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# Model of radio emission from spherically symmetric pulsar wind nebulae

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**Abstract.** We study radio emission from pulsar wind nebulae (PWNe) considering the observed spatial structure. We assume spherical symmetry of the PWN, and model the evolution of the magnetic field and the particle energy distribution. We do not consider the synchrotron cooling of particles but consider the adiabatic cooling, because we are mostly interested in the radio emission from PWNe. The model is applied to the Crab Nebula and succeeds to reproduce the observed spatially integrated spectrum in radio with a single power-law injection. In our previous work (a one-zone model), in contrast, the integrated spectrum of the Crab Nebula is reproduced by a broken power-law injection of particles. However, the spatial structure in radio is inconsistent with observations and we need a radial velocity profile which is very different from the model by Kennel & Coroniti. Further studies of the spatial structure of PWNe are important to understand the origin of the radio emission from young PWNe.

**Keywords.** radiation mechanisms: nonthermal, (ISM:) supernova remnants, ISM: individual (Cran Nebula), pulsar: general

## 1. Introduction

Pulsar wind nebulae (PWNe) are a cloud of magnetized plasma injected from a central pulsar. The magnetized plasma inside PWNe experienced many physical processes, such as the pair cascade at the pulsar magnetosphere, the bulk acceleration of the pulsar wind, and the particle acceleration at the termination shock of the pulsar wind (see reviews by e.g., Gaensler & Slane 2006; Kirk *et al.* 2006). We can study the physics of the pulsars and their surroundings from the emission from PWNe.

A number of studies investigating the broadband emission from PWNe were done in a one-zone evolution model (e.g., Gelfand *et al.* 2009; Bucciantini *et al.* 2011; Tanaka & Takahara 2010, 2011). They succeed in reproducing the broadband spectrum from radio to TeV $\gamma$-ray, and determine the parameters of magnetized plasma injected from central pulsars and of spin-down evolution of central pulsars. There are, however, some problems. Especially the origin of radio emitting particles is unclear in this model. We consider that the observed spatial structures of PWNe would be a important tool to understand the radio emitting particles and we need to construct the model beyond the one-zone approximation. Here we study spatially resolved radio emission from PWNe.

## 2. The model

The spin-down power of the central pulsar is divided into particle and magnetic energy as $L_{\rm spin}(t) = \dot{E}_p + \dot{E}_B$. We introduce the constant parameter $\beta_0$ which is the ratio of the particle to the magnetic energy density at injection radius $r_0$ and then $L_{\rm spin}(t) \approx \dot{E}_p(1 + \beta_0^{-1})$.

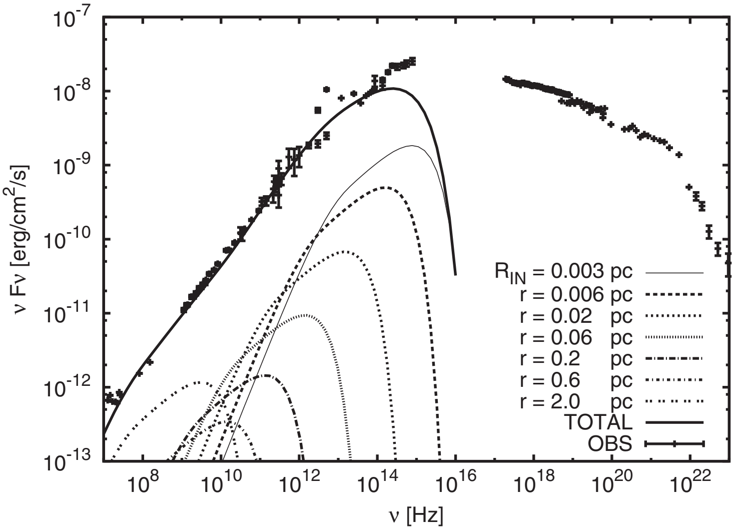

**Figure 1.** The integrated spectrum of the Crab Nebula from radio to X-rays. The observational data $< 10^{14}$ Hz are fitted by the total spectrum (thick line) which is the superposition of the emission from different radius (other lines).

The radio emitting particles stream from $r_0$ to the PWN. We ignore the diffusion of the radio emitting particles and they would transport by convection. Moreover, radio emitting particles are cooled by adiabatic cooling rather than radiative cooling. The spherical symmetry and isotropic distribution function of radio emitting particles $f(r, p, t)$ satisfies

$$\frac{\partial}{\partial t} f + \vec{u}\cdot\vec{\nabla} f - \frac{p}{3}(\vec{\nabla}\cdot\vec{u})\frac{\partial}{\partial p} f = Q_{\rm inj}, \tag{2.1}$$

where $\vec{u}(r)$ and $Q_{\rm inj}(r, p, t)$ represent the velocity of mean flow and the injection from the central pulsar. The velocity profile is assumed to have the power-law form $\vec{u} = u_0(r/r_0)^{1-\alpha_u}\,\vec{e}_r$.

For the injection from the central pulsar, we assume that the single power-law injection from the inner radius, i.e., $Q_{\rm inj}(r, p, t) = q(t)p^{-\alpha_p-2}\delta(r - r_0)\theta(p_{\rm max} - p)\theta(p - p_{\rm min})$. $q(t)$ is determined from $\xi \dot{E}_p(t) = \int_0^\infty 4\pi p^2 dp \int_0^\infty 4\pi r^2 dr \sqrt{p^2c^2 + m_{\rm e}^2c^4}\,Q_{\rm inj}(r, p, t)$, where $\xi \sim p_{\rm min}/p_{\rm max}$ accounts for the synchrotron cooling effect and we take $\alpha_p = 2$.

We assume the toroidal magnetic field inside the PWN, i.e., $\vec{B}(r, t^*) = B(t^*)(r/r_0)^{\alpha_B}\,\vec{e}_\phi$, where $t^*$ is the time the magnetic field injected at $r = r_0$. The normalization $B(t^*)$ is determined from $L_{\rm spin}$ and $\beta_0$. The power-law index $\alpha_B = \alpha_u - 2$ is determined from the induction equation

$$\frac{D\vec{B}(t^*, r)}{Dt} = \vec{\nabla}\times(\vec{u}\times\vec{B}(t^*, r)) + (\vec{u}\cdot\vec{\nabla})\vec{B}(t^*, r). \tag{2.2}$$

We calculate the synchrotron emission from a PWN assuming the radiation is isotropic.

## 3. Results

Figure 1 shows the result of the application to the Crab Nebula. We take the parameters of $\alpha_u = 1.16$, $r_0 = 3\times10^{-3}$pc, $\beta_0 = 10$, $p_{\rm max} = 500$GeV and $p_{\rm min} = 30$GeV which are fitted to reproduce the spatially integrated radio spectrum of the Crab Nebula. While the emission from infrared to X-rays reproduced by higher energy particles is not considered,

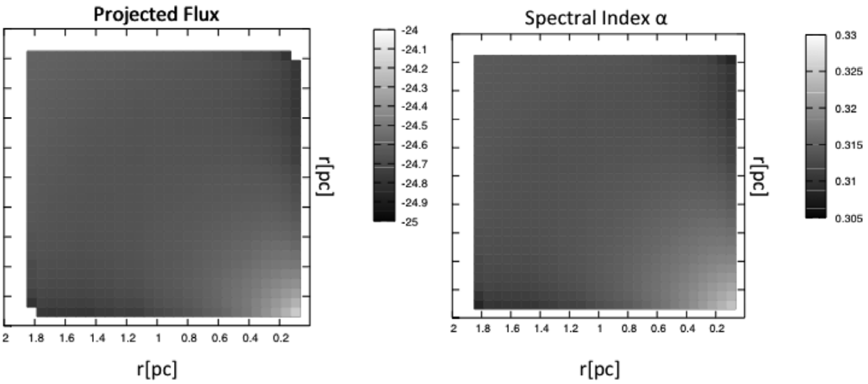

**Figure 2.** The calculated flux (left panel) and spectral index (right panel) projected on the sky. The used parameters are the same as Figure 1.

the radio emitting particles account for only $\xi = 1/20$ of total particle energy injected from the Crab pulsar.

The left panel of Figure 2 shows the projected flux in MHz calculated with the same parameters as in Figure 1. On the other hand, the radio observations of the Crab Nebula in 74 MHZ show almost uniform distribution of the surface brightness (Bietenholz *et al.* 1997). The right panel of Figure 2 shows the projected spectral index ($F_\nu \propto \nu^\alpha$) in MHz, calculated with the same parameters in Figure 1. The value $\alpha \sim 0.3$ in our calculation is different from the observed value $\alpha \sim -0.3$ (Bietenholz *et al.* 1997).

## 4. Conclusions

Radio emission from spherically symmetric PWNe is studied, and we applied the model to the Crab Nebula. In contrast to the one-zone model, the integrated spectrum of the Crab Nebula is reproduced without the injection of low energy particles of about 1 GeV. While the projected flux distribution in MHz seems consistent with the observation, the spectral index distribution is not. The advection of the particles (Eq. 2.1) and the toroidal magnetic field (Eq. 2.2) alone fail to reproduce the integrated spectrum and spatial structure simultaneously. The diffusion of the radio emitting particle may be important to reproduced the spatial structure of PWN. ¡

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