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1. INTRODUCTION

Basic features of observed Type I X-ray bursts are thought to be interpreted in terms of helium shell-flash near the surface of the accreted neutron stars (Lewin and Joss 1981). Numerical models of the helium shell-flash show that the luminosity grows very close to the Eddington luminosity just after the peak of energy generation.

The Eddington luminosity is given by

$$L_{Ed} = 4\pi cGM_r/\kappa \quad (1)$$

Here M_r is the mass contained within the sphere of the radius r , which is well approximated by the total mass of the neutron star M . The opacity κ is described by the Compton scattering opacity

$$\kappa = 0.20/(1+\alpha T); \quad \alpha = 2.2 \times 10^{-9} K^{-1}, \quad (2)$$

where the chemical compositions devoid of hydrogen is assumed, and T is the temperature. From equations (1) and (2) we see that the Eddington luminosity is a local quantity which is proportional to $(1+\alpha T)$; it decreases outward as the temperature decreases.

What happens when a radiation flux comes which exceeds the Eddington luminosity at the surface of the star $L_{Ed}^{(0)}$? It must push the outer shells to lead the mass loss from the star. It is the aim of the present paper to solve for the stellar structure with such steady mass outflow.

Many years ago Zytkov (1972) considered a similar problem and calculated stellar envelopes with mass outflow. However, her solutions could not be fitted consistently to any star or core of main-sequence- or

*Details of this work are submitted for publication by Ebisuzaki et al. (1982) and by Kato (1982).

of white-dwarf-type. Here, we will show that such envelopes can be fitted to the accreted neutron stars in the stages near the peak of the helium shell-flash.

We shall solve for the structure of the inner shells with temperatures as high as $10^8 - 10^9$ K through the outermost shells of lower temperatures, the latter of which is outflowing as Parker-type stellar wind. Ruggles and Bath (1979) computed such Parker-type stellar wind solutions which simulated the mass outflow at nova explosion of accreted white dwarfs. They were interested in the flow patterns etc. of the stellar wind. Therefore, they assumed that a given value of mass flux came from the interior, and they solved only for the part of the Parker-type stellar wind. In this sense their computations did not give any insight into the driving mechanism of the mass loss. On the contrary, the main aim of our present study is to clarify the driving mechanism of the mass loss operating rather deep interior where α_T of equation (2) is not negligible as compared with unity ($T \approx 10^8 - 10^9$ K). Therefore, our outer boundary conditions of the Parker-type solution is much less important than the solutions in the deep interior.

2. BASIC EQUATIONS AND ASSUMPTIONS

Stellar structure with a steady mass outflow with velocity \underline{v} is described by

$$\underline{v} \frac{dr}{dr} + \frac{GM}{r^2} + \frac{1}{\rho} \frac{dP}{dr} = 0 \quad , \quad (3)$$

$$\frac{d}{dr} (4\pi r^2 \rho v) = 0 \quad , \quad (4)$$

$$\kappa_L^{(dif)} = - \frac{16\pi a c r^2 T^3}{3\rho} \frac{dT}{dr} \quad , \quad (5)$$

$$vT \frac{ds}{dr} + \frac{\rho}{4\pi r^2} \frac{dL_r}{dr} = 0 \quad , \quad (6)$$

where \underline{P} , ρ , \underline{s} and $L_r^{(dif)}$ are, respectively, the pressure, the density, the specific entropy and the diffusive energy flux flowing through a shell at r . The energy is transported not only as the diffusive luminosity but also as the advection luminosity $L_r^{(adv)}$. Since we shall treat the shells exterior to the helium convective zone, equation of state is well described by those of the ideal gas plus radiation.

As the inner boundary conditions, the solution is fitted to the core of the neutron star at a shell where the ratio of the radiation pressure to the gas pressure \underline{y} is equal to unity; the temperature should be equal to T_b , and the radius should be equal to the radius of the inert neutron star. (We have found that the choice of the boundary value of \underline{y} affects but very slightly to our results.) As the outer boundary conditions, the solution is fitted to the Parker-type stellar wind at the sonic point. Thus, our problem is a boundary value problem which contains a parameter T_b at our disposal. The mass flux $\underline{\phi}$ is obtained as an eigenvalue. Then the velocity profile \underline{v} , the diffusive flux

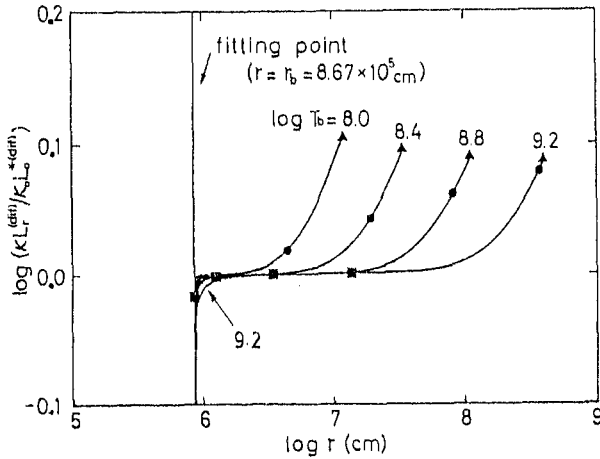


Figure 1. The ratio of the computed $\kappa_{Lr}^{(dif)}$ to the assumed value of $\kappa_{L0}^{*(dif)}$ is plotted against r for solutions with different T_b . Filled square, circle, and triangle denote the color photosphere, scattering photosphere (see below), and the sonic point, respectively.

$L_r^{(dif)}$ etc. are determined.

In general, equations (3)-(6) can be solved only numerically. However, we can solve them analytically if we approximate

$$\kappa_{Lr}^{(dif)} = \kappa_{L0}^{*(dif)} = \text{const.} \tag{7}$$

In addition to this equation we obtain analytical expressions of three integrals which describe the constancy of the mass flux, the Bernoulli's theorem, and a barotropic relation. Differentiating temperature distribution of such analytical solution, we can compute $\kappa_{Lr}^{(dif)}$ by means of equation (5). Then, we can check the consistency of our procedure, as is done in figure 1. We have found that equation (7) holds very precisely except for very outer region and for the region close to the inner boundary.

In order to check the validity of such approximation Kato (1982) has solved equations (3)-(6) numerically for similar problem. Her results show that $\kappa_{Lr}^{(dif)}$ is constant in the region above the shell where the local Eddington luminosity becomes almost equal to the incoming diffusive luminosity. This constancy holds upto the scattering photosphere which is defined below. Here we refer to such Kato's (1982) results in order only to demonstrate the validity of our approximation. Now we shall return to discuss the results of our analytical treatment, since it will give a better insight into physical processes involved in our problem.

3. RESULTS

For the inert neutron star we used the mass of $0.476 M_{\odot}$ and the radius of $r_b = 8.67 \text{ km}$. Numerical results are summarized in table 1, where the subscript cp and sp denote the color- and the scattering-photosphere, respectively. The color photosphere is defined with

$$\int_{r_{cp}}^{\infty} (3\kappa_{ff}\kappa_E)^{1/2} \rho dr = 1, \tag{8}$$

Table 1. Models for steady-state mass loss from neutron star.

model	1	2	3	4	
log T_b (K)	9.20	8.80	8.40	8.00	
log Φ ($g\ s^{-1}$)	18.76	18.35	17.94	17.51	
log r_s (cm) (sonic point).....	8.60	8.04	7.54	7.08	
color sph.	log T_{cp} (K)	7.29	7.64	7.91	7.98
	log τ_{cp}	2.30	2.51	2.68	2.61
scatt. photosph.	log r_{cp} (cm)	7.14	6.54	6.11	5.94
	log R_{bb} (cm)	6.03	5.33	4.79	4.65
	log T_{sp} (K)	5.89	6.22	6.54	6.84
	log r_{sp} (cm)	8.54	7.88	7.26	6.67
	log $L_{sp}^{(dif)}$ ($erg\ s^{-1}$)	38.08	38.08	38.08	38.08
	log $L_{sp}^{(adv)}$ ($erg\ s^{-1}$)	36.19	36.41	36.59	36.69
log v_{sp} ($cm\ s^{-1}$)	8.43	8.65	8.81	8.87	

where κ_{ff} and κ_E are the free-free and the electron scattering opacity, respectively. The scattering photosphere is defined by $\tau(r_{sp}) = 3/2$ where τ is the optical depth computed with κ_E .

Model 2 with $\log T_b(K) = 8.80$ is a typical solution for the star just after the peak of the helium shell-flash. We see that the diffusive luminosity is very close to the Eddington luminosity at the surface [$\log L_{Ed}^{(0)}$ ($erg\ sec^{-1}$) = 38.08]. The mass flux amounts to $10^{18}\ erg\ sec^{-1}$, but the velocity is rather low as compared with the light velocity. This is the reasons why the diffusive luminosity can not exceed the Eddington luminosity appreciably, and why the advection luminosity is relatively low at the scattering photosphere. We notice also that the surplus flux, i.e., $L_r^{(dif)}(T_b) - L_{Ed}^{(0)} = \alpha T_b L_{Ed}^{(0)}$, [$\approx 1.4 L_{Ed}^{(0)}$ for model 2], is almost equal to the rate of potential energy fed into the escaping gas, i.e., $(GM/r_b)\Phi$. This high conversion efficiency does not yield to any appreciable super-Eddington luminosity.

The observed color temperature will be rather close to T_{cp} than T_{sp} . The diffusive luminosity is expressed as $(16\pi/3) r_{cp}^2 T_{cp}^4 / \tau_{cp}$. If we interpret the luminosity in terms of the usual black-body law with the effective temperature equal to T_{cp} , the black-body radius is calculated as $R_{bb} = (4/3 \tau_{cp})^{1/2} r_{cp}$. Since we have $\tau_{cp} \approx 300$, the black-body radius is regarded as only about $r_{cp}/15$, and we can not recognize the large radius of the extended envelope directly from observations. Such effect has to be paid attention to when we analyze observed results.

REFERENCES

Ebisuzaki, T., Hanawa, T., and Sugimoto, D. 1982, submitted for publication to Publ. Astron. Soc. Japan.
 Kato, M. 1982, submitted for publication to Publ. Astron. Soc. Japan.
 Lewin, W.H.G., and Joss, P.C. 1981, Space Sci. Rev., 28, 3.
 Ruggles, C.L.N., and Bath, G.T. 1979, Astron. Astrophys., 80, 97.
 Zytkow, A. 1972, Acta Astron., 22, 103.

DISCUSSION FOLLOWING D. SUGIMOTO'S TALK

BATH: I did some work with a student of mine, Ruggles, in which we computed steady outflowing optically thick wind solutions from white dwarfs. With no approximations at all, the general Parker type solutions, but with the interaction of the radiation with matter included. Have you compared your solutions with your approximations with those solutions?

SUGIMOTO: We have not compared with your solution. We have compared with an old calculation by Zytkow.

BATH: The Zytkow work is not right. I would be interested to know how your work compares with ours. Because, when I was doing that work I was struck by the fact that the solutions were extremely sensitive to the eigenvalue nature of the problem and that unless you have your boundary conditions satisfied at the critical point and at the photosphere simultaneously, precisely, then the solutions were very disturbed.

SUGIMOTO: Concerning this problem, one of my colleagues, computed the same problem, without assuming that κL is constant. The result is essentially the same as ours. Concerning your discussion of the sensitivity of the solution to boundary conditions, we have to think of the physical situation, for example, if we change the boundary condition at the sonic point, it will make changes in the whole structure of the solution, but physically speaking, the layer is pushed up by surplus radiation, the pressure of the surface region is very low, so if we state, physically, that surface boundary condition has little to do with determining the mass flux, in such a case I think the assumption of steady mass flow is bad instead of that the boundary condition is important.

BATH: That may be true, but steady solutions are the only ones we can solve in a realistic way at the moment.

LAMB: I have a comment and a question. The comment refers to low mass neutron stars. While it is true that the problem of supernovae has not yet been solved, in all the cases that have been calculated it is impossible to form neutron stars with masses less than $0.7-0.9 M_{\odot}$. This is because when collapse occurs, part of the core falls in homologously and the reverse shock has to begin further out. In all calculations, the homologous core is in the range $0.7-0.9 M_{\odot}$. As for my question, I am trying to understand why the approximation $\kappa L_{\text{diffusion}} = \text{Const.}$ is roughly satisfied. I can imagine an electron scattering atmosphere where $\kappa = \text{Const.}$ and where the total luminosity might be constant in the envelope. But I would have thought that as soon as the advective part of the luminosity begins to dominate, the approximation must fail. Can you give us a physical picture of why the approximation is valid?

SUGIMOTO: The part of the advection energy and the energy which is necessary to push up the matter against the gravitational field, are all taken into account in the equation. People say that $\kappa L_T = \text{const.}$ is a polytrope of $n=3$, but the polytrope of $n=3$ is something like a singular point when the ratio of radiation pressure to gas pressure goes to infinity, so our solution is very different from the so-called $\kappa L = \text{const.}$, $n=3$. My understanding of the physical picture is that if

an excessive energy comes, it cannot be transported by diffusion, the layer will be pushed up in order to keep the diffusive luminosity just equal or lower than the Eddington luminosity, it implies $\kappa L = \text{const.}$

LAMB: The electron scattering optical depth from the photosphere is quite high in your solutions. Compton scattering is important if one is dealing with photon energies greater than mc^2/τ^2 . Even for $\tau \approx 20$, Compton degradation is going to be significant for the temperatures you are dealing with at the color photosphere. Therefore, I would think that degradation of the spectrum would be rather severe in the outer parts of the outflowing envelope and would affect the results significantly.

SUGIMOTO: Yes, that is true, it is only an order of magnitude estimate we are concerned with now, not the detailed quantitative solution, but the physical interpretation.

LAMB: It would seem that in the picture you are talking about one might expect strong magnetic fields and therefore opacities that could be significantly less than the usual Thomson scattering opacity. This would make possible super-Eddington fluxes.

SUGIMOTO: Yes, I agree that we should proceed to think about such situations.

MEYER: I think it is claimed that the situation with respect to a black body interpretation gives you a cooling solution, where you keep the surface area constant and you decrease the temperature. Would your models also fit to these observations within the range of uncertainties?

SUGIMOTO: Our models are concerned with only the very peak of the X-ray burst so we need very detailed observations near the peak of the burst, the black body radiation at constant radius observation concerns the decaying phase of the X-ray burst.