Interpretations of Solar Oscillations

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Abstract

The current theoretical status of understanding solar oscillations is reviewed. Interpretation of the thousands of well-determined frequencies refines our knowledge of the composition and convection structure of the Sun, since its mass, radius, luminosity, and age are better known from other sources. Recent issues that have been discussed are the solar center structure, bearing on the missing solar neutrino problem, the convection zone helium content, validating helium settling by diffusion, the variations of the oscillation frequencies over the solar cycle, indicating cyclical structure changes in the very outer magnetic layers, and the fine structure splittings of mode frequencies, revealing the internal rotation. Our ability to match observed frequencies to now within only a few microhertz has been enhanced by the recently improved MHD equation of state and the new Livermore OPAL opacities. Thus solar oscillations not only reveal solar structure data, but also they guide improvements for stellar astrophysics material properties. A new discussion of current investigations of the convection zone helium abundance and its depth is presented.

1. Current issues in solar oscillation research

Solar oscillation frequencies allow us to probe the Sun to determine its composition, convection, and rotation structure. Other parameters that are usually sought in stellar astrophysics, such as mass, radius, luminosity, and age are all well known for the solar case. After a short introduction, this review considers only issues about the solar convection zone in any detail. The references and other recent papers cover the many other solar oscillation topics of current interest.

A very important problem has been the case of the missing neutrinos. Now with data from the ^{37}Cl, electronic, and ^{71}Ga detectors from the Homestake mine, the Kamiokande detector and from the SAGE and Gallex experiments, it seems that indeed the ^{8}B, ^{7}Be, and $p-p$ reaction neutrinos are deficient relative to predictions for so-called standard solar models. Though even today there are statements that our understanding of solar structure could be defective and the cause of the solar neutrino deficiency, that position is really out of the question. It has been so for over 25 years. The Mikeyev, Smirnov, and Wolfenstein effect, where electron neutrinos undergo a transformation by interactions with the electrons in the solar material to change their electron flavor to muon or tauon flavors, seems to be the correct reason why less than the full neutrino flux is detected here on earth.

A related question has been whether there might be weakly interacting massive particles (WIMPs) orbiting within the inner 10% of the solar mass and radius that can more efficiently transport energy than radiation processes. Then the solar center would be significantly cooled. Two recent papers, including one by Cox, Guzik, and Raby (1990), have shown that this is not the case, even though at least three earlier ones concluded these WIMPs could effect the desired cooling. With such a cool and almost isothermal central temperature, predicted low angular degree solar oscillation frequencies just do not agree with those observed.

The lifting of the frequency degeneracy for modes with different ℓ values, caused by modes traveling with both prograde and retrograde motions, can allow mapping of the internal solar rotation structure. Interesting details about rotation near the surface and within the convection zone have been discovered in the last 10 years, but probing the expected rapid central rotation has not yet been accurately done. The problem is mostly due to the fact that only the lowest angular degree modes have any amplitude in the inner 20% of the solar radius, and they have only a few $(2\ell+1)$ separate modes. Separations between the frequencies of these distinct modes need to be measured to small fractions of a microhertz out of about 3000 μHz, and that has not been achieved yet.

Attempts have been made to discover solar-like oscillations in other stars. The best case is for Procyon, but this bright F5 IV star is still not bright enough to acquire adequate statistical accuracy to identify individual modes. Probing stars like we do for the Sun will allow similar results and will increase our knowledge of stellar structures and evolution.

Predictions of solar oscillation frequencies require very accurate material properties for the model construction. Fortunately equations of state (Däppen et al., 1988) and opacities (Rogers and Iglesias, 1992) are now available, and solar oscillations have inspired further refinements in these data. Guzik and Cox (1991) discuss this matter.

I do not discuss here the recently confirmed discovery that very high angular degree oscillation mode frequencies vary with the solar cycle. This correlation helps to map magnetic field and density structure variations with time in the solar surface layers. Also I leave out a discussion of the highest observed frequencies that are not like the usual trapped modes in stars.

2. Standard solar models

Table 1 lists only a few of the recent standard solar models. The latest models are by Guenther et al. (1992). Some of these listed have been constructed only to discuss the solar neutrino problem, but most have been used to calculate solar oscillation frequencies. Note that the most recent models, which use the most modern material properties, derive a helium mass fraction in the primordial composition of close to Y=0.28. They all predict about 8 solar neutrino units (SNU or captures in 10^{36} atoms per second). The widely quoted, and very unreliable Turck-Chieze et al. results, give only 5.8 SNU. Our 11.4 SNU was found to be caused by a poor approximation for the equation of state, and now our best result is 8.5 SNU.

The solar radius is obtained by models with adjustment of the mixing length in the mixing length theory of convection. Recent models, using the best material properties, need a mixing length of about 2.0, but not much theoretical significance should be attached to this number, since it merely is an adjustable parameter. In section 4, I show how arbitrarily changing this value changes the convection zone depth and significantly changes predicted solar oscillation frequencies.

Table 1

Some Recent Standard Solar Models

Authors	Y	SNU	ℓ/H_p
Bahcall, Ulrich, 1988	0.271	7.9	–
Lebreton, Däppen, 1988	0.278	7.6	2.16
Guenther, Sarajedini, 1988	0.240	–	1.35
Lebreton, Berthomieu, Provost, Schatzman, 1988	0.287	8.0	2.18
Lebreton, Berthomieu, Provost, Schatzman, 1988	0.291	8.4	2.11
Turck-Chieze, Cahen, Casse, Doom, 1988	0.276	5.8	1.55
Christensen-Dalsgaard, Däppen, Lebreton, 1988	0.237	–	–
Korzennik, Ulrich, 1989	0.271	8.2	–
Guenther, 1989	0.282	–	1.25
Guenther, Jaffe, Demarque, 1989	0.28	–	1.24
Cox, Guzik, Kidman, 1989	0.291	11.4	1.89
Cox, Guzik, Raby, 1990	0.28	8.0	1.89
Sackman, Boothroyd, Fowler, 1990	0.278	7.7	2.1
Sienkiewicz, Bahcall, Paczynski, 1990	0.280	7.7	1.62
Christensen-Dalsgaard, Gough, Thompson, 1991	0.273	–	2.184
Guzik, Cox, 1991	0.270	8.5	2.291
Guenther, Demarque, Kim, Pinsonneault, 1992	0.289	–	1.894

3. The convection zone helium content

Figure 1 shows the variations of the p-, f-, and g-mode frequencies versus angular degree (ℓ, or the number of node lines on the stellar surface) according to Christensen-Dalsgaard (1988). All the observed p- and f-modes can now be predicted to within a few microhertz with current precision solar models. The detection of g-modes is controversial, because they are so weak at the solar surface. If they do occur in the Sun, they must tunnel through the solar convection zone where they are evanescent, and their surface manifestation can barely be observed. Predictions of g-mode frequencies do agree closely with observations, and even rotation mode splittings seem reasonable. I do not discuss these modes any more here, even though a theoretical review of these g-modes is overdue.

Table 2 (Guzik and Cox, 1992) gives our predicted frequencies for modes that probe the part of the solar convection zone that is sensitive to the helium abundance. The helium content in the solar mixture varies throughout the Sun, since in the central regions the primordial value is enhanced by hydrogen burning, and in the convection zone, helium settles to deeper non-convecting layers. Recent studies of this diffusive settling are given by Cox, Guzik, and Kidman (1989) and by Proffitt and Michaud (1991). Kosovichev et al. (1992) present the most recent study for the convection zone helium abundance using many oscillation frequencies.

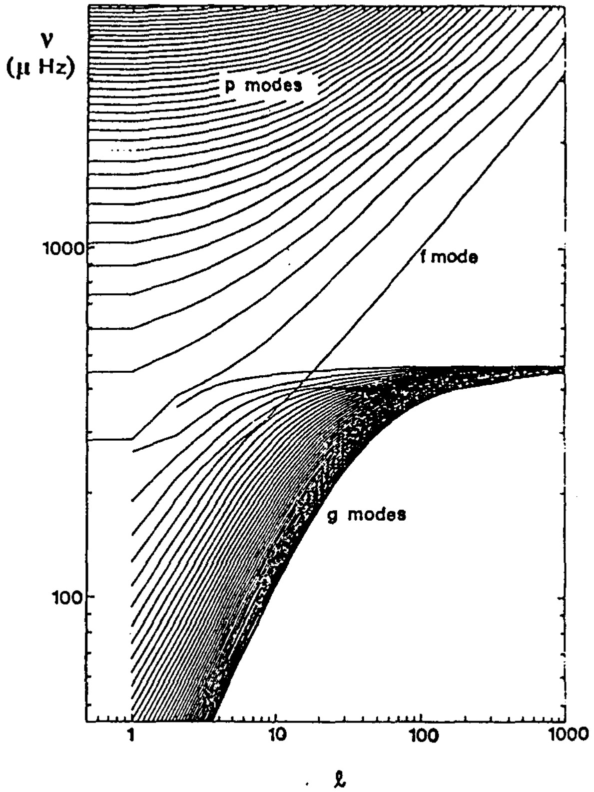

Figure 1 The p- f- and g-mode oscillation mode frequencies are plotted against the angular degree ℓ according to Christensen-Dalsgaard (1988). Individual mode points are connected for each ℓ value.

Modes with ℓ values between 300 and 600 seem to have sensitivity to the convection zone helium content. Those with ℓ smaller have their weight for period determination considerably deeper than around layers with a temperature near 40,000 K, where helium undergoes its second ionization. Higher ℓ modes are too shallow. Our best fit to the observed frequencies is for Y=0.24, and that is 0.03 in Y less than needed for the deeper layers just below the convection zone that reflect the primordial composition. This decrease is almost exactly what diffusion calculations give.

Table 2

FREQUENCY SENSITIVITY TO HELIUM ABUNDANCE

p-MODE		FREQUENCY (μHz)			FREQUENCY (μHz)			DIFFERENCE (μHz)	OBSERVATIONAL ERROR (μHz)	
l (1)	n (2)	$Y_{ee} = 0.27$ (3)	$(O-C)_L$ (4)	$(O-C)_K$ (5)	$Y_{ee} = 0.24$ (6)	$(O-C)_L$ (7)	$(O-C)_K$ (8)	$\nu(0.27) - \nu(0.24)$ (9)	L (10)	K (11)
200	1	1968.0	7.1	−1.0	1965.0	10.1	2.0	3.0	7.0	0.2
	2	2392.4	3.1	−0.0	2391.1	4.5	1.3	1.3	1.0	0.3
	3	2765.8	0.2	0.3	2765.1	0.9	1.0	0.7	0.4	0.2
	4	3134.4	−4.2	−1.3	3133.5	−3.3	−0.4	0.9	0.3	0.4
	5	3476.2	−4.1	−2.2	3475.9	−3.8	−1.9	0.3	0.4	0.4
	6	3803.2	−3.0	0.2	3802.9	−2.7	0.5	0.3	0.5	0.4
	7	4112.6	3.8	5.8	4111.8	4.6	6.6	0.8 (−2.5)	0.9	0.4
	8	4404.6	20.4	17.3	4401.9	23.1	20.0	2.7 (−4.7)	0.9	0.4
300	0	1740.9	2.7	...	1740.9	2.7	...	0.0	5.5	...
	1	2291.0	0.2	−2.3	2285.8	5.4	2.9	5.2	1.5	0.3
	2	2785.7	−5.4	−4.7	2779.6	0.7	1.4	6.1	0.4	0.2
	3	3239.0	−5.3	−2.8	3236.7	−3.0	−0.4	2.3	0.3	0.2
	4	3652.7	−2.6	−0.2	3652.6	−2.4	−0.1	0.1	0.4	0.3
	5	4055.3	6.3	6.9	4053.6	8.0	11.6	1.7 (−2.8)	0.7	0.3
	6	4429.3	29.5	25.6	4425.3	33.5	29.6	4.0 (−6.1)	1.0	0.6
395	1	2549.8	−4.6	...	2544.1	1.1	...	5.7	0.6	...
	2	3083.6	−9.2	...	3076.6	−2.2	...	7.0	0.3	...
	3	3606.2	−10.7	...	3598.7	−3.2	...	7.5	0.4	...
	4	4079.2	3.8	...	4074.9	8.0	...	4.3 (−3.6)	0.7	...
	5	4500.6	37.1	...	4494.8	42.9	...	5.8 (−7.9)	1.7	...
400	0	2006.7	−8.7	6.6	2006.7	−8.7	6.6	0.0	5.5	0.6
	1	2562.7	−16.0	−5.4	2556.9	−10.2	0.4	5.8	7.0	0.3
	2	3098.5	−8.9	−8.8	3091.3	−1.7	−1.6	7.2	12	0.2
	3	3623.9	−17.4	−9.3	3615.9	−9.4	−1.3	8.0	10	0.2
	4	4099.7	−7.6	6.2	4095.0	−2.9	10.9	4.7 (−3.7)	9.2	0.3
	5	4522.0	20.8	38.3	4515.7	27.4	43.4	6.3 (−8.4)	9.2	1.2
600	0	2452.6	−14.6	...	2452.5	−14.5	...	0.1	4	...
	1	3042.5	−17.5	−11.7	3037.8	−12.8	−7.0	4.7	4	0.8
	2	3645.5	−23.5	−12.4	3638.5	−16.5	−5.4	7.0	5	0.3
	3	4230.0	−5.0	9.18	4219.0	6.0	20.18	11.0 (−4.3)	5	0.5
800	0	2828.8	−7.8	...	2828.7	−7.7	...	0.1	4	...
	1	3479.7	−28.7	...	3477.1	−26.1	...	2.6	4	...
	2	4136.9	−16.9	...	4129.2	−9.2	...	7.7 (−3.4)	7	...
1000	0	3160.5	−20.5	...	3160.4	−20.4	...	0.1	3	...
	1	3890.2	−16.2	...	3889.4	−15.4	...	0.8	5	...
	2	4575.1	25.9	...	4563.9	37.1	...	11.2 (−8.8)	14	...

4. The depth of the solar convection zone

The depth of the solar convection zone can be determined to great accuracy by matching observed frequencies for high ℓ modes with predictions for various cases. Figure 2 shows how the weight for frequency determination is all concentrated near the bottom of the model convection zone. With only about 10 modes each for about 5 ℓ values that display great sensitivity to the structure at the convection zone bottom, its depth can be measured to about 0.002 in radius.

Figures 3, 4 and 5 show how the observed minus calculated (O-C) p-mode frequency differences vary with the mode frequency for various ℓ values and for various convection zone depths. The observed frequencies are from Libbrecht, Woodard, and Kaufman (1990) and Korzennik (1990).

Points for individual modes for each ℓ are connected by a line, but one can see the mode frequencies at the kinks. Modes for $\ell = 20$ and above seem to be too shallow for this depth measurement, but they do reveal model structure problems higher in the convection zone.

Inspection of these three figures shows that the best fit is for the intermediate case with the convection zone radius near 0.711 of the solar photospheric radius.

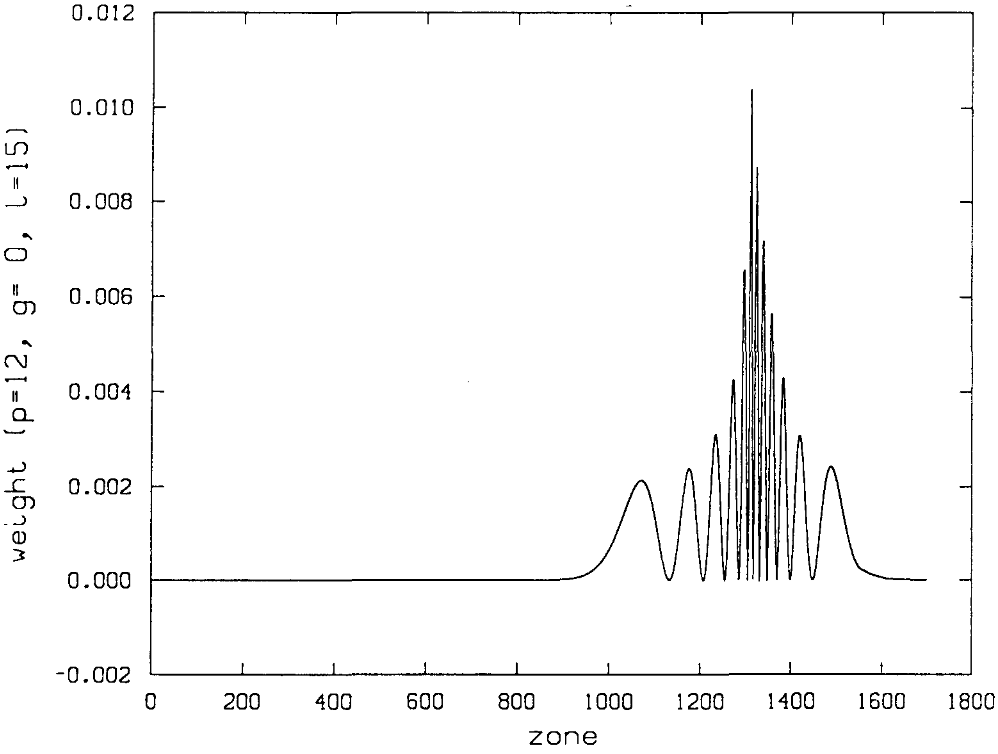

Figure 2 The weight for the p_{12} mode with $\ell=15$ versus mass zone number, showing that there is considerable weight at the bottom of the convection zone at zone 1234.

5. Discussion

Many results of probing the solar structure by comparing observational and theoretical oscillation frequencies are now available. Our new detailed results on the convection helium content and zone depth are now being followed by investigations of the shape of the helium composition structure in this layer. These results are reported by Guzik in another paper at this conference and in a paper being prepared for publication. For this work, we need nonadiabatic eigensolutions that include the effects of radiation diffusion in the upper convection zone to model the phenomena for the real Sun.

A long-standing question is how much overshooting occurs at the convection zone bottom. Another is the diffusion composition shape that results from the differing Y value in the convection zone and the deeper primordial value below. Some combination of these mechanisms, rotation induced turbulence, and the μ gradient, that stabilizes convection can now be mapped with good precision.

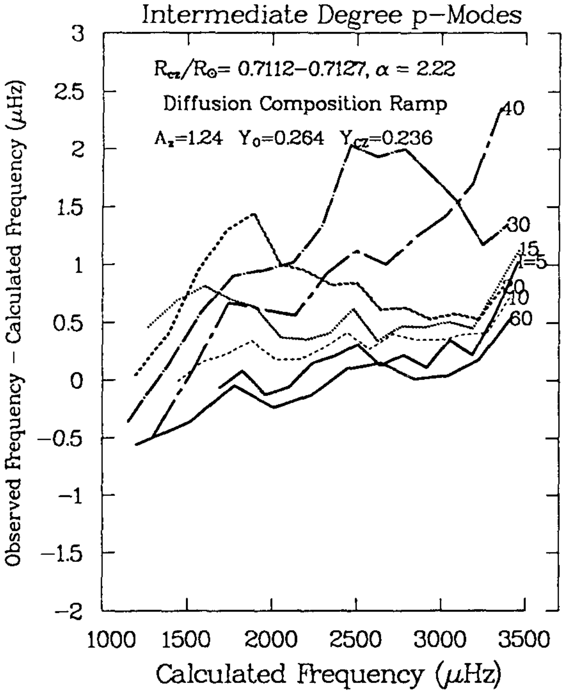

Figure 3 The O-C for intermediate ℓ p-mode frequencies versus mode frequency for the mixing length that produces a convection zone bottom at about 0.712 of the solar radius.

6. Questions

Hiromoto Shibahashi: How many free parameters do you have in your treatment of diffusion? I suppose that you adjust the radius of your solar model to the present solar value by changing the mixing length. Is the mixing length determined after fixing the parameters for diffusion?

The several parameters for the diffusion part of the solar evolution calculation are uncertain, but they are considered as given, just like the material properties. Indeed the mixing length required for the model to have the solar radius is somewhat influenced by these diffusion parameters, because the helium abundance in the convection zone is much smaller with diffusion than without. We now need an even larger mixing length.

Geza Kovacs: What happens with the pulsation spectrum if you completely neglect convection? Are there any calculations done with the Iglesias-Rogers (1992) opacities?

Solar models with no convection do not come anywhere near to matching the accurately known solar radius. Thus none have ever been calculated even 35 years ago! Such models will be much too large, and frequencies will be much too small. Surface convection cannot be ignored in the Sun, even though it often is for yellow giant pulsators. Since 1988, all solar models have used the then current Livermore OPAL opacities. As I discussed in my "Inside the Sun" review in Versailles, the only effect of these new opacities is in the few million kelvin region, where opacities 15% higher than the Los Alamos ones are needed to predict the correct intermediate ℓ p-mode frequencies. All the interesting blue and yellow giant variable star opacity effects are hidden in the extensive solar convection zone, where opacities are really very large, but they do not matter at all.

Jayme Matthews: Do the pulsation models you use to constrain the convection zone correspond to sectorial modes? Different $m \neq \ell$ modes will have different latitudinal weightings, so, for example, $m = \ell$ will be very sensitive to properties at the solar equator, while other modes will represent different latitude kernels.

The observers are aware of this behavior of spherical harmonics, and they consider these shapes when analyzing their data. Theoretically, the assumed spherical shape of a non-rotating Sun means that no consideration of the degenerate m parameter is necessary for calculating oscillation frequencies. Our solar oscillation frequencies depend on our spherical model approximations such as frozen-in convection and only on the radial order and the angular degree ℓ.

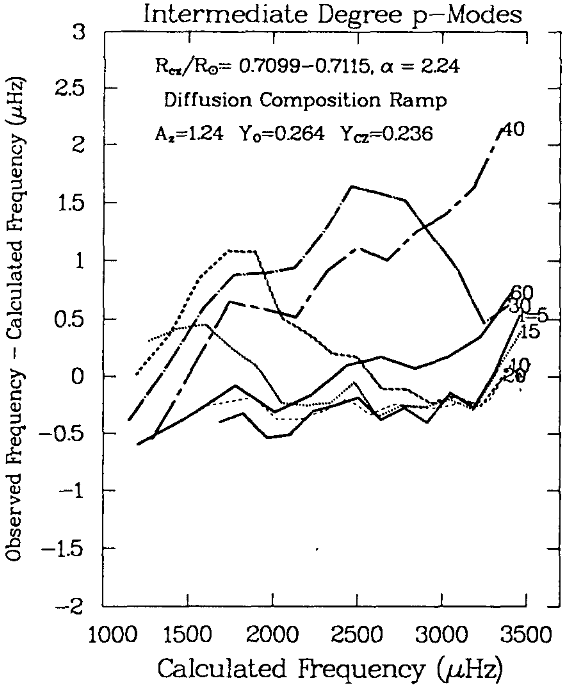

Figure 4 The (O-C) for intermediate ℓ p-mode frequencies versus mode frequency for the mixing length that produces a convection zone bottom at about 0.711 of the solar radius.

The high ℓ modes that display departures from sphericity are the ones that reveal the surface layer rotation.

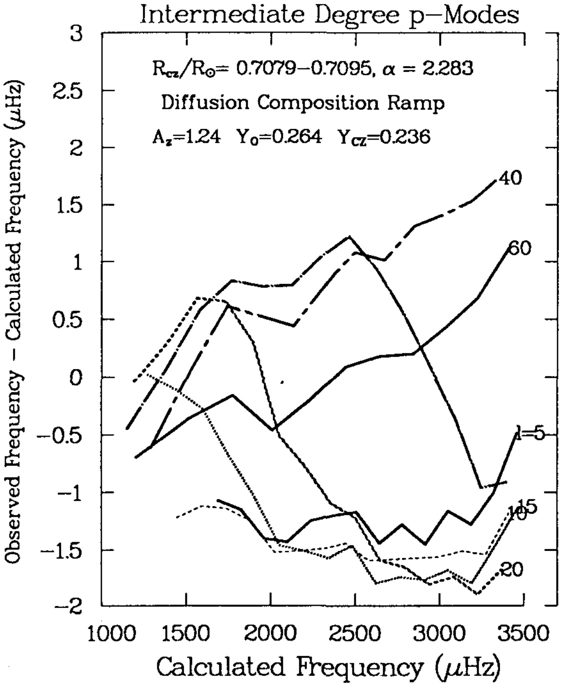

Figure 5 The (O-C) for intermediate ℓ p-mode frequencies versus mode frequency for the mixing length that produces a convection zone bottom at about 0.709 of the solar radius.

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