Correspondence

DEAR EDITOR,

In 'Moving the first digit of a positive integer to the last' (*Math. Gaz.* **83** pp. 216-220), Braza and Tong's treatment of the problem is impressive, but they fail to observe that there is an easy way to calculate by hand numbers such that moving the first digit to the last is equivalent to a division.

Choose the first digit, say 8, and the divisor, say 4. We proceed using the usual paper method for division; but at each step the latest digit of the quotient provides the next digit for the dividend (together with any carry digits).

$$4\,\overline{\big)\,8^0\;\;2^2\;\;0^0}\;\;(\text{quotient } 2\;0\;5) \;\rightarrow\; 4\,\overline{\big)\,8^0\;\;2^2\;\;0^0\;\;5^1}\;\;(\text{quotient } 2\;0\;5\;1)$$

The process terminates when the latest digit of the quotient equals the first digit chosen and there is no remainder outstanding. The given example will terminate at 820512, with quotient 205128.

This method shows why there can only be one basic answer for a given first digit. and why all other answers are concatenations of the basic answer.

Yours sincerely,

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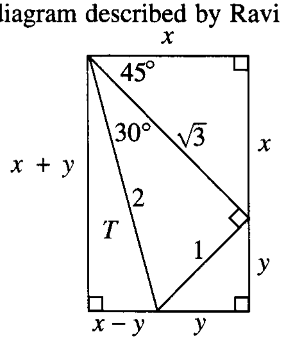
DEAR EDITOR,

I offer a couple of observations on items in the excellent July 1999 *Gazette*.

1. Readers of Robert J. Clarke's article might also be interested in a diagram described by Ravi Vakil in his lively book *A mathematical mosaic* (enthusiastically reviewed by Andre Toom in the January 1998 *American Mathematical Monthly*—'This is a book I would have like to have read as a boy'). Page 87 features what Vakil calls the Ailles Rectangle, named after an Ontario High School Teacher, Doug Ailles.



Here, from the 45°-triangles, x and y are immediately seen to be $\sqrt{3}/\sqrt{2}$ and $1/\sqrt{2}$ so that the trigonometric ratios for 15° and 75° can be read off from triangle T.

2. An alternative proof that J. A. Scott's recalcitrant series $\sum(n^{1/n} - 1)^p$ converges for $p > 1$ runs as follows: Fix a natural number k with $k > p/(p-1)$, so that $p > k/(k-1)$. Write $n^{1/n} = 1 + a_n$ with $a_n \geqslant 0$. Then, for all $n \geqslant 2k$, we have:

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$$n = (1 + a_n)^n > \binom{n}{k} a_n^k \quad \text{(ignoring all other terms of the binomial expansion)}$$

$$= \prod_{i=0}^{k-1} \frac{n - i}{1 + i} a_n^k$$

$$> \frac{\left(\frac{n}{2}\right)^k}{k^k} a_n^k$$

and so $a_n < \dfrac{2k}{n^{(k-1)/k}}$.

Thus $a_n^p < (2k)^p / n^{p(k-1)/k}$ and $\sum a_n^p$ converges by comparison with $\sum 1 / n^{p(k-1)/k}$ which is convergent because $p(k - 1)/k > 1$.

Keep up the fine editorial work!

Yours sincerely,

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DEAR EDITOR,

The *magic rectangles* discussed in a recent Note [1] by Marián Trenkler have a longer history than indicated there. Harmuth, in 1881, published two papers [2, 3], establishing necessary and sufficient conditions for the existence of magic rectangles in just this sense, that is, an arrangement of the integers 1, 2, … mn into an m by n rectangle where columns have the same sum, as do rows (the natural generalisation of magic squares).

Magic rectangles also have a more current interest than might be gathered from [1]. For example, as with magic squares, they have found some favour in statistics; see [4] for a digest of the statistical uses of magic squares, and [5] for some statistical work involving magic rectangles. Indeed, magic rectangles appeared in this *Gazette* in [6], in 1968, with an application of this sort in mind. The construction of magic rectangles continues to attract attention in research journals, as [7, 8, 9] attest.

Yours sincerely,

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References

1. M. Trenkler, Magic rectangles, *Math. Gaz.* **83** (1999) pp. 102-105.
2. T. Harmuth, Über magische Quadrate und ähnliche Zahlenfiguren, *Arch. Math. Phys.* **66** (1881) pp. 283-313.
3. T. Harmuth, Über magische Rechteche mit ungeraden Seitenzahlen, *Arch. Math. Phys.* **66** (1881) pp. 413-417.

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