the plethora of such terms riddles his papers like a disease. Let the conclusion of the last of Abraham's papers, coauthored by K. A. Scott and quoted here in full, serve as an illustration:

> "The occurrence of toral and bagel chaostrophes in forced van der Pol systems is established. It remains to draw the surrounding homoclinic tangles, in the wonderful style of Hayashi, by actual simulation instead of fantasy. But that is very difficult. The ubiquitous coincidence of two bifurcation events, called here the captive balloon catastrophe, also suggests further work, in search of a poppyseed bifurcation?"

In the search for a poppyseed bifurcation one is hardly aided by the index, which directs us to exactly this passage. The authors of these papers should have made much greater effort to link their work, interesting as it undoubtedly is, with the work going on in the fields of bifurcation and chaos in the rest of the world, but their self-indulgent use of such language is a serious obstacle.

Like all conference proceedings, this volume contains papers covering a considerable range of quality. But it is late, rather badly produced and unbelievably expensive and so, more than most such volumes, it raises the whole question of the merits of conference reports. Would the better papers here not have found their way into reasonably reputable periodicals in the normal way, and would we really have missed the poorer ones?

DOUGLAS C. HEGGIE

Cohn, P. M., *Free rings and their relations* (London Mathematical Society Monograph No. 19, Academic Press, 2nd ed. 1985), £66.50.

This book is a substantially extended and up-dated version of its first edition. The principal theme of the book is the work of the author, G. M. Bergmann and others upon free associative rings and algebras. Of the abundance of riches in the actual text it is only possible to give some impression without a review of inordinate length.

The text begins at Chapter 0 with some preliminaries which foreshadow the general level and direction of the text. Thus, for example, there appear Hermite rings, the matrix definition of a module, groups and rings of fractions, skew polynomial rings and free associative rings. Chapter 1 introduces the main classes of rings of the book, these being rings with conditions of freeness on their left or right ideals considered as modules. More specifically a free ideal ring, or fir for short, is a ring all of whose left ideals are free of unique rank. If this condition is assumed only for all m-generator ($m \leqq n$) left ideals or for all finitely generated left ideals one obtains an n-fir or a semifir respectively; by this classification an integral domain is precisely a 1-fir. It is shown that a ring is a semifir if and only if it is weakly semihereditary and projective-free. These notions of freeness are used throughout the subsequent chapters.

A generalisation of the Euclidean algorithm, called the weak algorithm, is used to characterise certain classes of firs. It is shown that a free associative algebra over a (commutative) field is a two-sided fir. The Hilbert series of a filtered ring is defined and for a filtered ring with a weak algorithm a formula of J. Lewin on the ranks of modules is obtained, this formula being the analogue of Schreier's formula for groups. Various generalisations of (commutative) unique factorisation domains and of the primary decomposition of Noetherian rings are considered. It is shown that over a $2n$-fir with left and right ascending chain conditions on left and right n-generator ideals every full $n \times n$ matrix admits a complete factorisation into atomic factors and any two such complete factorisations are isomorphic. An integral domain is shown to be rigid if and only if it is a 2-fir and a local ring. Rings, 2-firs in particular, having modules with a distributive submodule lattice, are examined. For matrices over an n-fir several notions of rank are investigated, the most important of which is the inner rank. The Sylvester domains of W. Dicks and E. Sontag are analysed, the name stemming from the fact that over a semifir a law akin to Sylvester's law of nullity is satisfied. A Sylvester domain is shown to have weak global dimension at most 2 and to be projective-free. Issues of commutativity and of centralizers in firs are raised. The centre of an atomic 2-fir is shown to be a Krull domain and every Krull domain is shown to occur as the centre of some principal ideal domain. Bergmann's theorem, which identifies the centralizer of a non-scalar element of a free k-algebra, k being a field, as a polynomial ring in one variable, is proved. There is a lengthy discussion of automorphisms of polynomial rings and free algebras leading up to a brief treatment of the Galois correspondence for free algebras and to the work of V. K. Kharchenko on so-called X-inner and X-outer automorphisms.

At this stage the reader reaches Chapter 7 which, as in the earlier edition, is one of the most significant chapters of the book. It is concerned with the embedding of rings in skew fields of fractions and contains many results of the author. As before it is shown that a ring can be embedded in a field if and only if the ring is an integral domain and no non-zero scalar matrix can be written as a determinantal sum of non-full matrices. The results here are more extensive than previously. Thus there is now a slick proof (due to V. Dlab and C. M. Ringel) of the theorem of Bergmann and Dicks that if R is a left hereditary ring and Σ is any set of square matrices then the universal localisation R_Σ is also left hereditary. The eighth and final chapter treats of skew, and iterated skew, polynomial rings and Laurent series. One pretty result is that if G is a free group then every element of the ordered series ring $K(G)$ is conjugate to a Laurent series in a single variable. As in the first edition each chapter ends with a commentary providing historical and mathematical background and there is an appendix summarising some results from lattices and homological algebra.

It is hoped that the above gives some flavour of this book which is concise but usually clear. If there are criticisms to be made one might observe that while theorems etc. stand out from the text the same is not true of definitions so that, on occasion, one has to search for an unambiguous delination. Perhaps however the main criticism must be reserved for the sterling price which is surely beyond the reach of all but the highly committed buyer.

D. A. R. WALLACE

Craik, A. D. D., *Wave interactions and fluid flows* (Cambridge Monographs on Mechanics and Applied Mathematics, Cambridge University Press, 1986), 322 pp., £35.

Waves have long been an important part of the theory of fluid mechanics. In the nineteenth century the linear theory of sound and of surface waves on water was developed, a few other waves were discovered, and the nonlinear theory was initiated. This century many more kinds of waves have been discovered, the linear theory has been refined, and the nonlinear theory has burgeoned. Also the quality and quantity of observations has improved greatly, so that the relationship of theory and experiment is much closer. The early ideas of Stokes on water waves of finite amplitude and of Rayleigh on acoustic streaming have, especially in the last three decades, grown to form a comprehensive theory of weakly nonlinear interaction of waves, not only the self-interaction of one wave.

Introducing nonlinearity, Craik observes that nonlinear theories are essentially of three distinct kinds. The first is of rigorous mathematical results (of, for example, the energy method) about arbitrary disturbances, the second is of weakly nonlinear theory in which the linear theory is used as a first approximation to waves of small amplitude, and the third is of numerical simulations. Craik might have added "laboratory simulations" to his list! Although observation is, by definition, not a kind of theory, numerical experiments are increasingly used with theory as laboratory experiments have been and are. However, Craik does not forget that understanding natural phenomenona is the aim, many observations being reported and related to theory throughout the book.

Although the second kind of theory is in practice restricted, because it is confined to basic flows with plane or axial symmetry in order to reduce the linear problem to an ordinary differential system and thereby make the weakly nonlinear problem tractable, it has dominated nonlinear theory so far and is the subject of this monograph. Craik treats chiefly the interaction of waves in incompressible fluids, but also treats the analogous interactions of waves in plasma physics and optics. He wisely limits his subject, because no book can cover everything well, the