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Short-Period Oscillations of Earth Rotation

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Abstract.

Sub-seasonal variations and especially sub-seasonal oscillations with periods of about 120, 60, 50, 40 days in polar motion and of about 120, 60–90, and 50 days in LOD are presented. Variations of amplitudes of these sub-seasonal oscillations of polar motion are shown. Maxima of these amplitudes are of the order of 2–4 mas. These oscillations are elliptical ones. The correlation coefficients between geodetic and atmospheric excitation functions in this range of the spectrum are variable and have annual variations. Maxima of correlation coefficients are of the order of 0.6–0.8.

Modern geodetic VLBI experiments provide very accurate results in polar motion and UT1−UTC with a temporal resolution of 3–7 minutes. Several irregular, quasi-periodic variations were found. In many UT1−UTC data sets, oscillations with periods around 8 hours and between 5 and 7 hours can be seen.

1. Introduction

Improvement of the accuracy and temporal resolution of the Earth rotation parameters (ERP) achieved by the application of satellite (SLR, GPS) and VLBI methods in the last two decades have enabled detection and study of sub-seasonal, diurnal, sub-diurnal and even irregular variations of ERP (IERS, 1988–1998). At present the accuracy of polar motion and length of day (LOD) determinations is of the order of 0.1 mas and 0.01 ms, respectively (see IERS Annual Report for 1998).

The new methods of spectral analyses and filtration, for instance the Fourier Transform Band Pass Filter (FTBPF), the Wavelet Transform, the Kalman filter have been developed and applied to these studies improving analyses of variations of ERP (Brzeziński, 1995; Gambis, 1992; Kosek, 1987, 1995; Kosek *et al.*, 1995a,b, 1998a; Morabito *et al.*, 1988; Petrov *et al.*, 1996; Popiński *et al.*, 1994, 1995 a, b; Schmitz-Hübsch and Schuh, 1999; Titov, 1999).

The sub-seasonal variations of polar motion and LOD containing oscillations with periods shorter than half a year were not only detected (Eubanks *et al.*, 1988; Feissel *et al.*, 1980; Hide *et al.*, 1991; Kołaczek *et al.*, 1987; Kołaczek, 1995; Kosek 1987, 1993; Nothnagel *et al.*, 1992; Schuh 1986, 1988) but their time variations have been found (Kołaczek 1992, 1993; Kołaczek *et al.*, 1993; Kosek *et al.*, 1995, 1998b, 1999a,b; Nastula, 1992; 1995; Popiński *et al.*, 1994; Schmitz-Hübsch and Schuh, 1999).

Correlations between sub-seasonal variations of Earth rotation and sub-seasonal variations of Effective Atmospheric Angular Momentum functions were determined and studied. Influences of El Niño events on this correlation have been established (Brzeziński, 1992; Chao, 1989, 1993; Dickey 1993; Dickey *et al.*, 1991, 1992; Eubanks *et al.*, 1985; Freedman *et al.*, 1994; Gambis, 1992; Hide *et al.*, 1980, 1991; Eubanks, 1993; Kołaczek *et al.*, 1999; Kosek *et al.*, 1995b; Kuehne *et al.*, 1993; Nastula, 1995; Nastula *et al.* 1997; Rosen and Salstein, 1983; Schmitz-Hübsch and Schuh, 1999; Schuh 1999).

Recently modern geodetic VLBI experiments provide very accurate results of polar motion and UT1−UTC with a temporal resolution of 3–7 minutes. Wavelet transformations of the time series obtained by VLBI allow us to detect several irregular quasi-periodic variations. In many UT1−UTC data sets oscillations with periods around several hours were found (Schuh and Titov, 1999). Using the Least-Squares Collocation Method (LSCM) also high-resolution polar motion series were computed and analysed. Oscillations with periods around several hours were also found in these series (Schuh and Titov, 1999).

In the paper the computed sub-seasonal variations of polar motion and LOD, especially their most energetic oscillations with periods of about 120, 60–90, 60, 50, 40 days, are presented and discussed. Results of recent VLBI determinations of quasi-periodic subdaily oscillations of UT1−UTC and polar motion are shown.

2. Sub-seasonal variations of ERP

Sub-seasonal variations of ERP are still quite energetic in comparison with the present accuracy of ERP determinations and contain important information about geophysical influences on ERP variations.

Sub-seasonal variations of polar motion filtered from the IERS C04 pole coordinate data using the Butterworth high pass filter (HPF) (Otnes *et al.*, 1972) with a cutoff period of 150 days have variable amplitudes reaching maxima of about 10 mas (Fig. 1). Perturbations of polhodia caused by these short-period variations are shown in Figure 2. Several energetic oscillations are visible in the spectra of these short-period variations computed by the FTBPF (Kosek, 1995; Popiński *et al.*, 1995b). The prograde oscillations with periods of about 120, 60, 50, 40 days are the most energetic ones (Fig. 3).

These oscillations were filtered by the FTBPF (Kosek, 1995) and are shown in Figure 4. Amplitudes of these oscillations vary in time and are modulated with a period of about 2–4 years. These elliptical oscillations are presented in Figure 5, where polhodia of these oscillations are drawn for some years for example. 120, 60 and 40 day oscillations can be of seasonal origin.

Time variable spectra of the short-period variations of the IERS C04 polar motion filtered by the Butterworth HPF with cutoff period of 90 and 150 days computed by the FTBPF are shown in Figure 6. We can easily notice amplitude variations of these oscillations and not very stable periods. These oscillations

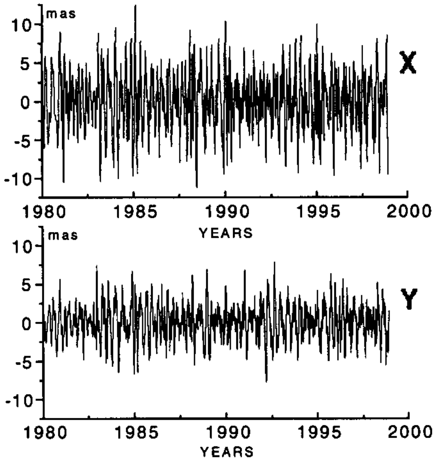

Figure 1. Short-period oscillations of polar motion IERS C04 filtered by the Butterworth HPF with 150-day cutoff period.

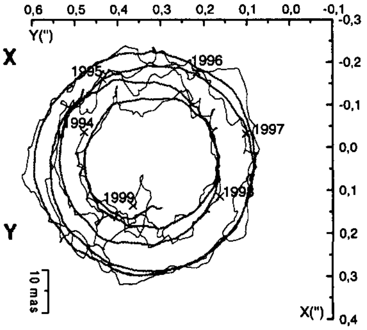

Figure 2. Polar motion IERS C04 determined in 1994–1999. The thin line represents the short-period variations with periods less than 150 days increased by 5 times. The scale of their amplitudes is given in the lower left corner.

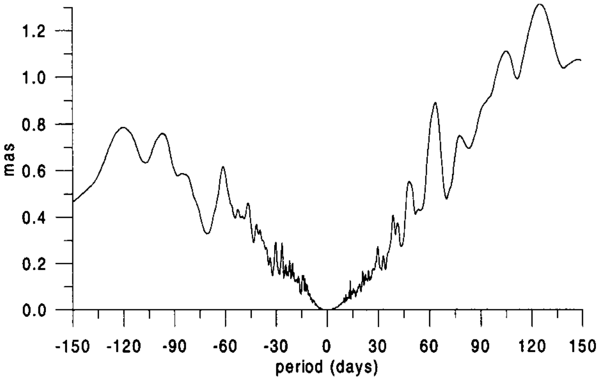

Figure 3. The FTBPF amplitude spectra ($\lambda = 0.001$) of x-iy IERS C04 pole coordinate data in 1984.0–1999.0 filtered by the Butterworth HPF with 150-day cutoff period (Kosek *et al.* 1998b).

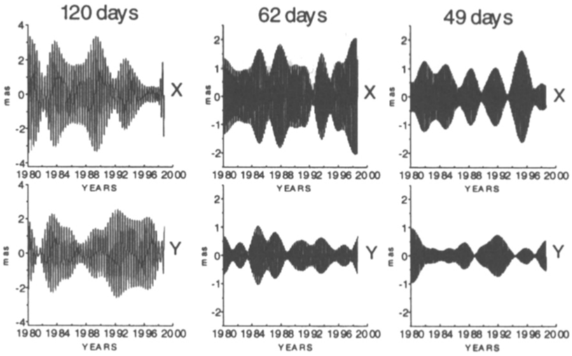

Figure 4. 120, 62 and 49 days oscillations filtered by the FTBPF ($\lambda = 0.001$) from the IERS C04 pole coordinates data.

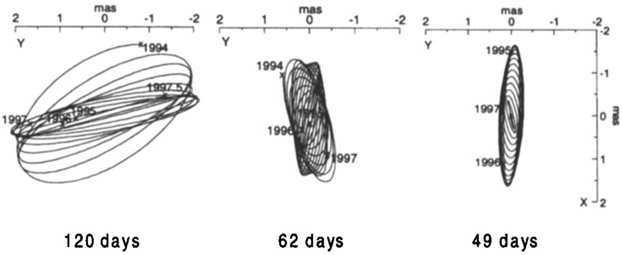

Figure 5. The polhodes of 120, 62 and 49 days oscillations filtered by the FTBPF ($\lambda = 0.001$) from the IERS C04 pole coordinate data.

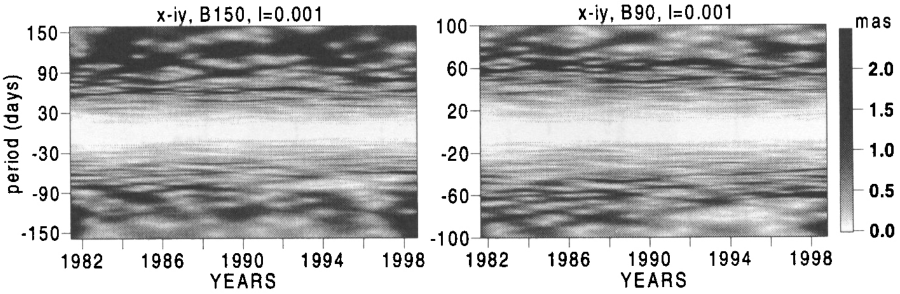

Figure 6. The time variable FTBPF amplitude spectra ($\lambda = 0.001$) of x-iy IERS C04 pole coordinates data filtered by the Butterworth HPF with 150 and 90 day cutoff periods (Kosek *et al.*, 1998b).

are not very strong. Thus they can be easily disturbed by different geophysical phenomena.

Sub-seasonal variations of polar motion are correlated with short-period variations of equatorial components of Atmospheric Angular Momentum (AAM). In the spectra of AAM there are short-period oscillations with periods of about 120, 90, 60, and 50 days, similar to those in polar motion (Kosek *et al.*, 1995). The running correlation coefficients between short-period variations of the atmospheric equatorial components (NCEP/NCAR w+p+ib) and geodetic IERS C04 excitation function of polar motion were computed (Fig. 7) (Kosek *et al.*, 1998b). The correlation coefficients are time variable and have seasonal variations. Their maxima reach values of 0.6–0.7.

LOD sub-seasonal variations obtained by filtering the IERS C04 data using the Butterworth HPF with cutoff period of 150 days have amplitudes reaching maxima of 0.5 ms. Before the filtering, the influence of solid Earth tides was removed according to the IERS Conventions (IERS, 1996).

The computed amplitude spectrum and time variable spectrum of the LOD sub-seasonal oscillations shows that the most energetic oscillations are the oscillations with periods of about 120, 60–90, 50 and 40 days (Fig. 8). The sub-seasonal variations of LOD are highly correlated with the variations of the axial component of the Atmospheric Angular Momentum and the correlation coefficients are close to 1 for oscillations with periods longer than 20 days (Kosek *et al.*, 1999a). This correlation was studied by many authors (Dickey, 1989; Eubanks 1993; Hide *et al.*, 1991). As in the case of polar motion the oscillations of LOD with periods of 120, 60–90 and 40 days can be of seasonal origin. The 50 day oscillation in LOD is considered to be connected with stratospheric winds.

3. Daily and sub-daily variations of Earth rotation parameters observed by VLBI

Today, a very high temporal resolution of the Earth rotation parameters can be achieved by VLBI using the least-squares collocation method (LSCM). It allows

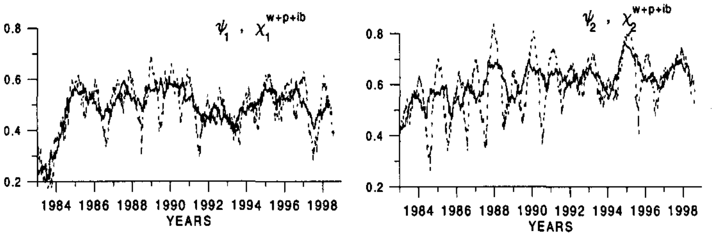

Figure 7. The running correlation coefficient between the atmospheric (NCEP/NCAR (w+p+ib)) and geodetic IERS C04 excitation functions filtered by the Butterworth HPF with a 150-day cutoff period, (thin and dotted lines denote 1 year and 0.5 year means, respectively) (Kosek *et al.*, 1998b).

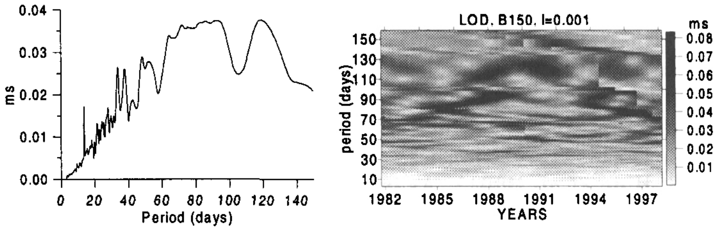

Figure 8. Amplitude spectrum and time variable amplitude spectrum ($\lambda = 0.001$) of LOD IERS C04 data filtered by the Butterworth HPF with 150-day cutoff period (Kosek *et al.*, 1998b).

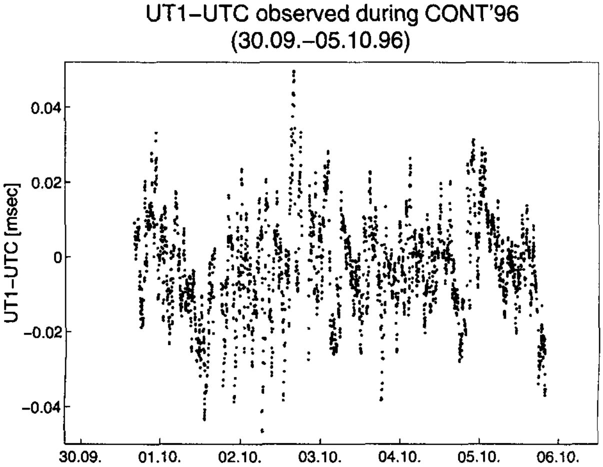

Figure 9. UT1-UTC parameters with high temporal resolution determined during five days of the VLBI session CONT which started on September 30, 1996 (ocean tidal terms were removed from the observations using the IERS Conventions (IERS, 1996) model).

us to estimate the Earth rotation parameters every 3–7 minutes (Titov, 1999). This paragraph concentrates on short-period variations of the Earth rotation parameters which can be seen when analysing the VLBI results. More than 300 VLBI experiments (NEOS-A from 1993 till 1998, CONT'96, CORE) each of them covering 24h or 120h were analysed by the OCCAM 3.4 VLBI software (Titov *et al.*, 1997). The high-resolution UT1–UTC data series and polar motion data series obtained by the VLBI solutions were analysed with respect to periodic variations. First, the main diurnal and semidiurnal variations due to the ocean tides were removed using the IERS Conventions (IERS, 1996) correction model. As an example Figure 9 shows UT1–UTC parameters determined during five days of the VLBI campaign CONT in 1996.

The residuals, *i.e.* after correction of oceanic tidal terms, offer the opportunity to search for even smaller variations in the rotation of the Earth. Those are supposed to be *e.g.* due to resonances with modes of the Earth, the oceans or the atmosphere or due to excitation by earthquakes or by strong typhoons. The wavelet transformation was applied to search for transient and irregular variations. Although ocean tidal influences had been already corrected, many of the wavelet spectra of the UT1 series show residual energy in the diurnal and semidiurnal period range. Additionally, most of the data sets revealed irregular quasi-periodic fluctuations which are non-diurnal and non-semidiurnal. Very often (but not always) periods at 20 hours and 40 hours, around 8 hours, between 5 and 7 hours and even down to 2 to 3 hours were found. To give an example of that, the wavelet spectrum for the CONT'96 UT1–UTC series shown in Figure 9 is plotted in Figure 10 for periods below 36 hours. Besides the residual diurnal and semidiurnal bands mentioned above, irregular periodic variations can be seen at 20h, 8h, 5h and 2h-3h that are above the error level of the individual UT1–UTC parameters of about 10 microseconds. The VLBI so-

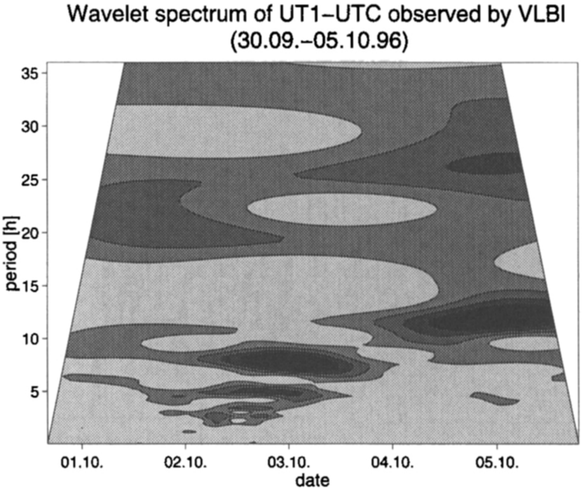

Figure 10. Wavelet spectrum for periods below 36 hours of the high-resolution UT1−UTC series shown in Figure 9.

lutions using the LSCM yielded also high-resolution polar motion series. These were analysed as complex values by the wavelet transformation. The wavelet spectra revealed irregular prograde and retrograde short-period fluctuations in polar motion, again very often around 8 hours and even with shorter periods, *e.g.* at 5–7 hours.

4. Conclusions

*In the paper the character of variations of the most energetic sub-seasonal os*cillations of polar motion with periods of 120, 60, 50 days and of LOD with periods of 120, 60–90, 50 and 40 days is described. They have variable amplitudes modulated with a period of 2–4 years. The sub-seasonal variations of polar motion and LOD are correlated with sub-seasonal variations of AAM. The running correlation coefficients are variable with maxima ranging between 0.6 and 0.8. They have seasonal variations.

High resolution Earth rotation data observed by VLBI have begun to reveal hitherto unseen phenomena. Irregular, quasi-periodic variations can be seen in the wavelet spectra besides the well-known diurnal and semidiurnal periods. Possible excitation mechanisms for the observed variations of the Earth rotation parameters such as terdiurnal tides, the free oscillations of the oceans (10h, 20h, 40h, 50h), atmospheric modes and resonances with free oscillations of the Earth (*e.g.* the Slichter mode) are under discussion. A triggering by strong earthquakes or by a typhoon should also be considered. Although the tiny variations which could be seen are above the error level of the observations careful tests have to be done to check their significance. In particular, the correlation with tropospheric refraction at the observing stations is going to be investigated.

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