francisco, CA 94103.
E-mail: schweber@berkeley.edu.

The Harrison order $\mathcal{H} \cong \omega_1^{CK}(1+\eta)$ is an important (counter)example in computable structure theory. We will show how $\mathcal{H}$ can be construed as a kind of "admissible ultraproduct" of computable ordinals. This provides an interesting parallel with the behavior of ultraproducts with respect to countably complete ultrafilters in set theory, and strongly contrasts with a more-studied effective analogue of ultraproduct and ultrapower constructions, the "cohesive product/power." We will examine the basic properties of these ultraproduct-like constructions, and highlight some questions they raise about structures of high Scott rank.

► RUSSELL STETSON, *Characterizing sofic groups.*
Department of Mathematics, Rutgers University, Hill Center, 110 Frelinghuysen Rd, Piscataway, NJ 08854, USA.
E-mail: rps132@scarletmail.rutgers.edu.

For $n \geq 1$, let S_n be the symmetric group on $\{1, 2, \cdots, n\}$ equipped with the normalized Hamming distance metric d_n. Then a group G is said to be *sofic* if for every finite subset $F \subseteq G$, and every real $\varepsilon > 0$ there exists an injection $\varphi : F \to S_n$ for some $n \geq 1$ such that whenever g, h, $gh \in F$, then $d_n(\varphi(gh), \varphi(g)\varphi(h)) < \varepsilon$; if $1_G \in F$ then $d_n(\varphi(1_G), 1) < \varepsilon$; and for some fixed $c > 0$ if $g \neq h \in F$ then $d_n(\varphi(g), \varphi(h)) \geq c$.

It is unknown whether every group is sofic. In the group theoretic literature, sofic groups are usually characterized in terms of embeddings into metric ultraproducts of finite symmetric groups. It is natural to ask whether there is a characterization in terms of the more concrete notion of a metic reduced product of finite symmetric groups. In more detail, let $P = \prod_{n\geq 1} S_n$ be the full direct product and let N be the normal subgroup of elements $(\pi_n) \in P$ such that $d_n(\pi_n, 1) \to 0$ as $n \to \infty$. Then the metric reduced product is the quotient $P_0 = P/N$.

We have shown that it is neither provable nor disprovable from the *ZFC* axioms of set theory that if G is a group such that $|G| \leq 2^{\aleph_0}$, then G is a sofic group if and only if G embeds into P_0.

Abstracts of talks presented by title

► JOACHIM MUELLER-THEYS, *Buchholz Quotients.*
Independent; Heidelberg, Germany.
E-mail: mueller-theys@gmx.de.

We may think of *functions* as valuations. As is well-known, any $f\colon M \to N$ induces the unique *equi-valence* $\equiv_f$, permitting to define equivalence classes depending on representatives $a \in M$. We now define these classes (and $\emptyset$) in the absolute way by means of values $v \in N$. Both generalises our *exact definitions of the chemical elements* through the proton-number function.

I. Values bundle arguments. Accordingly, the *Buchholz quotient* $\langle v\rangle_f := \{a \in M : f(a) = v\}$ is the v-bunch. $\langle v\rangle_f = \emptyset$ iff $v \notin f[M]$. *The set* $\langle M\rangle_f := \{\langle v\rangle : v \in f[M]\}$ *of all (non-empty) bunches partitions* M: (i) If $\langle v\rangle \in \langle M\rangle$, $\langle v\rangle \neq \emptyset$; (ii) Since $a \in \langle fa\rangle$, $a \in \bigcup\langle M\rangle$; (iii) Different bunches are disjoint: if $a_0 \in \langle fa\rangle, \langle fb\rangle$, $fa = fb$, whereby $\langle fa\rangle = \langle fb\rangle$.

We define $a \sim_v b$ by $a, b \in \langle v\rangle_f$. $a \sim_v b$ is symmetrical and transitive, but *not* reflexive generally. Eventually, $a \equiv_B b$:iff $a \sim_v b$ for some $v \in N$.

II. $a \equiv_f b$:iff $f(a) = f(b)$. $\equiv_f$ is an *axiomatic equivalence relation*, viz. reflexive, symmetrical, and transitive. $[a]_f := \{b \in M : b \equiv_f a\}$ is the *equi-valence class to which* $a \in M$ *belongs*. $[a]_f = [b]_f$ *if and only if* $a \equiv_f b$. The quotient set $[M]_f := \{[a]_f : a \in M\}$ partitions M. Since $[a] = [a']$ implies $f(a) = f(a')$, $F([a]) := f(a)$ is well-defined. Since $F : [M] \to N$ is injective, the values $F([a])$ *characterize* the classes $[a]$. (Cf. "Equivalence", The Bulletin of Symbolic Logic 28 (2022), pp. 564–5.)

III. $[a]_f = \langle v\rangle_f$ *if and only if* $f(a) = v$. Particularly, $[a] = \langle f(a)\rangle$. Consequently, $[M]_f = \langle M\rangle_f$. Moreover, $a \equiv_f b$ *iff* $a \equiv_{\mathrm{B}} b$.

Let, for example, M be some set of persons to which f assigns their ages, including Alex, 27. Then [Alex] $= \langle 27\rangle$. Further applications arise (height, weight, ...).

IV. Let $M := \mathcal{A}$ be any set of (chemical) atoms. Furthermore, $f = \pi$ assign to $A \in \mathcal{A}$ its number of protons. Then $\langle 1\rangle_\pi$, $\langle 2\rangle_\pi$, $\langle 3\rangle_\pi$, ... define the chemical elements independently of representatives, whereas $[A]_\pi$ is the element to which A belongs. $\langle 1\rangle$ e. g. is also called hydrogen—logically posteriorly.

Atoms may be analyzed as triples $A := (P, N; E)$, where $P \neq \emptyset$, N, E be finite sets of protons, neutrons, electrons resp. Then $\pi(A) := |P|$ further specifies $\langle n\rangle_\pi$. $\nu(A) := |N|$ and $\varepsilon(A) := |E|$ make possible exact definitions of (atomic) *ions* by $\varepsilon(A) \neq \pi(A)$ and of the *isotope* of the n-th element with $m \geq 0$ neutrons as $\langle m\rangle_\nu \,/\, \langle n\rangle_\pi := \{A \in \langle n\rangle_\pi : \nu(A) = m\}$. For instance, $\langle 2\rangle_\nu \,/\, \langle 1\rangle_\pi$ is too called tritium.

Notes. These considerations contain advances with respect to the *Logic Colloquium 2023* abstract "A Mathematical Model of the Atom" (in the "Book of Abstracts" and to appear in BSL) and online talk "Exact Definitions for Chemical Elements". We add Yuri A. Shukolyukov, Giovanni Duca, Francesco A. Genco, Laurent Dubois, Michèle Friend to the mentioned.