

## Bayesian Joint Estimation of Non-Gaussianity and the Power Spectrum

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**Abstract.** This paper gives a summary of our rigorous, non-perturbative, Bayesian framework which enables one jointly to test Gaussianity and estimate the power spectrum of CMB anisotropies. A detailed and thorough presentation of this new technique has been given in Rocha et al. 2000. There we apply our method to Very Small Array (VSA) simulations based on signal Gaussianity, showing that our algorithm is indeed not biased.

### 1. Introduction

In this paper we summarize the work presented in Rocha et al. 2000, where we set up a proper Bayesian framework with which to tackle the issue of non-Gaussianity. There we derived a general form of the likelihood from the Hilbert space of a linear harmonic oscillator. The space of all distributions can then be spanned by the amplitudes,  $\alpha_n$ , of the various energy eigenstates, with a general distribution taking the form of a Gaussian times the square of a (possibly finite) series of Hermite polynomials. Closer comparison with the Edgeworth expansion reveals that indeed the amplitudes  $\alpha_n$  can be written as series of cumulants (Contaldi, Bean, Magueijo, 2000); these are the combinations of cumulants which can be varied independently. In particular these are the combinations which can be independently set to zero without mathematical inconsistency. In some sense the  $\alpha_n$  generalize cumulants to non-perturbative situations. We thus arrive at a well-defined mathematical framework for conducting Bayesian tests of Gaussianity, which jointly produces  $C_\ell$  estimates. Its interest is twofold. Firstly there is the obvious interest in finding out whether the CMB fluctuations are Gaussian or not. Secondly there is the issue of whether  $C_\ell$  estimates themselves may vary if non-Gaussian degrees of freedom are allowed into the likelihood. We applied this formalism to VSA simulations, pending actual data. It is interesting to note that if we take it for granted that inflation is realized in its simplest form, and is triggered by quantum cosmology, then measuring the parameters  $\alpha_n$  amounts to mapping the wave-function of the Universe as it emerged out of a quantum epoch.

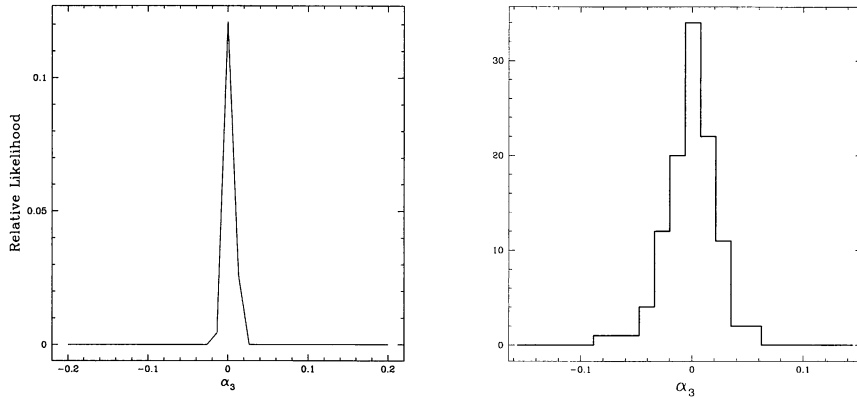


Figure 1. Left hand side: The joint likelihood for  $\alpha_3$  obtained by multiplying the individual likelihoods in  $\alpha_3$  for the 10 spectral bins. Right hand side: The distribution of the peak of the likelihood in  $\alpha_3$  for simulated VSA observations of several Gaussian CMB realizations.

## 2. Application to VSA simulations

The non-Gaussian likelihood formalism presented in Rocha et al. 2000, works most simply when applied to a series of independent variables. Therefore we combine our method with the technique of signal-to-noise eigenmodes. We consider for each S/N eigenmode the more general situation in which all  $\alpha_n$  are set to zero, except for the real part of  $\alpha_3$ . The reason for this is that such a quantity reduces to the skewness in the perturbative regime. We have applied this method to a  $30 \times 12$  hour simulated VSA observation of a Gaussian CMB realisation drawn from a standard inflationary model. In Fig 1, right hand side, we plot the distribution of the peak of the likelihood for  $\alpha_3$  obtained from Monte Carlo simulations. In each VSA simulation the CMB is a realization drawn from an inflationary model. The CMB fluctuations are thus Gaussian. The distribution peaks around a value of  $\alpha_3 = 0$  confirming that our algorithm is indeed not biased.

## 3. Conclusions

In this paper we presented a brief summary of our rigorous Bayesian framework for testing non-Gaussianity, and jointly estimating the power spectrum. Our main achievement was to convert testing Gaussianity into a problem of Bayesian estimation. We also showed that our method is unbiased .

## References

- Contaldi, C., Bean, R., Magueijo, J., 1999, Phys. Lett., B468, 189  
 Rocha, G., Magueijo, J., Hobson, M., Lasenby, A., 2000, astro-ph/0008070