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OBITUARY

JEFFREY DENNIS WESTON (1919–2000)

Jeffrey Dennis Weston was born on 15 September 1919 in London. He died suddenly on 10 March 2000, the 55th anniversary of his marriage to Mary.

At his funeral, tributes were paid by Aubrey Truman (published, appropriately because of Weston's involvement with the Institute of Mathematics and its Applications, in *Mathematics Today* **36** (2000) p. 69) and by David Williams, whose speech included the following.

> Today, we mourn the loss of a remarkable man, and we celebrate a wonderful life.
>
> Jeffrey's very creative early career in radio engineering has been described by Aubrey.
>
> Mathematics then took over Jeffrey's life, inspiring in him an almost-religious sense of awe and wonder. In his chosen field, mathematical analysis, he proved a number of results of lasting importance. Right up until he died, he was working on a book and papers on the theory of Lie groups, a subject fundamental for understanding some awe-inspiring wonders of our Universe. The appearance of the symbol for infinity, ∞, on the Order of Service reflects Jeffrey's fascination with the amazing rigorous theories of infinite numbers.

> At Swansea, he built up a very strong Mathematics Department, a superb mathematics library, and the magnificent Reading Room facility which must indeed bear his name.
>
> Jeffrey played very important rôles in the London Mathematical Society and in the British Mathematical Colloquium, of which he was a founder member. In his very influential editorial work, he strove to ensure that the *writing* of mathematics share the perfection of its concepts.
>
> If there is one word which sums up Jeffrey, it is integrity. He always stood up for what he believed in; and what he believed in was usually right. He believed that some things are absolute, and that one should never compromise on them. How one wishes that other people in schools and universities had taken a similar stance! The modern anthem, 'Compromise, compromise, compromise', has led to British education's being a mockery of its former self. On some small issues, Jeffrey was perhaps too intransigent; but in the things that really matter, he was a beacon of integrity who puts the rest of us in education to shame.
>
> Though he always fought for what he considered right, Jeffrey was a shy and reserved man. His shyness hid from many the warmth, the caring, the humour, which lay in such measure behind it. I only hope that he knew what high regard, and what fondness, so many of us felt towards him.
>
> In Mary, he found the perfect soul-mate. Theirs was a wonderful marriage. Mary could rejoice in Jeffrey's never-diminished enthusiasm for mathematics. They shared a passion for music, and played an important part in musical activities at Swansea. Some celebrated musicians became their close friends.
>
> Several people here today can witness that at very difficult times for them, Jeffrey and Mary were there to help, and *did* help greatly. It is wonderful that at this tragic time for her, Mary's friends (Sue, Betty, Carmel, Jane, Robin and Giselle, and undoubtedly others whom I don't know) are showing such kindness to her.

In 1936, Weston began an Engineering degree course at the University of London, graduating in 1939. During the war, he did important work, acquiring several patents, as a Radio Development Officer with Standard Telephones. Colleagues there included E. W. Earp, C. Strong, Shaun Wylie and Christopher Strachey; and his contact with these contributed to his developing a deep and abiding interest in Pure Mathematics, particularly Analysis. He gained his PhD in Electrical Engineering at Sheffield in 1946, and became a lecturer, first at Woolwich Polytechnic and then at Sheffield University, where he chose to work on Functional Analysis. He was then head-hunted to join W. W. Rogosinski and F. F. Bonsall as part of the celebrated Analysis group at King's College, Newcastle, then part of the University of Durham. During his time in Newcastle, he published 33 papers in five years. He was awarded a DSc in Mathematics by the University of London in 1960.

In 1961, he became Head of the Pure Mathematics Department at Swansea; and he remained in this post until his retirement in 1982. He strengthened the department, particularly in Analysis and Topology, via appointments including those of Alan Ellis, Nigel Kalton, Elmer Rees and some very able younger mathematicians. At Swansea, he concentrated on helping the research of others rather than on his own research: caring for his staff, fighting their corner when necessary, playing a key part in the design of the award-winning Mathematics building with its superb Reading Room, and going to great lengths to build up what was then one of the finest British mathematical libraries outside Oxbridge and London. We record our own gratitude that he appointed us to a thriving and particularly happy department.

He contributed significantly to mathematical life in several other ways. The London Mathematical Society, the British Mathematical Colloquium, the Institute of Mathematics and its Applications, the Mathematical Association, and meetings at Gregynog: these were very big things for him. His legendary concern for precise English (we suspect that he knew 'Fowler' by heart) was well utilised during his six years as Editor of the LMS Proceedings. He was a conscientious member of the LMS Council, and became a Vice-President of the LMS during 1968–69. He attended very nearly all the meetings of the BMC held during his lifetime. He was President of the South Wales Mathematical Association for many years, and had very strong ideas about how Mathematics should be taught. He edited the volume [**53**] in tribute to J. E. Littlewood, with a galaxy of contributors. He was a generous and effective supervisor of research students.

He kept fully in touch with the detailed workings of the technology which allowed ever more faithful reproduction of the richness of sound of the Bruckner he enjoyed, but the music that meant most to him was that of Beethoven, the subject of many discussions he and Mary had with John Lill.

Mathematical work

Having converted to pure mathematics, Weston embraced its capacity for abstraction with zeal. Bertrand Russell's

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show

became his creed. However, those who knew Weston are well aware that he always maintained a keen interest in applications (an interest reflected in his deep involvement with the IMA), and that, in secret, he loved concrete calculations such as those found in the Wiener–Hopf theory associated with wave guides.

Many of us feel sad that his skill with calculations is rather rarely reflected in his work, and would have welcomed more papers along the lines of [**7**]. That paper, part of which relies on work of Ingham, provides tight bounds on a quadratic form which had been studied by Schur. To give the flavour, here is one result which is given a nice direct proof: *When λ is real and not an integer, and the real numbers x_n are such that $\sum_1^\infty x_n^2 = 1$, the upper and lower bounds of*

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{x_m x_n}{m-n+\lambda}$$

are $\pi\operatorname{cosec}\pi\lambda$ and $\pi\cot\pi\lambda$ when $|\lambda|<1$, and are $\pm\pi|\operatorname{cosec}\lambda|$ when $|\lambda|>1$; when all the x_n are positive, $\pi\cot\pi\lambda$ is the upper bound of the quadratic form when $-1<\lambda<0$ and the lower bound when $0<\lambda<1$. The bounds are not attained. [The 'upper bound' is, of course, the supremum over the described sequences (x_n). That there is a missing square-root sign in line 4 of the paper is obvious, and does not detract from the paper in any way.]

The cold and austere beauty of abstraction appealed to Weston more than did concrete formulae. A main motivation in his work was to find the most general context in which a concept may be defined, and then the most natural definition of the concept within that context. (One N. Bourbaki is often cited in his work!) Weston saw a definition set in the right context as already having intrinsic beauty akin to that of a well-cut diamond in an elegant setting. Of course, he also appreciated that a good definition has the immense advantage over the diamond that the former *leads* somewhere; you don't *just* look at it. Yet the stern perfection of a 'right' definition did indeed in itself appeal to Weston's aesthetic sense; and he found it hard to understand that in many students it did not evoke the same response. He had his own sense too of what constitutes the right proof of a result, as several short papers indicate. Paper [**18**] shows his almost mischievous determination to hold forever the Guinness record for the shortest deduction of Zorn's lemma from the axiom of choice. No competition!

Papers on the operational calculus of perfect operators [**22**, **29**, **31**, **37**, **38**, **46**, **48**]. This is the topic which most of Weston's longer papers addressed; and one must see the work as illustrative of his aesthetic response to a mathematical structure in and of itself. That he saw a particular beauty in the structure of 'perfect operators' on 'perfect functions' is perhaps to some extent suggested by his terminology. He wanted us, while we are reading the papers, to share his response to the *structure* rather than to consider how we might solve a particular concrete problem via Laplace transforms. He believed that a student en route to Schwartz's masterpiece could profitably look first at this simpler topic which, although much more limited in scope, has in its *field* structure (explained below) a particular perfection.

To set the background, consider situations where we do not have a field structure. Let f and g be C^∞ functions on $\mathbb{R}$ with disjoint (non-empty) compact supports. Clearly, for standard 'pointwise' multiplication, each of f and g is a divisor of zero in that $fg = 0$. If f_1, g_1 are the inverse Fourier transforms of f, g, respectively, then the convolution $(f_1 * g_1)$ is identically zero, so that we have (in the space $\mathscr{S}$ of 'rapidly decreasing' functions) divisors of zero for the convolution product for functions on $\mathbb{R}$. Next, a Schwartz distribution is a *functional* on a space of test functions, a classical function x acting on a test function p via $p \mapsto \int xp$. One *cannot* multiply functionals. However, one *can* multiply *operators* on spaces of test functions.

A study of Heaviside's operational calculus led Weston to consider the algebra $\mathscr{E}$ of continuous functions x on $[0, \infty)$ of exponential growth ($|x(t)| \leqslant K\mathrm{e}^{ct}$ for some K and c), the product being the convolution product

$$(x * y)(t) := \int_0^t x(s)y(t-s)\,\mathrm{d}s.$$

The product is associative and commutative, and, by a theorem proved by Titchmarsh without the 'exponential growth' condition, $\mathscr{E}$ has no divisors of zero. (Of course, the fact that a Laplace transform of a function in $\mathscr{E}$ is analytic in a half-plane helps in the case of functions of exponential growth.) One can therefore construct the associated field $\mathscr{F}$ of fractions.

Weston defines a *perfect function* (his typical test function) to be an element p of $\mathscr{E}$ which is infinitely differentiable with all derivatives in $\mathscr{E}$ and such that $p^{(n)}(0) = 0$ for $n = 0, 1, 2, \ldots$. For an element x of $\mathscr{E}$, denote its Laplace transform (the integral of $\mathrm{e}^{-zt}x(t)$) as $\hat{x}(z)$ for $\Re(z)$ sufficiently large. Call an operator A on the space $\mathscr{D}_0$ of perfect functions a *perfect operator* if there exists a function $\hat{A}$, holomorphic on some right half-plane, such that for each perfect function p, there exists a real number $c(p)$ such that $\widehat{Ap}(z) = \hat{A}(z)\hat{p}(z)$ when $\Re(z) > c(p)$. We call the function $\hat{A}$ the Laplace transform of A. An element x of $\mathscr{E}$ determines the perfect operator X,

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where $Xp = x * p$, so that $\hat{X}(z) = \hat{x}(z)$ (in a right half-plane). (Later, we identify X and x).

If, as in Weston's notation, h in $\mathscr{E}$ is the constant function equal to 1, then $(Hy)(t) = \int_0^t y(s)\,\mathrm{d}s$, so that H^{-1} is the differential operator D on $\mathscr{D}_0$. Of course, D has Laplace transform z, and the fractional derivative D^α has Laplace transform z^α (principal value). We can think of a function x as $\hat{x}(D)$ in the field $\mathscr{F}$, so that, for example, $\mathrm{e}^{at} = (D-a)^{-1}$; and indeed for a perfect p the perfect solution q of $(D-a)q = p$ is $q = e_a * p$ where $e_a(t) = \mathrm{e}^{at}$. You can puzzle out how constant-coefficient equations with general initial conditions fit into this pattern.

The deepest theorem in this context, the representation theorem [**31**], states that if A is a perfect operator, then there exist an integer n and an element x of $\mathscr{E}$ such that

$$(Ap)(t) = \int_0^t x(s)p^{(n)}(t-s)\,\mathrm{d}s,$$

so that $A = XD^n = D^nX$. A well-known analogue of the representation holds for Schwartz distributions; and indeed, Weston [**37**] later found that Schwartz had earlier obtained a result equivalent to the described representation result for a perfect operator A. The representation theorem identifies the ring of perfect operators as that generated by $\mathscr{E}$ and D. Here is another characterization of perfect operators: a linear operator on $\mathscr{D}_0$ is perfect if and only if it commutes with every operator $p \mapsto x * p$ where $x \in \mathscr{E}$. Results of this type help explain why Weston retained his fascination with this topic until the end of his life.

Comments on some other papers. A nice illustration of the benefits of abstraction, typical of Weston's ability to pick out the essence of an argument and use it in other circumstances, is the paper [**25**]. By extending the notion of *cluster set*, he proves a theorem that unifies Collingwood's results on radial cluster points and boundary cluster points of analytic functions. He then uses the same theorem to prove the following generalization of a result of Baire: if f is a function on $X \times Y$ which is separately continuous, where X is of the second category and Y satisfies the first axiom of countability, then the points of $X \times Y$ at which f is jointly continuous are dense.

Paper [**23**] is another nice use of abstraction, proving an extension of Goldstine's theorem (on the X^* denseness of X in X^{**}) with two applications to Measure Theory.

Paper [**9**] aims to provide the natural definition of an almost periodic function from a topological group T into a complete metric space M, and to construct an associated Bohr compactification of T. Some of these ideas were then used in [**12**] to construct a 'mean value' $\mu[\phi]$ of a function from a compact topological group into a complete metric space X in situations where it is possible to construct a 'mean' $\mu(x_1, x_2, \ldots, x_n)$ of any finite subset $\{x_1, x_2, \ldots, x_n\}$ of X, the function μ (from the set of finite subsets of X into X) satisfying certain conditions. An example is where X is a Banach space and $\mu(x_1, x_2, \ldots, x_n)$ denotes the arithmetic mean of $\{x_1, x_2, \ldots, x_n\}$.

Papers [**4**] and [**5**] are studies of Paley–Wiener functions, functions in $L^2(\mathbb{R})$ with Fourier transform vanishing outside $(-\pi, \pi)$, and of their relevance in communication theory. (The L^2 property is a finite-energy requirement and the Paley–Wiener property is related to finite bandwidth.)

Non-Standard Analysis. Weston was always deeply interested in Non-Standard Analysis, and always believed that it would prove of great use. He was therefore delighted when the work of Perkins, Loeb and others began a series of spectacular applications to Probability.

Recent work. Both Truman and Williams understood from remarks made more than once by Weston in the last year of his life that he had recently finished a book with the theory of Lie groups as a major topic, something mentioned in both funeral tributes. Sadly, this work has not yet been found. What *was* found is the first chapter of a different book – on perfect operators, now (for reasons he explains) renamed 'timeless'.

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