THE EVOLUTION OF OLDER REMNANTS

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Abstract. An older remnant will be defined as one in which radiative cooling occurs somewhere and has swept up enough mass for the details of the explosion to be less important than the state of the interstellar medium in which the explosion occurred. Without discussing any particular remnant in detail, I will consider how large and small scale density variations in the ambient medium affect the appearance and energetics of such remnants. Finally I will show that radiative instabilities can modify the emission spectrum of radiative shocks in such a way that a naive interpretation of these spectra can be very misleading.

Introduction. In the days when supernova remnants, or at least the theoretical ones, were spherical and expanded into a uniform medium, it was easy to decide what was meant by an older remnant. Remnants had three phases, free expansion, Sedov and radiative (Woltjer 1972) and a remnant was regarded as old if it had entered the radiative phase.

Unfortunately, such a simple picture is no longer adequate even as an idealisation of what happens. We know that the interstellar medium is inhomogeneous on many scales, some of which correspond to the sizes of supernova remnants and we also know that Type II supernovae can significantly modify the interstellar medium in their neighbourhood. All this means that there may not be a simple division of supernova remnant evolution into three phases.

For the purposes of this article, I will call a remnant old if radiative cooling is important somewhere, but will not insist that a

https://doi.org/10.1017/S0252921100102787 Published online by Cambridge University Press

significant part of the explosion energy has been radiated away. Observationally this means that a remnant is old if it has a filamentary structure whose emission is characteristic of radiative shocks. Examples are the Cygnus Loop, Shan 147 and Vela. As far as theory is concerned we have to look at the effects of radiative cooling on the dynamics and appearance of the remnant.

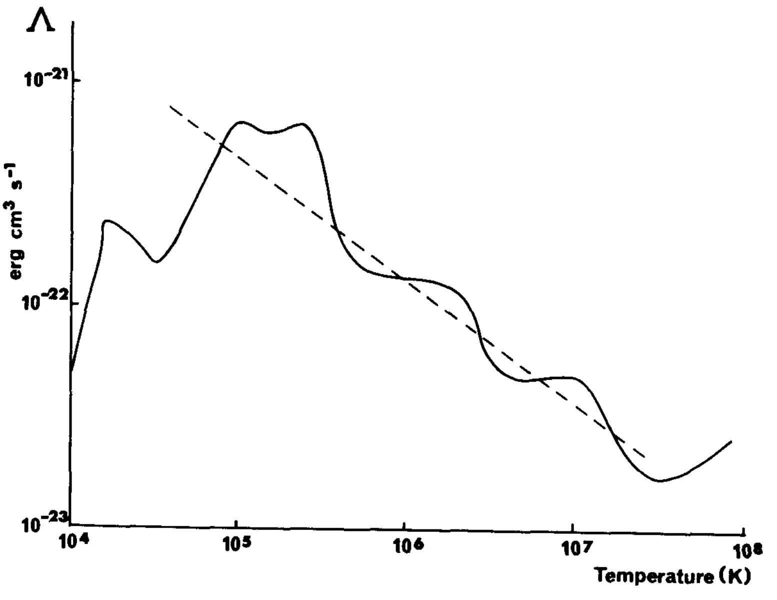

Figure 1. The radiative cooling rate per unit volume for an optically thin plasma. The straight line is the approximation (1) (Kahn 1976).

The radiative cooling rate for an optically thin gas in the relevant temperature range is shown in figure 1. Although this is not the most recent calculation, it has a maximum at about 10^5 K , which is what is important as far as the dynamics is concerned. Note that it does not include the effect of dust cooling which might well dominate above 10^5 K, depending upon the dust to gas ratio (Dwek these proceedings).

One of the nice things about this cooling law is that in the range 5×10^4 K < T < 5×10^7 K it is very well approximated by $T^{-1/2}$ power law,

$$\Lambda = A\rho^2(p/\rho)^{-1/2} \quad (A = 3.9\times10^{32} \text{ c.g.s}). \qquad (1)$$

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Kahn (1976) showed that this assumption makes it possible to calculate the effect of radiative cooling on the overall energetics independently of the details of the dynamics. I will first describe how this works for spherical remnants, and then show how it can be extended to supernovae in plane stratified media.

Spherical Remnant. Suppose that a supernova explosion has energy E_0, ejects mass M_e and occurs in a uniform medium with density ρ_0. Then there will be a Sedov phase provided

$$\left[\frac{E_0}{10^{51}}\right]^{-0.74} \left[\frac{M_e}{M_\odot}\right]^{5/6} \left[\frac{\rho_0}{10^{-24}}\right]^{0.2} < 4 \ . \qquad (2)$$

This is simply the condition that the remnant enters the Sedov phase before radiative cooling becomes important. It is based on Gull's (1973) calculations, which show that it looks like a Sedov solution once it has swept up about 50 M_e, combined with Cox's (1972) estimate of when radiative cooling becomes important.

One would prefer a Sedov phase to exist, because then all the details of the original explosion can be ignored and only the energy E_0 matters. Condition (2) is satisfied for all plausible values of E_0, M_e and ρ_0, but unfortunately it ignores the fact that a Type II supernova can modify its surroundings, either because of its ionizing radiation (Shull, Dyson, Kahn & West 1985), or its stellar wind (Charles, Kahn & McKee 1985). There is also a good deal of observational evidence that this occurs (Braun these proceedings).

I am going to ignore these complications and assume that the original state of the ambient medium is more important than the details of the explosion. Then in a uniform medium radiative cooling becomes important when the post shock temperature is

$$T = T_{sg} = 1.2\text{x}10^6 \left[\frac{E_0}{10^{51}}\right]^{0.1} \left[\frac{\rho_0}{10^{-24}}\right]^{0.2} \text{K} \qquad (3)$$

Cox (1972). For almost all supernova remnants this is in the range for which the approximation (1) is valid.

If we now ignore magnetic fields and assume that all shocks are

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strong, then as long as $T \geq 5\times10^4$ K everywhere, things only depend on E_0, ρ_0 and A. From these we can form a characteristic mass, length and time given by

$$m_c = \frac{(2.02E_0)^{6/7}}{\rho_0^{2/7}A^{3/7}} = 7.3\times10^{36}\left[\frac{E_0}{10^{51}}\right]^{6/7}\left[\frac{\rho_0}{10^{-24}}\right]^{-2/7} \text{gm},$$

$$l_c = \frac{(2.02E_0)^{2/7}}{\rho_0^{3/7}A^{1/7}} = 1.9\times10^{20}\left[\frac{E_0}{10^{51}}\right]^{2/7}\left[\frac{\rho_0}{10^{-24}}\right]^{-3/7} \text{cm}, \quad (4)$$

$$t_c = \frac{(2.02E_0)^{3/14}}{\rho_0^{4/7}A^{5/14}} = 1.2\times10^{13}\left[\frac{E_0}{10^{51}}\right]^{3/14}\left[\frac{\rho_0}{10^{-24}}\right]^{-4/7} \text{s}.$$

We expect radiative cooling to become important when the swept up mass is about m_c and the radius and age will then be approximately l_c and t_c respectively. Notice that these numbers are about right for the Cygnus Loop and IC443.

If $\gamma = 5/3$ and the cooling rate is given by (1), then the energy equation becomes

$$\frac{dK}{d\tau} = -\frac{2}{3}K^{-1/2}, \quad (5)$$

where $\tau = t/t_c$ is a dimensionless time and

$$K = \frac{pt_c^2 m_c^{2/3}}{\rho^\gamma l_c^4}$$

is the dimensionless adiabatic constant.

From (5) we can see that the rate of change of K of a fluid element is independent of its dynamics as long as the approximate cooling law holds. The time at which an element of gas cools is then

$$\tau_{cool} = \tau_s + [K(\tau_s)]^{3/2}. \quad (6)$$

Here τ_s is the time at which the element passed through the shock and the second term is the time it takes for K to decrease to zero according to equation (5).

In the Sedov phase we have

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$$M = \frac{4\pi}{3}\tau_s^{6/5}, \quad K(\tau_s) = \frac{4}{25}\frac{\upsilon(\gamma)}{\tau_s^{6/5}},$$
$$\upsilon(\gamma) = \frac{2(\gamma-1)^{\gamma}}{(\gamma+1)^{\gamma+1}} = 0.07 \quad \text{for } \gamma = 5/3. \tag{7}$$

Here M is the mass interior to the fluid element in units of m_c. Inserting (7) in (6) gives.

$$\tau_{cool} = \left[\frac{3M}{4\pi}\right]^{5/6} + \left[\frac{4\upsilon}{25}\right]^{3/2}\left[\frac{4\pi}{3M}\right]^{3/2} . \tag{8}$$

Hence at time τ all the gas for which $\tau > \tau_{cool}$ will have cooled. From figure 2a one can see that cooling first occurs at $\tau = 0.18$ and that its onset is sudden in the sense that most of the mass of the remnant cools at times only slightly later than τ_{cool}. At later times only a small fraction of the mass near the centre remains hot.

Plane Stratified Density Distribution. It is obviously important to see how the ideas of the previous section are modified if the supernova explodes in a non-uniform environment. To get a feel for what happens, let us consider a plane stratified exponential density distribution

$$\rho(z) = \rho_0(0)e^{-z/h} , \tag{9}$$

and assume that the scale height h is large enough for there to be an initial spherical Sedov phase.

Laumbach and Probstein (1969) derived approximate equations for the motion of the shock by assuming that all the energy and mass in each solid angle is conserved and concentrated near the shock. These equations are

$$Z = \frac{r_s}{h}\cos\theta , \quad \frac{dZ}{d\tau'} = U ,$$
$$\frac{dU}{d\tau'} = \left[\frac{i - G(Z,\phi)U^2}{F(\phi)\phi}\right], \quad \frac{d\phi}{d\tau'} = \frac{\rho_0(Z)}{\rho_0(0)} Z^2 U , \tag{10}$$
$$\tau' = t\left[\frac{E_0|\cos\theta|^5}{4\pi\rho_0(0)h^5}\right]^{1/2}, \quad \begin{array}{ll} i = 1 & 0 \le \theta < \pi/2 \\ i = -1 & \pi/2 \le \theta < \pi . \end{array}$$

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Here r_s is the radius of the shock and θ is the angle between the shock and the z axis. Notice that these equations need only be integrated twice, once for each value of i, to get the shock position for all θ and t.

Integration of these equations shows that the velocity of the upward moving shock ($0 \le \theta < \pi/2$) initially decreases, reaches a minimum at $\tau' = 2.4$ and then increases again. Garlick (1983) and Falle, Garlick and Pidsley (1984) have shown that the acceleration of the rising shock calculated from the approximate solution agrees remarkably well with the results of full numerical calculations. We can therefore use the approximate solution to estimate the effects of cooling.

There is now a dimensionless parameter which we can define to be

$$\beta = \frac{l_c}{h} = \frac{1.9\times10^{20}}{h}\left[\frac{E_0}{10^{51}}\right]^{2/7}\left[\frac{\rho_0(0)}{10^{-24}}\right]^{-3/7}. \qquad (11)$$

Clearly for $\beta \ll 1$ cooling occurs while the remnant is still spherical, while for $\beta \gg 1$ there will be no cooling in the top part of the remnant since the rising shock will have begun to accelerate long before cooling sets in.

The energy equation is as before except that the entropy behind the shock is now a function of θ as well as time. In fact

$$K(\tau'_s,\theta) = \frac{(\beta|\cos\theta|)^3}{8.08\pi}\,\frac{\upsilon(\gamma)U^2}{[\rho_0(Z)/\rho_0(0)]^{\gamma-1}}, \qquad (12)$$

where $U(\tau')$ and $Z(\tau')$ are determined from the approximate equations (10). Using our previous approximation for τ_{cool}, we get

$$\tau_{cool} = \frac{(8.08\pi)^{1/2}}{(\beta|\cos\theta|)^{5/2}}\tau'_s + \frac{(\beta|\cos\theta|)^{9/2}}{(8.08\pi)^{3/2}}\left[\frac{\upsilon(\gamma)U^2}{[\rho_0(Z)/\rho_0(0)]^{\gamma-1}}\right]^{3/2} \qquad (13)$$

As before we really want τ_{cool} as a function of the mass interior to the fluid element. From (10) we have

$$M = \frac{4\pi}{(\beta|\cos\theta|)^3}\,\frac{1}{\rho_0(0)}\int_0^Z \rho_0(x)x\,dx = \frac{4\pi}{(\beta|\cos\theta|)^3}\,\phi(Z), \qquad (14)$$

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where $M/4\pi$ is the mass per unit solid angle interior to the shock.

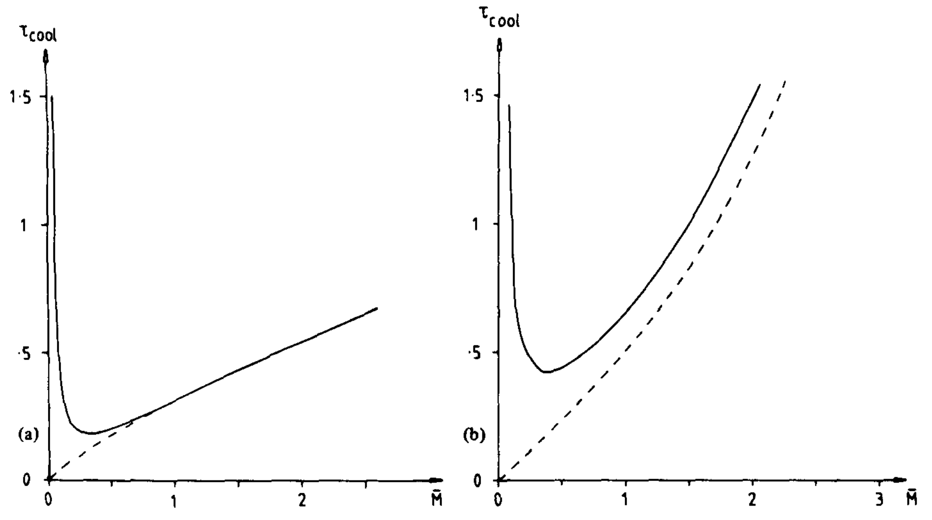

Figure 2. Cooling time (solid line) as a function of the Lagrangean coordinate $\bar{M}$. Mass interior to the shock (dashed line). a) Spherical remnant, b) $\beta|\cos\theta| = 2$. (Falle, Garlick & Pidsley 1984).

Figure 2b shows the result for the upwards moving shock ($\theta < \pi/2$) for $\beta|\cos\theta| = 2$. In contrast to the spherical case, there is now hot gas immediately behind the shock as well as in the interior of the remnant. The amount of hot gas behind the shock depends very sensitively on $\beta|\cos\theta|$ since it is determined by the ratio of the two terms on the right hand side of equation (13). This means that for $\beta|\cos\theta|$ greater than a certain value the gas does not cool behind the upward moving shock, while for smaller values it does.

In fact for $\beta|\cos\theta| > 2.8$, cooling occurs after the rising shock has begun to accelerate and so it is reasonable to suppose that the gas does not cool in a cone with semi-angle θ_0 given by

$$\beta|\cos\theta_0| = 2.8 \ . \tag{15}$$

The amount of hot gas in this cone can be considerably greater than that produced by a spherical remnant in the same mean density, so these

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effects ought really to be taken into account when making estimates of the rate at which supernovae inject energy into the interstellar medium.

Although I have only discussed an exponential density distribution, similar results are obtained provided the density decreases sufficiently rapidly with z for the rising shock to accelerate. Furthermore in all such cases the top part of the remnant disconnects from the bottom part so that for many purposes they can be considered separately. Falle & Garlick (1982) have exploited this fact to construct a model of the Cygnus Loop in which the explosion occurs on the dense side of a plane density discontinuity.

Many authors have carried out full numerical calculations of single or multiple explosions in plane stratified media (e.g. Chevalier & Gardner 1974; Tenorio-Tagle, Rozyczka & Yorke 1985; Tomisaka & Ikeuchi 1986; McCray these proceedings). In theory such calculations should give us much more information than the kind of analysis I have just described. Unfortunately there is a major snag, namely that in none of them is the cooling region adequately resolved. The result is that not only is the radiative energy loss incorrect, but the various instabilities are not correctly modelled. This does not mean that these calculations are useless, but it does mean that they must be treated with some caution.

Small Scale Inhomogeneities. In the previous section I looked at the effect of density variations whose scale was of the same order as the size at which the remnant becomes radiative. However, the appearance of remnants like the Cygnus Loop suggests that there are also irregularities on much smaller scales. Indeed McKee & Cowie (1975) have argued that in the Cygnus Loop we only see optical filaments when shocks propagate into small clouds.

The interaction of a plane shock with density inhomogeneities has been looked at by many authors (e.g. Sgro 1975; Chevalier & Theys 1975; Woodward 1976; Nittmann, Falle & Gaskell 1982; Hamilton 1985; Heathcote & Brand 1983). Although we have a rough idea of what happens, at least in the adiabatic case, there are a number of important details which are

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not clear. Since the propagation of shocks in non-uniform media is discussed by McKee (these proceedings), I will concentrate on some laboratory experiments on adiabatic flow past rigid bodies and suggest how these might help us to understand the much more complicated astrophysical problem.

Figure 3. Shadowgraphs of shock diffraction on a rigid wedge (van Dyke 1982, p. 146).

Figure 3 shows a series of shadowgraphs of a shock incident on a rigid wedge. Although this is adiabatic flow and a wedge is perhaps not the most relevant shape, it does show a number of features of interest. Firstly there is a reflected shock which becomes the bow shock at large times provided the flow is supersonic behind the incident shock. There is also a diffracted shock which reflects off the symmetry axis at the rear of the wedge. Initially this is a regular reflection, but as the angle of incidence increases it becomes a Mach reflection with a post shock pressure which is substantially higher than that behind the incident shock. Finally intense vortices are formed at the corners and then move downstream at the local fluid velocity.

What we have to decide is how much of this is relevant to the

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astrophysical problem. First consider the geometry of the object. Provided the object is not streamlined for supersonic flow, all the features that I have described will be present whatever the shape of the object. For example the flow past a cylinder or sphere is very similar except that two sets of vortices are formed (Bryson & Gross 1961).

Of course supernova remnants do not encounter rigid objects, at least not of significant size. However, from pressure balance we have

$$V_c = V_e \left[\frac{\rho_e}{\rho_c} \right]^{1/2}, \tag{16}$$

where V is a shock velocity and the subscripts e and c refer to the exterior and cloud respectively. So if the cloud is much denser than its surroundings, it deforms slowly compared to the timescale of the exterior flow. The cloud therefore behaves like a rigid body, at least as far as the transient stage of the exterior flow is concerned.

Once the transients in the exterior flow have died away, we get a quasi-steady flow past a slowly deforming body. In principle one could calculate this flow, find the pressure distribution on the cloud and so determine how it deforms. In practice this is very difficult, but we can nevertheless draw some qualitative conclusions.

For almost any shape, the minimum pressure will be at the widest part of the body and the difference between this pressure and the maximum pressure will be of the order of the ram pressure. So, provided the exterior flow is not very subsonic, the cloud will expand sideways at something like its sound speed and will therefore disrupt in about a sound crossing time. That such a sideways expansion occurs can be seen in van Dyke (1982 p.86) which shows a water drop suddenly immersed in an air stream whose ram pressure is considerably higher than that due to surface tension. One consequence of this disruption is that clouds cannot be coherently accelerated to anything like the exterior flow speed (Nittmann, Falle & Gaskell 1982).

From this we can see that laboratory experiments are really very useful as long as the exterior flow is adiabatic and the density contrast is large. In particular we can use them to check the reliability of numerical simulations such as those in Woodward (1976)

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and Nittmann, Falle & Gaskell (1982). Unfortunately none of the calculations in the astrophysical literature are for cases for which the experiments are relevant. It would be interesting to do a high density contrast adiabatic calculation with a modern numerical scheme just to see how good the results are.

Finally let us consider the influence of cooling on shock-cloud interactions. The first thing is that cooling can amplify small density fluctuations in the undisturbed gas (Chevalier & Theys 1975). This means that inhomogeneities that would be unimportant in the adiabatic case can have observable effects once cooling occurs. Apart from this we might get high velocity dense regions where the shocks intersect at the rear of the cloud (Tenorio-Tagle & Rozyczka 1984). There are also various radiative instabilities which would cause corrugations in the shock fronts so that both the interior and exterior flow is much more complicated than in the adiabatic case.

The trouble is that, although we have some idea of the consequences of cooling, we are unable to come up with reliable quantitative results which can be compared with the observations. Numerical simulations, at least those carried out so far, are too crude to be much use. What is needed is high resolution numerical calculations supported by the sort of analysis described earlier. In this context I think that Whitham's area rule for shock propagation might be very useful (Whitham 1974).

Despite all these theoretical difficulties, we can say something. In the first place the dominant shocks in the cloud will tend to propagate parallel to the primary shock since the lateral shocks are much slower. So if only the cloud shocks are radiative, we will tend to see tangential filaments such as those in the Cygnus Loop. On the other hand if the exterior shocks cool, we should see large scale lacy structures such as those in Shan 147 and Vela.

Radiative Instabilities. We can write the cooling rate shown in figure 1 in the form

$$\Lambda = A\rho^2\Phi(p/\rho c_*^2) \ , \tag{17}$$

where

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$$c_*^2 = \frac{kT_*}{\mu m_h}, \tag{18}$$

and T_* is a reference temperature. T_* can be chosen to be the temperature at the maximum of Φ ($T_* = 10^5$ K). If we then set $\Phi(T_*) = 1$, we get $A = 2\text{x}10^{26}$ c.g.s.

For a spherical remnant the flow is now governed by the parameters E_o, ρ_o, A, and c_* and from these we can form a dimensionless parameter

$$\alpha = \frac{T_{sg}}{T_*} = 10\left[\frac{E_0}{10^{51}}\right]^{0.11}\left[\frac{\rho_0}{10^{-24}}\right]^{0.22}. \tag{19}$$

Here T_{sg} is the temperature defined by equation (3). α only affects the evolution of the remnant if there is radiatively cooling gas at temperatures below T_*.

Let us now look at the stability of gas whose cooling rate is given by (17). Suppose that $\Phi(T) \propto T^s$. Then if cooling occurs at constant pressure, the cooling time increases with increasing temperature if $s < 2$, while for constant density this is true for $s < 1$. This suggests that there is instability if $s < 2$ for constant pressure cooling and $s < 1$ for constant density.

Now the pressure will remain roughly constant if the cooling time $t_{cool} \gg t_{dyn}$ where t_{dyn} is some dynamical timescale. Conversely the density will remain constant if $t_{cool} \ll t_{dyn}$. Suppose that a region of initial size ℓ begins to cool and that the resulting compression is one dimensional. Then

$$\ell(t) \propto \frac{1}{\rho(t)}.$$

The relevant dynamical time is obviously the sound crossing time, so

$$t_{dyn} = \frac{\ell}{c} \propto \frac{1}{\rho T^{1/2}}.$$

On the other hand we have for the cooling time

$$t_{cool} \propto \frac{p}{\rho^2 T^s} \propto \frac{1}{\rho T^{s-1}}.$$

Hence

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$$\frac{t_{dyn}}{t_{cool}} \propto T^{s-3/2} , \tag{20}$$

and so increases as the gas cools if $s < 3/2$. If t_{dyn} ever becomes much smaller than t_{cool}, then we expect a large pressure imbalance to occur which will lead to the formation of shocks. A necessary condition for this is $s < 3/2$. For the interstellar cooling law this condition is satisfied for $T > T_*$ and so we expect this kind of instability for spherical remnants if $\alpha > 1$. From (19) we can see that this should happen for almost all such remnants.

Various authors have looked at radiative instabilities. Both Avedisova (1974) and McCray, Stein & Kafatos (1975) carried out a linearised stability analysis with the post shock pressure held fixed. They found that density fluctuations grow if $s < 3$ for perturbations with wavelength much greater than the cooling length, while $s < 2$ is required if the wavelength is much shorter than the cooling length. However, these are isobaric instabilities and do not lead to the formation of additional shocks.

In my numerical calculations of thin shell formation in spherical remnants (Falle 1975, 1981), I found that cooling led to the formation of multiple shocks which caused large variations in the speed of the primary shock. Langer, Chanmugam and Shaviv (1981,1982) found a similar effect in their calculations of radiative accretion onto white dwarfs.

These results have stimulated a lot of interest in such instabilities. Chevalier & Imamura (1982) used a linearised stability analysis to show that the shock speed will not be constant if $s < 0.8$, even if it is driven by a constant speed piston. To some extent this is confirmed by numerical calculations (Imamura, Wolff & Durisen 1984). Recently Bertschinger (1986) has extended this analysis to two dimensions and shown that in that case instability occurs if $s < 1$.

It has become common practice to deduce the velocity of radiative shocks by comparing the observed optical and UV line ratios with those predicted by steady shock models with various shock speeds (e.g. Raymond et.al. 1980). Unfortunately the above considerations suggest that radiative shocks will not be steady if the shock speed is high enough for cooling to occur in the unstable region of the cooling curve.

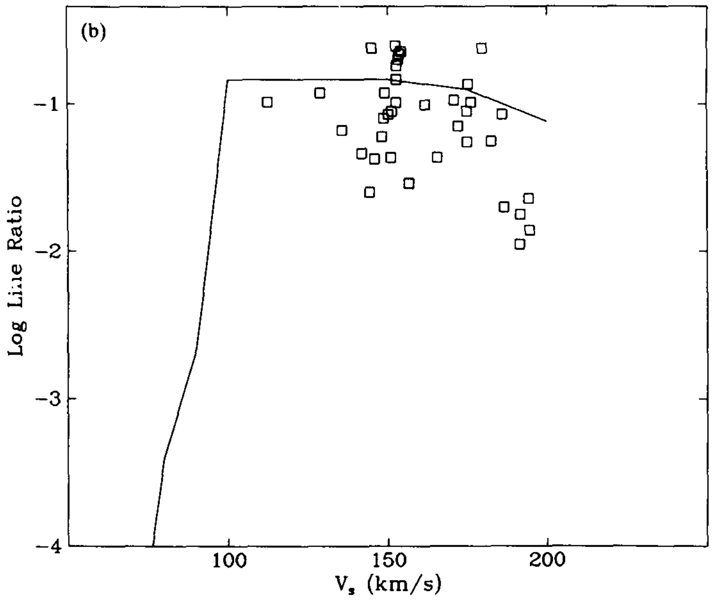

Figure 4. Instantaneous [OIII]5008/[OII]3728 line ratio for an unsteady shock whose mean speed is 175 km s^{-1}. The solid line is the ratio for a steady shock (Innes, Giddings & Falle 1987).

Recently Innes, Falle & Giddings (1987) have shown that, if the detailed atomic physics is included, then radiative shocks will be unsteady if their speed is greater than 130 km s^{-1}. The line ratios then do not correlate with the primary shock speed, nor even with the mean fluid speed, but vary dramatically on the cooling timescale. This effect can be seen in figure 4 which shows the instantaneous [OIII]5008/[OII]3728 line ratio plotted against the instantaneous primary shock speed for a shock driven by a constant speed piston such that the mean shock speed is 175 km s^{-1}. The variations in shock speed were induced by a single sinusoidal density perturbation upstream. The perturbation had an amplitude of 50% of the upstream density and a wavelength 1.4 times the thickness of the cooling region.

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Conclusions. I have discussed some of the effects that radiative cooling and density inhomogeneities can have on the evolution of supernova remnants. As far as the overall energetics is concerned, it is clear that the Kahn (1976) and Laumbach & Probstein (1969) approximations give us reasonable estimates of the efficiency with which supernovae inject energy into the interstellar medium.

Interactions with small scale inhomogeneities are more difficult to deal with, but we can use laboratory experiments, numerical simulations and perhaps Whitham's area rule (Whitham 1974) to deduce how clouds of various sizes and densities affect the appearance of remnants. We clearly need a reliable quantitative theory of such interactions in order to make the proper use of the detailed observations that are now possible.

Finally I have indicated that radiative instabilities must exist in radiative remnants and that these make it very difficult to interpret the spectra of radiative shocks. They may also be responsible for at least some of the complex structure seen in old remnants.

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