## THESIS ABSTRACTS

favour of a long-standing conjecture asking whether Turing determinacy implies the axiom of determinacy.

Abstract prepared by Clovis Hamel *E-mail*: chamel@math.toronto.edu

ANDREAS LIETZ, *Forcing* "NS $_{\omega_1}$  is  $\omega_1$ -Dense" from Large Cardinals. Universität Münster, Münster, Germany. 2023. Supervised by Ralf Schindler. MSC: Primary 03E57, Secondary 03E35, 03E55, 03E60, 03E25. Keywords: nonstationary ideal, forcing axioms, large cardinals, axiom (\*).

### Abstract

We answer a question of Woodin [3] by showing that "NS<sub> $\omega_1$ </sub> is  $\omega_1$ -dense" holds in a stationary set preserving extension of any universe with a cardinal  $\kappa$  which is a limit of  $<\kappa$ -supercompact cardinals. We introduce a new forcing axiom Q-Maximum, prove it consistent from a supercompact limit of supercompact cardinals, and show that it implies the version of Woodin's (\*)-axiom for  $\mathbb{Q}_{max}$ . It follows that Q-Maximum implies "NS<sub> $\omega_1$ </sub> is  $\omega_1$ -dense." Along the way we produce a number of other new instances of Asperó–Schindler's MM<sup>++</sup>  $\Rightarrow$  (\*) (see [1]).

To force Q-Maximum, we develop a method which allows for iterating  $\omega_1$ -preserving forcings which may destroy stationary sets, without collapsing  $\omega_1$ . We isolate a new regularity property for  $\omega_1$ -preserving forcings called respectfulness which lies at the heart of the resulting iteration theorem.

In the second part, we show that the  $\kappa$ -mantle, i.e., the intersection of all grounds which extend to V via forcing of size  $<\kappa$ , may fail to be a model of AC for various types of  $\kappa$ . Most importantly, it can be arranged that  $\kappa$  is a Mahlo cardinal. This answers a question of Usuba [2].

# REFERENCES

[1] D. ASPERÓ and R. SCHINDLER, *Martin's Maximum*<sup>++</sup> implies Woodin's axiom (\*). Annals of Mathematics (2), vol. 193 (2021), pp. 793–835.

[2] T. USUBA, *Extendible cardinals and the mantle*. *Archive for Mathematical Logic*, vol. 58 (2019), pp. 71–75.

[3] W. WOODIN, *The Axiom of Determinacy, Forcing Axioms, and the Nonstationary Ideal*, Walter de Gruyter, Berlin, 2010.

## Abstract prepared by Andreas Lietz

*E-mail*: andreas.lietz@tuwien.ac.at.

URL: https://andreas-lietz.github.io/resources/PDFs/AJourneyGuidedByThe
Stars.pdf.

ZHANSAYA TLEULIYEVA. *Algorithmic Properties of Rogers Semilattices* Nazarbayev University. Supervised by Manat Mustafa and Nikolay Bazhenov. MSC: 03D45. Keywords: theory of numberings, computable numbering, Rogers semilattice, limitwise monotonic, analytical hierarchy, projective determinacy, types of isomorphism.

#### Abstract

The thesis uses various approaches to explore the algorithmic complexity of families of subsets of natural numbers. One of these approaches involves investigating upper semilattices



