THE CONSTANT OF NUTATION AND THE SYSTEM OF ASTRONOMICAL CONSTANTS

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RÉSUMÉ. — L'auteur discute l'influence sur les constantes de la nutation et de la précession, des valeurs d'autres constantes fondamentales, comme l'aplatissement dynamique de la Terre, la masse de la Lune et la parallaxe solaire, en prenant deux modèles théoriques différents de la Terre.

ABSTRACT. — The author discusses the dependence of the constants of nutation and precession upon the values of other fundamental constants such as the dynamical ellipticity of the Earth, the mass of the Moon and the solar parallax in the case of two different theoretical models of the Earth.

ZUSAMMENFASSUNG. — Ausgehend von zwei verschiedenen theoretischen Erdmodellen untersucht Verf. die Abhängigkeit der Präzessions- und Nutationskonstanten von den Werten anderer fundamentaler Konstanten wie der dynamischen Erdabplattung, der Mondmasse und der Sonnenparallaxe.

Резюме. — Пользуясь двумя разными теоретическими моделями Земли автор обсуждает влияние значений основных постоянных (как динамическое сжатие Земли, масса Луны и солнечный параллакс) на постоянных нутации и прецессии.

One of the outstanding discrepancies in the system of astronomical constants has been the discordance between the observed and the computed value of the constant of nutation N, that is, the amplitude of the principal term in the nutation in obliquity. The values obtained from the observations have always been smaller than the values computed from the theory of the motion of the Earth, considered as a rigid body.

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This theory leads to the following expressions, de Sitter [1]:

$$(1) \qquad \begin{cases} P = H\left(A + B\dfrac{\mu}{1+\mu}\right), \\ N = HC\dfrac{\mu}{1+\mu}\cos\Theta H, \end{cases}$$

where P is the constant of precession, H the dynamical ellipticity, and the values of A, B, C and $\cos \Theta H$ are given by Sitter.

Another difficulty that appears in the present system of astronomical constants is the value obtained for the Moon's mass μ by different methods. One way of computing the mass of the Moon is to use the expression of the lunar inequality

$$L = \frac{\mu}{1+\mu}\,\frac{\pi_{\odot}}{\sin \pi_{☾}},$$

where $\pi_{\odot}$ is the solar parallax and $\pi_{☾}$ the lunar parallax. Another way of determining μ is to use the expressions (1) of the constants of precession and nutation.

Research on the internal constitution of the Earth offers the possibility of explaining the disagreements mentioned. It is necessary to adopt Earth models in agreement with the observations but, at the same time, offering the possibility of solving the equations of motion obtained.

The theory developed by Jeffreys and Vicente [2] considers Earth models that take into account the elasticity of the shell and the existence of a liquid core. The comparison of the observed values of the several nutations with the theoretical values predicted by this theory shows that the discrepancy is much reduced, especially in obliquity [3].

We are concerned here only with the constants of nutation and precession. The observed values of these constants correspond to the actual Earth, and so we start from the best observed values. Then apply the corrections computed by Jeffreys and Vicente to determine the theoretical values corresponding to a rigid Earth.

In the case of the constant of nutation we take as the best values the general mean $9''.207$, given by Clemence [4], and Fedorov's [5] value $9''.198 \pm 0''.002$ which is the most accurate so far obtained.

Considering these values for the actual Earth (ζ), affected by the elasticity of the shell and fluidity of the core, we have to divide them by the correcting factors $\frac{\zeta}{\zeta_0}$ in order to obtain the values corresponding to a rigid model of the Earth (designated by ζ_0). The theory developed by Jeffreys and Vicente considers two models of the Earth, and we can be fairly confident that the actual behaviour of the Earth will be between

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these two models. The results obtained for the constant of nutation N are (for the correcting factors see, for instance, Vicente [6]) :

		N.	
Model.	Correcting factor.	9″.207.	9″.198.
Central particle	0.9964	9″.240	9″.231
Roche	0.9989	9″.217	9″.208

These results show immediately the importance of accurate observed values agreeing within their probable errors. If that does not happen the consequence is to obtain large differences for the same model.

In the following calculations we only consider Fedorov's value, because the other known results from the observations do not refer to so many complete periods of the 19-year nutation. They have, nevertheless, the advantage that they were made with different instruments at different latitudes.

We can consider two values, for the constant of precession P, derived from the observations : 5 493″.156 ± 0″.175 (de Sitter [1]) and 5 493″.793 ± 0″.077 (Rabe [7]). They do not agree within their probable errors, but Rabe's value is the most recent and it will be used in the calculations.

The constant of precession is not affected by the elasticity of the shell and the fluidity of the core, that is, the correcting factor $\frac{\zeta}{\zeta_0} = 1$ ([6], table 2). In this case, the values corresponding to the rigid Earth are the same as the observed values.

The expressions (1) give the possibility of determining the values of H and μ, when we consider for P and N the values above computed for a rigid Earth. These values were derived from the observed ones taking in consideration the appropriate correcting factors. The solutions are

	Central particle.	Roche.
P	5 493″.793	5 493″.793
N	9″.231	9″.208
H	0.003 271	0.003 288
μ^{-1}	81.20	81.84

An increase of 0″.02 in the value adopted for the constant of nutation will give a variation of 0.6 in the value of the reciprocal of the Moon's mass.

Another way of calculating the mass of the Moon is from the lunar inequality and the solar parallax. There are a number of determinations of the lunar inequality, and some of them do not agree within their probable errors. Following Rabe [8], I adopt the mean

$L = 6''.437 \pm 0''.002$ of his solution and Jeffreys' rediscussion of Spencer Jones results. I also consider the value $L = 6''.4430 \pm 0''.0017$ obtained by Delano [9].

We can consider two values for the solar parallax, obtained by different methods : Jeffreys' rediscussion of Spencer Jones value $8''.7888 \pm 0''.0011$ (s. e.) and Rabe's value $8''.79835 \pm 0''.00039$. The values computed for μ^{-1} are

	$\pi_\odot$	
L.	8''.7888.	8''.79835.
6''.437..............	81.29	81.38
6.443..............	81.21	81.30

These results show the importance of the values adopted for the solar parallax and the lunar inequality. A difference of $0''.01$ in L or $\pi_\odot$ gives variations of about 0.1 in the computed value of μ^{-1}. We can see that it is possible for the value of the reciprocal of the Moon's mass to have differences of 0.2, depending on the set of values adopted for L and $\pi_\odot$.

The comparison between the values of μ^{-1}, computed by the two methods mentioned, shows that there are no disagreements if we remember that the behaviour of the Earth will be between the two models adopted.

It is possible to say that the theory, taking into account the elasticity of the shell and the existence of a liquid core of the Earth, leads to values of the Moon's mass in agreement with the values determined from the lunar inequality. In this way disappears one of the outstanding difficulties in the system of astronomical constants.

The above results emphasize the importance of accurate values for the several observed quantities that appear in the calculations. Unfortunately, as it was remarked before, the best determinations do not agree with each other. The discordance among the determinations of the observed values of the constant of nutation, the lunar inequality and the solar parallax shows that some of them are affected by systematic errors.

If we want to get greater precision, we must have, not only more accurate observations but, also, a critical discussion of all known results. But, as it is known from past experience, there are many difficulties in order to improve the results so far obtained.

Any revision of the system of astronomical constants should consider the constants of nutation and precession as fundamental constants, after we have shown that it is possible to explain the differences between the observed values and the values computed from the rigid body theory of the Earth's motion.

The existence of artificial satellites and asteroids give the possibility of improving some of the fundamental constants, with all the advantages of a method that is different from the ones so far employed.

The determination of distances in the solar system, by radar methods, will contribute to a better knowledge of the solar parallax in the next few years.

The researches on the motions of artificial bodies, and radar observations in the solar system, show that it will not be convenient to change now the system of fundamental constants, and it is better to wait for the results of the researches in progress.

These investigations confirm the advisability of the resolution of the 1950 conference, on the system of astronomical constants, stating that no change be made in the conventionally adopted value of any constant.

The revision of the system of astronomical constants will need the co-operative efforts of astronomers and geodesists, and it will take a number of years to complete.

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