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ON MAXIMAL SETS OF MUTUALLY ORTHOGONAL IDEMPOTENT LATIN SQUARES

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It is a well-known trivial fact that for a given integer n there exists at most $n-2$ pairwise orthogonal idempotent latin squares. In the following note we prove that for n a prime power there always exists $n-2$ such squares.

THEOREM. *Let $n=p^r$ be a prime power. Then there exist $n-2$ pairwise orthogonal idempotent latin squares.*

Proof. The latin squares will be represented as multiplication tables of idempotent quasigroups. The elements of the quasigroups will be those of $GF(p^r)$ and the ith quasigroup will have its multiplication given by $A *_i B = iA+(1-i)B$. Here A and B range over $GF(p^r)$ and i takes on all values in $GF(p^r)$ except 0 and 1.

In order to show that $*_i$ and $*_j$ are orthogonal operations it is simply necessary to show that the equations $X *_i Y = A$ and $X *_j Y = B$ have unique solutions for X and Y where A and B are given elements of $GF(p^r)$ and $i \neq j$.

But these equations become $iX+(1-i)Y=A$ and $jX+(1-j)Y=B$ with determinant $i-j$. Hence they have a unique solution.

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