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Multivariate Bonferroni-Type Inequalities and Optimality.

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This thesis is concerned with multivariate Bonferroni-type inequalities and their optimality. We derive bivariate Sobel-Uppuluri-Galambos type bounds and present bivariate optimality theory which is a new development in the analysis of multivariate bounds.

In Chapter 1 we set the scene, outlining the development of bivariate Bonferroni-type bounds and surveying univariate and bivariate optimal upper and lower bounds. The investigation on optimality in later chapters is motivated by the fact that even though the use of optimality appears in the literature, a clear definition of optimality is lacking.

Chapter 2 aims at the extension of univariate Sobel-Uppuluri-Galambos bounds to the bivariate case. We introduce a new idea, *marginal Bonferroni summation*, with which results sharper than Galambos and Lee [**1, 2**] bounds are produced.

Chapter 3, the foundation of bivariate optimal bounds theory, firstly addresses three notions of bivariate optimality: Fréchet optimality, linear optimality and one in linear programming language. Attention is devoted to seeking the relation between these definitions, investigating the existence of each kind of optimality, and the practicalities of assessing the associated optimal bound when it exists. An explicit expression of a Fréchet optimal upper bound is formed. Chapter 4 centres on applying linear programming theory to construct Fréchet optimal lower bounds.

Chapter 5 develops a bivariate non-Bonferroni-type upper bound, a hybrid upper bound, and demonstrates that in some cases, bivariate hybrid bounds may be sharper than bivariate Fréchet optimal Bonferroni-type upper bounds.

The thesis contains many numerical examples to illustrate different facets of optimality bounding theory. These involve the construction of appropriate probability spaces.

REFERENCES

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