Any number consisting of

1.	One figure repeated is divisible by					11.
2.	Two figures	,,	,,	,,	,,	101.
3.	Three	,,	,,	,,	,,	7, 11, 13.
4.	Four	,,	,,	,,	,,	73, 137.
5.	Five	,,	,,	,,	,,	11, 9091.
6.	Six	,,	,,	,,	,,	101, 9901.
7.	Seven	,,	,,	,,	,,	11, 909091.
8.	Eight	,,	,,	,,	,,	17, 5882353.
9.	Nine	,,	,,	,,	,,	7, 11, 13, 19, 52579.

The proofs of these properties are very simple. Take for example the third.

The smallest number consisting of three figures repeated is 001001, and all the others are multiples of it.

But $1001 = 7 \times 11 \times 13$.

Dr Booth's proof is:

A number N of six places may be thus written:

$N = 100000a + 10000b + 1000c + 100d + 10e + f,$

which, when divided by 7, will give a quotient q, and a remainder $5a + 4b + 6c + 2d + 3e + f$.

Now if $d = a$, $e = b$, $f = c$, this remainder may be written $7(a + b + c)$, which is divisible by 7, whatever be the values of a, b, c.

Similarly for the divisors 11 and 13.

On consulting Dr Booth's paper the other day I find that he states 17 to be a divisor of any number consisting of eight figures repeated. He does not appear to have observed that the other divisor 5882353 is a prime.

Projective Geometry of the Sphere.

By R. E. Allardice, M.A.

[*Abstract.*]

If A and B be two fixed points on a great circular arc and P a variable point on the arc, there are two and only two possible positions of the point P corresponding to a given value of the ratio sinAP/sinBP, provided arcs measured in one direction from A or B be considered positive, and in the opposite direction negative; and these two points are antipodal.

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In the case of the determination of the position of a ray of a pencil of great circular arcs by means of the corresponding ratio, a complete circular arc takes the place of the two antipodal points in the previous case. This theorem is the polar of the last.

Most theorems relating to transversals in plane geometry may be transferred to the sphere by conical projection from the centre of the sphere on the spherical surface. This was illustrated in the paper in the case of Menelaus's theorem. The same method may be applied to prove that the anharmonic ratio of any transversal of a pencil of four rays is the same; or this may be proved more simply still by means of the theory of projective ranges, or directly in a manner analogous to that employed in plane geometry, by means of the following theorem.

If h be the perpendicular from the vertex A on the base BC of a spherical triangle, then $\sin a \sin h = \sin b \sin c \sin A$.

It may be noticed that the anharmonic ratios of a range and of the polar pencil are the same.

It is obvious that such theorems as that which gives the properties of the complete quadrilateral follow at once for the sphere.

Application to Sphero-conics.

A sphero-conic is the intersection of a quadric cone with a sphere which has its centre at the vertex of the cone.

A complete sphero-conic consists of two closed curves, antipodal to one another, each of which may be considered a spherical ellipse; or a great circle, drawn through the centre of each perpendicular to one of the axes may be considered as dividing the complete conic into two spherical hyperbolas.

The chief properties of these curves may be deduced at once from the theory of projective pencils. This method gives in succession

Pascal's and Brianchon's theorems;

The properties of the inscribed and of the circumscribed quadrilateral;

The theory of pole and polar by means of the complete quadrilateral;

The properties of conjugate diameters (which however do not bisect their ordinates);

The properties of the foci by means of the theory of involution.

[The principal properties of sphero-conics, a number of which were enunciated in the paper, may be found in Chasles's *Memoir on Cones and Spherical Conics* translated by Rev. Charles Graves.]

Transformation of Figures.

Similar Figures. According to the ordinary notion of similar figures in plane geometry, such cannot exist on the surface of the sphere, since the area of a figure is determined by its curvature. The following is, however, analogous to the method of transformation by similarity in plane geometry.

If T be a fixed point, P a point on a given curve, Q a point such $\tan\frac{1}{2}TQ/\tan\frac{1}{2}TP$ is constant, the locus of Q is said to be a curve similar to the locus of P.

The curve similar to a circle is another circle.

The tangents at corresponding points on two similar curves are equally inclined to the common radius vector.

The polar of the above method of transformation gives a second method, which is applicable in plane geometry.

Inversion. If in the method of transformation given above the product $\tan\frac{1}{2}TQ\tan\frac{1}{2}TP$ be substituted for the quotient $\tan\frac{1}{2}TQ/\tan\frac{1}{2}TP$, the curve so derived may be called the inverse of the given curve.

In the case of the sphere however the methods of transformation by similarity and by inversion are really not distinct.

To the fact that in transformation by similarity or by inversion tangents at corresponding points are equally inclined to the radius vector, corresponds in the polar methods of transformation the fact that corresponding tangents are equal; and to the fact that the angle of intersection of two curves is unaltered in the one method of transformation, the fact that the distance between the points of contact of common tangents is unaltered in the second method.

[Some of the foregoing results are given in Mulcahy's *Modern Geometry.*]

Statical proofs of some Geometrical Theorems.

By JOHN ALISON, M.A.

1. Figure 54. If ABC be a triangle, P any point, then the system of forces PA, PB, PC is equivalent to the system PH, PK, PL, where H, K, L are the middle points of BC, CA, AB.

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