The Theory of Lebesgue Measure and Integration, by S. Hartman and J. Mikusinski. Pergamon Press, Oxford, London, etc. 1961. 176 pages. 30 shillings (= $5.00).

The aim of this book is to set out the essential facts of Lebesgue measure and integration. No knowledge is assumed beyond the differential and integral calculus. Every graduate in mathematics should be acquainted with the concepts and theorems of this book, even if he has no special interest in the theory of real functions. Any worker in probability theory or in theoretical physics is likely to need the ideas and results of the Lebesgue theory.

We say at once that the book is well printed and attractively bound and that the text does cover what it sets out to cover. Any critical remarks that follow about particular features are to be read against this background of general satisfaction.

It is now sixty years since the ideas of a few French mathematicians and one Englishman (W. H. Young) put the theory of measure and integration on a firm basis; Lebesgue, who set out his work in the clearest way, is acknowledged as having played the biggest part.

A number of expositions of measure and integration have been written. When a new one appears, the reader asks whether it has any virtues that the previous ones lack. Here we must say that the presentation is inferior to that of some of the existing texts, and the authors and translator between them hardly attain the standard expected of them. There is an old-fashioned air about the book; most of it could have been written in 1910, and though there have been no revolutions since then, there have been advances in presentation.

The following specific points illustrate these criticisms. (1) In Chapter I there is no mention of the empty set. (2) The axiom of choice is mentioned only in reference to non-measurable sets. The authors should explain that it pervades the whole theory. A quite gratuitous freedom of choice is allowed on page 13. (3) The wording of the definition of the Lebesgue integral leaves the reader with the impression that the essential feature is the division of the y-interval instead of the x-interval. What is in fact essential is that the x-interval is divided into measurable sub-sets. (4) Absolute continuity (Chapter VII) should be envisaged as a property of set-functions rather than of point-functions. (5) The conditions for equality in Hölder's and Minkowski's inequalities should be given. (6) Chapter XIII on the Stieltjes integral is too sketchy to have value. (7) Exercises (with hints for solution) are necessary for the student in a text-book at this level; there are none.

There are numerous un-English touches, such as the use of irrevocable in the middle of page 42, differently than at the bottom of page 84, and intervene at the top of page 169. The article the is frequently omitted as in Theorem 6 on page 168.

J. C. Burkill

A Short Account of the History of Mathematics, by W. W. Rouse Ball. Dover Publications Inc., New York, 1960. xxiv + 522 pages. $2.00

The History of the Calculus and its Conceptual Development, by Carl B. Boyer. Dover Publications Inc., New York, 1959. 346 pages. $2.00.

The Dover series of inexpensive reprints of important books in mathematics, the sciences, and many other fields of learning, has long won widespread recognition by students and scholars alike. While some of the reprinted works are of recent origin, others date back as far as the middle of the 19th century.

In mathematics, the prospective buyer of a treatise such as, say, Cantor's "Transfinite Numbers" or Knopp's "Theory of Functions" usually knows what he is about to acquire: not a volume filled with the latest results of mathematical research, but a classic in its field. But when it comes to the history of science and mathematics, the situation is altogether different. Apart from perhaps a handful of specialists, books in these fields will be sold mostly to students, teachers, and interested laymen, who will use them as an important - or even as the only - source of information. Unfortunately, there seems to be a widespread belief that our knowledge in the history of the sciences, and particularly of mathematics, is rather static, so that a book written a generation ago will be able to inform us just as well as one published more recently. This is, of course, quite untrue. One need, for instance, only think of the most remarkable findings of the last 30 years concerning pre-Greek and medieval mathematics.

On these grounds this reviewer concludes that it would have been better not to reprint Ball's "History of Mathematics". When the 4th edition, which is reproduced here, was published in 1908, Ball remarked in a note: "No material changes have been made since the issue of the second edition in 1893." - There is no point in elaborating on the shortcomings of the book when compared with the present state of knowledge in this field.

The "History of the Calculus" by Professor Boyer was first published in 1939. A second printing in which, according to the author, "a few minor errors in the text have been corrected", appeared in 1949.