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Corrigendum: on the mean 3-rank of quadratic fields

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Professor Dongho Byeon (Seoul National University) pointed out a mistake in the statement of Theorem 1.2 in my paper [Bel99]. The mean value 1/2 should be replaced by 1/4 there and in the last formula on p. 8; the proof is otherwise correct. Explicitly, we have

$$\sum_{\Delta\in\Delta^+_{\mathrm{fund}}(X)} \frac{3^{r_3(-3\Delta)}-3^{r_3(\Delta)}}{2} \sim \frac{(H^- - H^+)X}{\zeta^2(2)} = \frac{X}{\pi^2},$$

$$\sum_{\Delta\in\Delta^+_{\mathrm{fund}}(X)} 3^{r_3(\Delta)} \sim \frac{4}{3} \sum_{\Delta\in\Delta^+_{\mathrm{fund}}(X)} 1 \sim \frac{4}{3}\times\frac{3X}{\pi^2},$$

where the first line appears on p. 8, and the second is the Davenport–Heilbronn theorem. Dividing the first relation by the second yields the result.

Using properly Cohen–Lenstra heuristics and Dutarte's conjecture, we are also led to this value 1/4 as follows.

Let $q := 1/3$ in what follows. For $|q| < 1$ and $n \geqslant 0$, one defines

$$(q)_n = \prod_{i=1}^{n}(1-q^i).$$

Summing heuristically over all possible 3-rank values r, we get

$$\begin{aligned}\sum_{\substack{\Delta\in\Delta^+_{\mathrm{fund}}(X)\\ \delta(\Delta)=0}} 3^{r_3(\Delta)} \Big/ \sum_{\Delta\in\Delta^+_{\mathrm{fund}}(X)} 1 &\overset{?}{\to} \sum_{r\geqslant 0} q^{-r} P(\delta=0 \mid r_3=r)P(r_3=r)\\ &\overset{?}{=} \sum_{r\geqslant 0} q^{-r}\times q^{r+1}\times\frac{q^{r(r+1)}(q)_\infty}{(q)_r(q)_{r+1}}\\ &= q,\end{aligned}$$

where we use [Bel99, Conjecture 3.2] and [CL84, Conjecture C9] in the second line, and a well-known q-identity in the last; see e.g. [CL84, Corollary 6.7]. Applying once again Davenport–Heilbronn's result, we expect a weighted mean value of $3q/4 = 1/4$, in accordance with the corrected theorem.

References

Bel99 K. Belabas, *On the mean* 3*-rank of quadratic fields*, Compositio Math. **118** (1999), 1–9.

CL84 H. Cohen and H. W. Lenstra, Jr., *Heuristics on class groups of number fields*, in *Number theory, Noordwijkerhout 1983 (Berlin)*, Lecture Notes in Mathematics, vol. 1068 (Springer, Berlin, 1984), 33–62.

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