
CORRIGENDUM

‘The Asymptotic Number of Connected d -Uniform Hypergraphs’ — CORRIGENDUM

MICHAEL BEHRISCH^{1†}, AMIN COJA-OGHLAN^{2‡}
and MIHYUN KANG^{3§}

¹ Institute of Transportation Systems, German Aerospace Center, Rutherfordstrasse 2, 12489 Berlin, Germany
(e-mail: michael.behrisch@dlr.de)

² Goethe University, Mathematics Institute, 60054 Frankfurt am Main, Germany
(e-mail: acoghlan@math.uni-frankfurt.de)

³ TU Graz, Institut für Optimierung und Diskrete Mathematik (Math B), Steyrergasse 30, 8010 Graz, Austria
(e-mail: kang@math.tugraz.at)

doi:<http://dx.doi.org/10.1017/S0963548314000029>, Published by Cambridge University Press, 13 February 2014.

The authors would like to rectify a mistake made in Theorem 1.1 of their article (Behrisch, Cojaa-Oghlan & Kang 2014), published in issue 23 (3). The text below explains the changes required.

1. Correction to Theorem 1.1

The formula for the probability that the random hypergraph $H_d(n, m)$ is connected given in [1, Theorem 1.1] is incorrect. With $H_d(n, m)$ denoting the random d -uniform hypergraph with n vertices and m edges, the correct version of Theorem 1.1 reads as follows.

Theorem 1.1. *Let $d \geq 2$ be a fixed integer. For any compact set $\mathcal{J} \subset (d(d-1)^{-1}, \infty)$ and for any $\delta > 0$ there exists $n_0 > 0$ such that the following holds. Let $m = m(n)$ be a sequence of integers such that $\zeta = \zeta(n) = dm/n \in \mathcal{J}$ for all n . There exists a unique number $0 < r = r(n) < 1$ such that*

$$r = \exp\left(-\zeta \cdot \frac{(1-r)(1-r^{d-1})}{1-r^d}\right). \quad (1.1)$$

[†] Supported by the DFG Research Center MATHEON in Berlin.

[‡] Supported by DFG CO 646.

[§] Supported by the Deutsche Forschungsgemeinschaft (DFG Pr 296/7-3, KA 2748/3-1).

Let $\Phi_d(r, \zeta) = r^{\frac{r}{1-r}}(1-r)^{1-\zeta}(1-r^d)^{\frac{\zeta}{d}}$ for $d \geq 2$. Furthermore, define, for $d > 2$,

$$R_d(n, m) = \frac{1 - r^d - (1 - r)(d - 1)\zeta r^{d-1}}{\sqrt{(1 - r^d + \zeta(d - 1)(r - r^{d-1}))(1 - r^d) - d\zeta r(1 - r^{d-1})^2}} \cdot \exp\left(\frac{(d - 1)\zeta(r - 2r^d + r^{d-1})}{2(1 - r^d)}\right) \cdot \Phi_d(r, \zeta)^n,$$

and for $d = 2$,

$$R_2(n, m) = \frac{1 + r - \zeta r}{\sqrt{(1 + r)^2 - 2\zeta r}} \cdot \exp\left(\frac{2\zeta r + \zeta^2 r}{2(1 + r)}\right) \cdot \Phi_2(r, \zeta)^n.$$

Finally, let $c_d(n, m)$ denote the probability that $H_d(n, m)$ is connected. Then for all $n > n_0$ we have

$$(1 - \delta)R_d(n, m) < c_d(n, m) < (1 + \delta)R_d(n, m).$$

2. Correction to the proof of Theorem 1.1

The mistake in [1, Theorem 1.1] derives from an error in [1, Lemma 2.1]. Specifically, the expression given for v in [1, equation (2.4)] has to be replaced by

$$v = \exp\left(\frac{(d - 1)rc}{2(1 - r)}(1 - 2r^{d-1} + r^{d-2})\right).$$

With this correction, the argument given in [1, Section 2] yields the correct result as stated above.

The erroneous formula [1, equation (2.4)] stems from [3, Lemma 10], where the expression

$$\exp\left[b_5m - \mu - \frac{(d - 1)(1 - a_5)b_5c}{2}\right]$$

has to be replaced by

$$\exp\left[b_5m - \mu - \frac{(d - 1)(1 - a_5)b_5c}{2a_5}\right].$$

The a_5 in the denominator slipped into [3, Section 3.2] in the step from equation (22) to the equation following (23).

Acknowledgement

We thank Béla Bollobás and Oliver Riordan for drawing our attention to the mistake, which they noticed in the context of preparing their recent article [2].

References

[1] Behrisch, M., Coja-Oghlan, A. and Kang, M. (2014) The asymptotic number of connected d -uniform hypergraphs. *Combin. Probab. Comput.* **23** 367–385.

- [2] Bollobás, B. and Riordan, M. (2014) Counting connected hypergraphs via the probabilistic method. [arXiv:1404.5887](https://arxiv.org/abs/1404.5887)
- [3] Coja-Oghlan, A., Moore, C. and Sanwalani, V. (2007) Counting connected graphs and hypergraphs via the probabilistic method. *Random Struct. Alg.* **31** 288–329.