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THE FORMATION OF STRUCTURE IN THE UNIVERSE

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THE THEORY OF THE LARGE SCALE STRUCTURE OF THE UNIVERSE

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INTRODUCTION

The God-father of psychoanalysis Professor Sigmund Freud taught us that the behaviour of adults depends on their early childhood experiences. In the same spirit, the problem of cosmological analysis is to derive the observed present day situation and structure of the Universe from certain plausible assumptions about its early behaviour. Perhaps the most important single statement about the large scale structure is that there is no structure at all on the largest scale - 1000 Mpc and more. On this scale the Universe is rather uniform, structureless and isotropically expanding - just according to the simplified pictures of Einstein-Friedmann....... Humason, Hubble.....Robertson, Walker. On the other hand there is a lot of structure on the scale of 100 or 50 Mpc and less. There are clusters and superclusters of galaxies.

Much work has been done on the classification of these bodies into "richness classes" and attempts have been made to deduce from observations a "mass function" giving the distribution of matter among clumps of various sizes and masses. There is a firmly established division between regions with enhanced, higher-than-average density of stars and radio-sources and regions with density lower than average. In recent years correlation functions have been used to characterize the relation between density enhancements and the linear scales of the distribution of galaxies in space.

A systematic effort of measuring the redshifts (optical and 21 cm radio) of thousands of galaxies has resulted in confirmation of Hubble's law. Surprisingly it is approximately valid for smaller distances than those characteristic of the density distribution. The redshift measurements have opened the way for disentangling the three-dimensional structure of the universe, as opposed to the two-dimensional projection of astronomical objects on the celestial sphere.

The present symposium has really opened up a new direction in the

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search for the geometrical patterns governing the distribution of luminous matter in space. We heard about ribbons or filaments along which clusters of galaxies are aligned; the model of a honeycomb was presented with walls containing most of matter; the presence of large empty spaces (holes - not black holes of course) was emphasized. Cosmological theory must be aware of this information and try to use it in order to discriminate between various proposed schemes. Let us briefly characterize those schemes which seem to us most promising at the present time. There are two extreme assumptions which can be made about the initial density perturbations. The first concerns an ideal unperturbed metric connected by General Relativity with ideal homogeneity of the overall density in the early radiation dominated Universe. The perturbations consist of an inhomogeneous distribution of "matter" - of baryon excess - on a background of homogeneous radiation. Therefore the ratio γ/B (photons per baryon) varies from place to place. But the specific entropy of matter is proportional to this ratio and therefore those perturbations are called "entropy perturbations".

The second type of perturbation consists of common motion of photons and baryons. These perturbations conserve entropy and therefore they are called "adiabatic".

A departure from the main line of this report is permissible in the introduction. The actual value of the ratio γ/B which is of the order of 10^8 or 10^9, is most important for cosmology. The closed or open geometry of the Universe as a whole depends on this number.

Is it possible that in due time this number will be calculated by elementary particle theorists, taking into account the lack of exact symmetry between particles and antiparticles as indicated by laboratory experiments (so-called CP - violation, 1964) and also baryon non-conservation predicted by some theories? In this case it is conceivable, that the γ/B ratio is constant everywhere, just because the physical constants are everywhere the same. But this argument is not very strong. It is equally possible that γ/B depends on the interrelation of external physical constants and the local properties of the space-time metric; in this case the γ/B ratio must not be a constant.

Let us return to conventional cosmology. At the present moment we do not see any better policy than to make plausible assumptions about the size, amplitude and character of the initial perturbations, to develop logically and mathematically all the consequences of these assumptions and to compare them with observations.

In this report we shall investigate adiabatic perturbations - the second type, according to the classification given above. This investigation has been carried out during approximately the last ten years by our group, which includes Doroshkevich, Sunyaev, Novikov, Shandarin, Sigov, Kotok and others. We use important theoretical results obtained by Lifshitz, Bonnor, Silk, Peebles, Yu and others.

We consider several phases in the development of the perturbations:

1) acoustic oscillations of the radiation-dominated plasma and their attenuation before and during recombination;

2) the growth of small perturbations in the neutral gas;

3) the non-linear growth of perturbations leading to the formation of compressed gas layers - pancakes;

4) the further fate of pancakes, the interaction of pancakes, their decay into galaxies and protoclusters of galaxies.

The first two points are investigated using the "merry old" linear theory of perturbations. In 3) and 4) an approximate nonlinear theory is widely used and also numerical simulation. The statistical side of the problem is considered. Radio astronomical predictions are made. The main result is most encouraging: the adiabatic perturbation spectrum possesses a definite cut-off wavelength as already pointed out by Silk. We now see that this critical wavelength is also reflected in the cell structure of the Universe.

1. THE THEORY OF PERTURBATION GROWTH

A plausible featureless initial spectrum of density fluctuations in the radiation-dominated plasma is assumed. Due to photon viscosity and damping during recombination the final Fourier spectrum of growing perturbations in the neutral gas is given by

$$\overline{\left(\frac{\delta\rho}{\rho}\right)^2_K} = b_K^2 = a_K^2 K^n e^{-KR_c}$$

The critical length R_c depends on 1) the radiation density during recombination taking account of the specific effects of Ly-α reabsorption through the $2s \to 1s+2\gamma$ metastable hydrogen decay, 2) the Compton cross-section for scattering of photons by electrons, 3) the matter density or γ/B ratio. The best calculations give the characteristic length (multiplied by $(1+z_{rec})$ in order to account for the expansion from recombination to the present epoch)

$$R_c = 8 \text{ Mpc for } \Omega = 1 \text{ and } R_c = 40 \text{ Mpc for } \Omega = 0.1.$$

The wavelength λ_c is determined by $\lambda_c = 2\pi/R_c$, $K_cR_c = 1$ so that $\lambda_c = 2\pi R_c$.

The index n and the average value of a_K^2 are adjusted to fit the observed picture. But independent of this adjustment, due to the exponential damping factor e^{-KR_c} we are sure that the surviving fluctuations are very smooth. It is immediately clear that in the adiabatic theory early formation of stars or globular clusters or even of galaxies is impossible. First large-scale density enhancements must grow, and only thereafter is their fragmentation in smaller units possible.

The second qualitative feature is gas motion under the influence of gravitation only. The pressure forces, which depend on gradients, are negligible on the large scales involved. In this case the growth of perturbations is especially simple: they grow in amplitude due to gravitational instability and increase in linear dimensions, conserving their form. The density perturbations and the peculiar velocity (the excess over the Hubble velocity) are given by

$$\frac{\delta\rho}{\rho} = f\left(\frac{r}{R}\right)\phi(t) \quad ; \quad \underline{u} = \underline{U}\left(\frac{r}{R}\right)\psi(t)$$
$$\phi(t) = t^{2/3} \propto (1+z)^{-1}, \quad \psi(t) = t^{1/3} \propto (1+z)^{-1/2} \quad \text{for } \Omega = 1$$
$$R = \text{constant} \times t^{2/3} \propto (1+z)^{-1}$$

Obviously the density field and velocity field are connected by the continuity equation

$$\frac{\partial}{\partial t}\left(\frac{\delta\rho}{\rho}\right) \propto f \propto \operatorname{div}\underline{u} \propto \operatorname{div}\underline{U}$$

and by the equation of motion in which the perturbation of the potential by the perturbed density is included.

It is important to realize that already in the linear theory the extra compression in places with positive and growing $\delta\rho/\rho$ is anisotropic: the three components of the divergence of the peculiar velocity are not equal

$$\frac{\partial u_x}{\partial x} \neq \frac{\partial u_y}{\partial y} \neq \frac{\partial u_z}{\partial z}$$

There is also shear, $\partial u_x/\partial x \neq 0$ etc. - but of course no vorticity $\partial u_x/\partial y - \partial u_y/\partial x = 0$ because the motion is due to potential (gravitational) forces. The anisotropy of the deformation due to peculiar velocities is easily understood by tidal action. The nearby density distribution distorts the motion at the point under consideration.

A natural way to build an approximate theory, exact in the linear region and also good enough in non-linear situations, is to use the Lagrangian formulation. The position of every particle in space (i.e. its Eulerian coordinates) $\vec{r}$ is given as a function of time t and the initial position (i.e. Lagrangian coordinate) of the particle $\vec{\xi}$.

The solution with growing perturbations only is written

$$\vec{r} = a(t)\left[\vec{\xi} + b(t)\vec{\psi}(\vec{\xi})\right]$$

The first term $a\vec{\xi}$ describes the Hubble expansion $\dot{a}/a = H$, the second term $ab\vec{\psi}(\vec{\xi})$ describes the displacement of every particle from its legitimate unperturbed position. $b(t)$ is a growing function, $b(t) \propto t^{2/3}$. The perturbation due to gravitation $\vec{\psi}(\vec{\xi})$ is of potential type $\vec{\psi} = \mathrm{grad}_\xi \phi$.

Analytical and numerical studies confirm that this is a good approximation - less than 20-30% errors occur in highly nonlinear situations; the proofs are in our original papers.

Given the formula for $\vec{r}(\vec{\xi},t)$, it is easy to write down the velocity of every particle

$$\vec{u} = \left.\frac{\partial \vec{r}}{\partial t}\right|_{\vec{\xi}} , \quad \vec{u}_{pec} = \vec{u} - H\vec{r} = a\dot{b}\,\vec{\psi}(\vec{\xi})$$

and also the density of matter

$$\rho = \rho(\vec{\xi},t) \propto \left(\frac{\partial^3 \vec{r}}{\partial \vec{\xi}^3}\right)^{-1}$$

Here $\partial^3\vec{r}/\partial^3\vec{\xi}$ is the Jacobian i.e. the determinant of the partial derivatives.

Using $\vec{\psi} = \mathrm{grad}_{\xi}\phi$ and choosing coordinate axes which diagonalize the deformations, using the notation

$$\frac{\partial^2\phi}{\partial\xi_1^2} = -\alpha \; ; \quad \frac{\partial^2\phi}{\partial\xi_2^2} = -\beta \; ; \quad \frac{\partial^2\phi}{\partial\xi_3^2} = -\gamma \; ; \quad \alpha>\beta>\gamma$$

we obtain

$$\rho = \bar{\rho}\,\frac{1}{(1-b\alpha)(1-b\beta)(1-b\gamma)}$$

With ϕ determined by the initial small perturbation field we find the particles where α has local maxima α_{m1}, α_{m2} .... The condition $1 - b(t_i)\alpha_{mi} = 0$ determines the moment when infinite density is obtained for the i-th particle.

From the density formula we see that this infinity is due to the intersection of trajectories of adjacent particles lying on the ξ_1 coordinate axis. At the moment t_i the perturbation along the other two axes ξ_2, ξ_3 is finite.

The approximate theory predicts the formation of thin dense gas clouds. They grow due to fresh gas falling onto their flat boundary and being compressed and heated by shock waves. They also spread sideways due to new intersections of trajectories of adjacent particles.

The picture outlined above was already known at the time of the Krakow IAU Symposium No.63, "Confrontation of cosmological theories with observational data". Qualitatively they are described in the report by Doroshkevich et al; formulae and detailed analyses were given in our original papers, and also in the book by Zeldovich and Novikov "Structure and Evolution of the Universe", published in Russian in 1975 and prepared for publication in English by Chicago University Press. These are mentioned in this report for the sake of completeness and to make it possible to read this report without using references.

Now we turn to the results obtained after the Krakow Symposium, partly published in Astronomical Journal (USSR) and partly in preprints of the Institute of Applied Mathematics. These results are most important in connection with optical and radio astronomical observations.

2. LATE PHASES OF THE DEVELOPMENT OF PERTURBATIONS AND CELL STRUCTURE

Numerical calculations were pushed to a late phase, when more than half the matter is brought together into the dense phase. During the lecture in Tallinn a movie was shown made by an electronic computer display. Here, in the written form, only a small number of selected pictures can be shown.

There is one movie (corresponding to Fig.1) calculated using the approximate nonlinear theory for two-dimensional perturbations. The initial spectrum has a sharp cut-off on the short wave - and also long wave end; it is flat (on the average) within the excited interval. The individual Fourier coefficients in this interval are taken at random according to a normal Gaussian distribution.

Comparing these calculations with others it must be stressed that the potential, the velocity and density contributions are calculated for a continuous medium, not for a finite number of discrete point masses. The calculations are not exact and the initial conditions somewhat artificial (two dimensions, flat spectrum). But these departures from the ideal calculation are not of the sort which arise when a finite number of discrete masses is considered.*

The pictures in the movie contain a finite number of points. But those points are test particles for visualising the motion and density distribution. The potential used in the calculation corresponds to a continuum or, in other words, to a calculation with an infinite number of particles with inertial and gravitational mass.

The calculation is continued to the moment after the first intersection of trajectories occurs. It is assumed that the particles are non-interacting and one layer can penetrate through another. The sticking together of particles, their physical, non-gravitational collisions and the formation of shockwaves are not included. Therefore the pancakes in this picture are somewhat thicker than would be found in a real gasdynamical calculation. Still they are rather thin, distinctly different from the spherical or irregular clumps predicted in a simplified approach.

As time goes on, the pancakes spread laterally and they intersect. In Figure 1 a typical net structure is seen: matter is mostly concentrated in thin filaments, the inner regions are empty and divide up the network.

It seems plausible that a three-dimensional calculation will lead to a cell or honeycomb structure with matter concentrated in the walls

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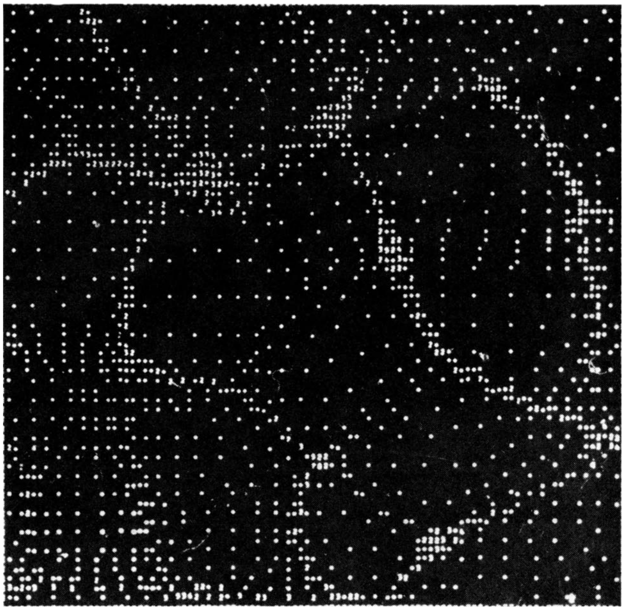
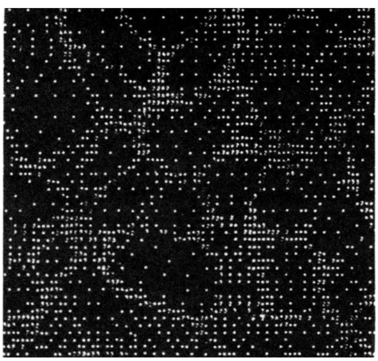
*It is $\sqrt{N}$ in two dimensions and $\sqrt[3]{N}$ in three dimensions which are important in incorporating shortwave perturbations involuntarily in N-body calculations just due to the discrete character of the mass distribution.



Approximate theory



Approximate theory

Numerical simulation

Figure 1

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surrounding large disconnected empty spaces. The intersection of walls could give enhanced density along lines.

It is not yet clear if such types of structure, or at least its remnants are discernible in the observational data of Joeveer, Einasto, Tago and Seldner, Siebers, Groth and Peebles. It is well known that the human eye has the property of finding lines and other patterns in random assemblies of points. One example is the Sciapparelli channels on Mars, but even more striking are the constellations - figures of humans and beasts found by the ancients in the distribution of stars on the sky. Therefore one must be very cautious in interpreting the observations. One must find some mathematical algorithm to distinguish between superclusters as quasispherical clumps and the honeycomb structure. It is possible that correlation functions (two points, three points etc.) are not the best method for this particular task.

There are obvious difficulties: 1) the walls must fragment into separate galaxies and clusters of galaxies. The turbulence inside the pancakes and the gravitational interaction of the fragments must partly wash out the structure. 2) In investigations of the three-dimensional structure we use redshift as a measure of distance. Because of peculiar velocities, this procedure is not exact. These points need further investigation. But with all these uncertainties, one point must be stressed: the occurrence of cells in theoretical calculations is not an artefact due to the use of an approximate theory.

Calculations of another type were carried out and used to make the second part of the movie. The motion of $(128)^2 \approx 16000$ points in two dimensions was calculated numerically. The potential for every distribution of points was calculated using Poisson's equation $\Delta\phi = 4\pi G\rho$ with some smoothing and interpolation on the smallest scale. Periodicity on the largest scale was assumed: points intersecting from inside the wall of the square reappeared on the opposite wall. The periodicity condition was also used in the potential calculation.*

Again a flat spectrum of perturbations with cut-offs from both sides was used in formulating the initial conditions. The results of numerical simulations are practically indistinguishable from the results of the approximate theory. The characteristic pattern with thin walls and disconnected empty spaces depends on the cut-off of the short waves - this is our firm conclusion. It is confirmed by the fact that the average linear dimension of the empty spaces are approximately equal to the cutoff wavelength, $2\pi/k_{max}$.

Therefore this pattern is characteristic of the adiabatic theory with the exponential cut-off at short wavelengths, which results from the matter-radiation interaction.

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*The force $\propto r^{-1}$ and potential ∞ log r is characteristic of two dimensional gravitation; to be exact we are working with infinite bars, not points.

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For entropy perturbations, there is no cut-off except the Jeans' mass for neutral hydrogen corresponding to masses of the order of globular clusters. Therefore probably no net or cell-structure will occur in this case. Corresponding numerical computations with sufficient accuracy are still lacking. Perhaps the two dimensional case will be easier to handle and be still meaningful.

We are optimistic about the prospects for discriminating between entropy and adiabatic perturbations by means of investigations of the large-scale structure of the Universe.

A three dimensional calculation was done for adiabatic perturbations with an exponentially cut-off spectrum, but it is the visualisation of results which is the bottleneck in this case. This report is written at the moment when this work is still in progress.

3. STATISTICAL PROPERTIES OF THE BIRTH OF PANCAKES AND COUNTS OF QSO

The birth of an individual pancake occurs at the moment of intersection of trajectories, i.e. mathematically speaking at the moment when the smallest denominator in the density expression $(1-b(t).\alpha)$ vanishes so that $\rho \to \infty$. Therefore we must find the local maxima of $\alpha = -\partial^2\phi/\partial\xi_1^2$. Due to the statistical character of the problem the answer is also given in statistical terms. The function $P(\alpha_m)$ gives the density of local maxima of given amplitude

$$dN = P(\alpha_n)\, d\alpha_n \qquad (cm^{-3})$$

In the case of a cut-off spectrum P is proportional to R_c^{-3}. The dependence on α_m is universal, given the normal Gaussian law of density perturbations. But $P(\alpha_m)$ is not a simple Gaussian function, because in calculations of α_m we are performing the nonlinear operation of diagonalisation of the $\partial^2\phi/\partial\xi_i\partial\xi_k$ matrix and we are choosing the maxima. Doroshkevich has obtained

$$P(\alpha_n) \propto \alpha^5 e^{-n\alpha^2} \qquad (\alpha > 1/\sqrt{n})$$

Using the connection between the amplitude of the maximum α_m and the moment t_m we can obtain the birth function F(t) or f(z) giving the number of pancakes born per unit comoving volume per unit time or unit of z.

The newborn pancakes have small vorticity and low temperature. The formation of compact objects and brightest galaxies is easiest just at the birth-place of the pancake and even before the growth of the ends of the pancake.

Therefore it is plausible to identify the birthrate of pancakes with the birthrate of brightest known compact objects - quasars. If the

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life time of quasars is short and independent of their absolute age, then at every epoch the concentration of quasars is proportional to their birthrate. The high power α^5 before the Gaussian factor leads the $(1+z)^7$ dependence of the density of quasars as an intermediate asymptote in the case of a flat Universe. At high z the power law is cut off by the Guassian exponent. By the choice of a single constant (corresponding to the amplitude of perturbations) it is possible to obtain a good fit of the pancake birth rate curve to Longair's results on radio source counts and Schmidt's data on quasar evolution.

Still, the similarity between the radio source and quasar evolution and the birthrate of pancakes should not be overestimated. The power laws involved refer to different regions of z. The birth of cold pancakes occurs from some high z (of the order of 10 or 20) up to $z \sim 4 \div 3$. It is well known that for $z < 4$ the gas is totally ionized; therefore even if pancakes are formed, their physical properties are totally different as compared with genuine pancakes formed from cold initial gas. On the other hand, the observed counts of radio sources and quasars refer to the range $0 < z < 4$; at $z > 4$ instead of evolution there is a cut-off or stagnation. This question needs further investigation.

Another statistical test concerns the two-point correlation curve. The adiabatic pancake theory does not contradict the most interesting part of Peebles' correlation function $\delta \sim \xi^{-1.7}$ in the region near $\delta \sim 1$. We refer to original papers for quantitative confirmation.

The general outlook seems to be that the adiabatic theory does not contradict the observations.

4. THE CRUCIAL TESTS AND FURTHER PROBLEMS

Still the absence of contradiction is not positive proof. In order to distinguish between the entropy and adiabatic theories one needs direct observation. The observation of very early globular clusters and galaxies at $z > 30$ to $z \sim 100$ or 200 would be strong evidence in favour of the entropy perturbation theory with further clumping of the initial small mass objects into clusters of galaxies. If hot gas clouds of primordial composition (H + He) are found, identifiable with pancakes, this would be a strong argument for the adiabatic theory. Fully ionized very hot gas could be detected by its X-ray emission and by distortions of the Planckian background radiation spectrum (cooling in the Rayleigh-Jeans region). The medium-hot hydrogen gives redshifted 21 cm radiation.

In any case, the controversy with the observed limits on $\Delta T/T$ of the relic radiation fluctuations must be solved - but this is needed for all variants. Entropy perturbations predict $\Delta T/T$ only 2 or 3 times less than adiabatic perturbations. The study of those perturbations which are directly connected with the structure of the Universe is the most rewarding part of the problem.

Extrapolating from Krakow through Tallinn to the next symposium somewhere in the early eighties one can be pretty sure that the question of the formation of galaxies and clusters will be solved in the next few years.

What remains is the wider question of the overall spectrum of perturbations including the smallest scale damped in the very early radiation dominated or hadronic era and of the longest perturbations, whose amplitude remains small even now. Is the power law spectrum without any characteristic length valid? New, indirect observational tests are needed. Still the major theoretical questions remain unsolved: what is the fundamental theory of the initial perturbations? And what is the ultimate reason for the homogeneous and isotropic expansion from the singularity which is the background for the perturbations?

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DISCUSSION

Suchkov: There are quite distinct knots in your array of pancakes. Now, if the pancakes are destined to be superclusters or clusters of galaxies, what kind of future do you foresee for these knots?

Zeldovich: The numerical calculations need to be pushed further in order to obtain unambiguous answers. Possibly the filaments along which clusters of galaxies are aligned (if this effect is statistically verified) will be identified with intersection lines but it is not yet clear theoretically.

Chernin: What kind of relaxation could lead to the evolution of a flat pancake into a cluster like Coma with more or less spherical form?

Zeldovich: Pancake formation is due to compression on one axis, but this does not exclude less dramatic compression (without intersection) in one or two other directions. Therefore at least a part of pancakes can transform into rather dense clumps. Turbulence inside the pancake and also its curvature tend to make the clump thick. The last effect tending to make the cluster spherical is gravitational interaction.

On the other hand, there must also be pancakes which are expanding in the two directions tangential to the pancake surface and in this case one should observe Hubble's law in a region with strongly enhanced density. Of course, the Hubble constant for this region is different from the genuine long-range H; the local H is subject to quadrupole perturbations. One should ask Prof. de Vaucouleurs and Profs Sandage and Tammann if perhaps we are living in such a region.

Binney: One cannot but be impressed that Dr Zeldovich's beautiful film gives a better representation of the sky as published recently by Dr Peebles and collaborators than does that shown earlier by Dr Aarseth (Peebles et al. 1977). Further strong evidence in support of the picture, based on a spectrum biassed towards large masses, are the facts that both most rich clusters of galaxies and elliptical galaxies are as often as not nearly as aspherical as a slowly rotating body can be (Klingworth 1977, Rood and Chincarini 1974, Macgillivray 1976, Schipper and King 1977).

I should like to ask Dr Zeldovich, however, whether he believes large-scale shock formation is a necessary part of this picture. I ask this because I have difficulty in believing that the cold cosmic gas will fail to fragment soon after it starts to contract in one dimension. This will destroy the pressure-balance required across the centre of the pancake. My belief is that one may retain the cellular structure and the aspherical cluster formation even without large-scale shock formation. Certainly one cannot overemphasize the importance of anisotropic collapse on a large scale.

Zeldovich: Dr Binney is making a statement rather than a question. I should point out that the film was made by Doroshkevich, Shandarin, Sigov and Kotok; I would also add Einasto and Joeveer to the list of people observing large scale structure.

As to the origin of the structure: it is the cut-off of short wave perturbations which is most important. The cell structure remains (perhaps somewhat weaker, with thicker walls) in the collisionless case with trajectories continuing without break after intersection, i.e. in the absence of the shock. Concerning fragmentation, when the perturbations are small (linear regime) the exponent of the gravitational instability has no maximum; it is an increasing function of wavelength. The cut-off short wave perturbations do not outgrow those of long wavelength. The compression time before pancake formation is so short that it does not compensate the handicap due to short wave damping. We feel that the overwhelming part of fragmentation occurs after shock wave compression - if there are no primordial short wave entropy perturbations of course.