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OBITUARY

MARY WYNNE WARNER (1932–1998)

Mary Warner, as she was mainly known in the mathematical world, died in April 1998. At a time when few women mathematicians reached the top in their profession, she succeeded in doing so through her ability and determination. Her research contributions were commemorated at a recent international conference on fuzzy topology, the field in which she was one of the pioneers and recognized as one of the leading figures for the past thirty years. She was also an outstanding teacher. But to understand her achievements properly it is necessary to know something of her life.

Mary Wynne Davies was born on 22nd June 1932 in Carmarthen, a country town in South Wales. She and her younger sister were the only children of Sydney and Esther Davies. Her father was at that time a teacher at Queen Elizabeth Grammar School in Carmarthen. In 1938 he was appointed head of another such school in the nearby market town of Llandovery, and it was there that Mary began her education. Ten years later the family moved to Holywell, in North Wales, where Mary's father had been appointed headmaster of the local grammar school. This was a larger, more important school, where he remained for the last 25 years of his career. He was a remarkable man whose influence on his family was profound and lasting. An old-fashioned Welsh grammar school headmaster, one of the last of the kind, his standards were very high. He was a very good, patient, but most exacting teacher. For many years he was a leading member of the Welsh Secondary Schools

Association, and was the first Welsh head to become President of the Headmasters Association of England and Wales. He was also much involved with the University of Wales, and received an honorary doctorate from Aberystwyth.

Mary grew up in Wales and was very proud of her Welsh background. She spoke the Welsh language and could recite faultlessly long portions of the Welsh Bible. She was educated at her father's school in Llandovery until after taking School Certificate (with ten Distinctions – the best result in Wales for her year) she transferred to Howell's Girls Boarding School in Denbigh, since Llandovery did not teach physics. In 1951 she went up to Oxford to read mathematics, having crowned a highly successful school career by winning an Open Scholarship to Somerville College and a Drapers' Company Exhibition.

At Somerville the Principal, Janet Vaughan, took a special interest in her progress. A contemporary of Mary's at college remembers her as a quiet, unassuming person. 'Only the scholar's gown suggested the high academic ability which was to bring her great distinction. Close friends, however, knew her quick, self-deprecating, ironical wit and realized that the laid-back manner concealed deep intellectual interest in her subject, as well as unusual dedication to it.' In spite of a tendency to suffer from travel sickness Mary spent the summers backpacking on the continent, a much more adventurous thing to do in those days than it is now. A close friend describes what it was like to go to the cinema with her:

'One's hopes of being totally absorbed by the film were soon dashed. A fierce whisper from Mary would shatter one's concentration. "Who is he? What's she doing that for?" – and she expected an immediate answer. Delaying tactics were useless, her questions were repeated, with additions and accompanied by a sharp dig in the ribs. This went on throughout the performance. Mary just couldn't enjoy the film unless she knew exactly what was happening, and why. Loose ends of plot, which she spotted immediately, caused her great irritation.'

Mary graduated with second class honours in 1953, although in Moderations she had achieved a first and she also won college and university prizes for her work. Her academic potential was such that she was awarded a DSIR grant, the equivalent of a research council studentship, and a Research Scholarship by her college. She also undertook some tutorial work for several of the Oxford colleges. In those days there were just two Professors of Pure Mathematics at Oxford, the analyst Titchmarsh and the topologist Whitehead. Mary became a student of the latter, who led a lively research group, most of the members of which distinguished themselves in their subsequent careers. It was a particularly exciting time in the development of algebraic topology, and a succession of leading researchers came from all over the world to lecture about the latest results at the Whitehead seminar. Mary was welcomed into this friendly and stimulating circle and made good progress with graduate work.

However, to the surprise of her friends Mary, who never enjoyed any domestic pursuit, started knitting a sweater for a 'poor boy' she knew. This was her husband-to-be Gerald Warner, known to everyone as Gerry, who had come up to Oxford in 1951, after National Service, to read history at St. Peter's. After graduating three years later he had already embarked on what proved to be a highly distinguished career in the Diplomatic Service, specializing in Intelligence work, and was about to be posted to Beijing. Soon after their wedding in 1956 Mary and Gerry left for China by boat and train, a journey of 7 weeks. Mary prepared herself to undertake the duties of a diplomat's wife, equipped (as instructed) with five evening dresses and five cocktail dresses, amongst other necessities. She very much enjoyed the travel and other aspects of diplomatic life, and played a full part in supporting Gerry in his work, in the various countries to which he was posted. However Mary always looked for opportunities to pursue her mathematical interests, whatever the difficulties, as she did in Beijing.

Fortunately another member of the Whitehead research group, the distinguished Chinese topologist Chang Su-chen, had recently returned to Beijing University, and for a time she was able to enjoy discussing mathematics with him. Later, when the political situation became more tense, he told her that it would be unwise for him to see her any more. Eventually Chang Su-chen, like so many other intellectuals, suffered much in the Cultural Revolution. It was in Beijing that the Warners' first child, Sian, was born in 1958.

Shortly afterwards they returned to England for several years and took a house in London, where their second child Jonathan was born in 1959. During this period Mary held a part-time Lectureship at Bedford College. Soon, however, Gerry was again posted overseas, this time to Burma for year, and it was in 1961, while the family were living in Rangoon, that the third child Rachel was born. Still determined to pursue her academic career Mary, having been appointed Senior Lecturer at Rangoon University, designed, administered and led the first MSc course in Mathematics to be given there.

On their return to London Mary resumed her post at Bedford College for another two years. However in 1964 Gerry was again posted overseas, this time to Warsaw. Again the Whitehead connection helped to open doors, and Mary became a member of the distinguished circle of topologists at Warsaw University presided over by Karol Borsuk. As Visiting Research Fellow she organized a topology seminar, under the watchful eyes of the secret police, and she embarked on a doctoral thesis, with Balynicki-Birula as her research adviser.

Two years in Warsaw were followed by two more in Geneva, where the following incident occurred. Once when the Warners were giving a diplomatic dinner party at a restaurant, noted for its tartes à la creme, a guest was mocking Welsh poetry, of which Mary was very fond. Becoming more and more indignant, but restrained by protocol from expressing her feelings to the speaker, she finally turned on Gerry, throwing at him one of the specialities of the house and bringing the conversation to a complete stop.

It was while she was in Geneva that Mary completed her thesis, 'The homology of Cartesian product spaces', and was awarded a doctorate in mathematics by the Polish Academy of Sciences. The examiners were Borsuk and Kuratowski. Apparently the room where her viva was held (she had managed to gain exemption from the test of proficiency in Marxist–Leninist theory) was packed with friends and fellow students, waiting to rejoice with her in her success. This was a turning point in her career.

After ten years of almost successive overseas postings Gerry was transferred back to London, and in fact there was only one more such posting to come. The family were able to settle down and Mary was in a position to hold a regular academic appointment. This was at the City University, where she became a Lecturer in 1968, and this remained her professional base for the rest of her career. The research she had done previously, while perfectly respectable, was not of a kind to attract much attention. Homotopy theory had been developing rapidly since she was at Oxford and the effort involved in catching up would have been daunting. Instead of attempting to do this she turned her attention to what was then a new kind of topology, fuzzy topology. Before long she had mastered this new field and was contributing a steady stream of research publications.

Gerry's last overseas posting, in 1974, was to Malaysia for two years. City University agreed to hold Mary's post open while she was away. As usual she quickly forged contacts with the local mathematicians, and became the only person to hold teaching appointments at both the Malaysian and Chinese universities in Kuala Lumpur. It was during this period that she began to collect blue-and-white Chinese porcelain; later this interest broadened to include a study collection of South East Asian pots, which for some years was displayed in Oxford at the Ashmolean Museum. However, although enjoying life in the tropics Mary felt increasingly that she should be back in England, where her children were going through school. The diplomatic life, with its long, enforced separations, often creates problems for children and their parents. For Mary there was the further problem that she was determined to show what she could do professionally, to pursue her career in a way which was hardly possible in a place like Kuala Lumpur. It was only after the Warners returned to London that she began to establish her reputation for research with a succession of major papers, and to play a prominent role at international conferences.

While Mary seemed to be enjoying life to the full, there were already signs of strain as if she had some foreboding of what lay ahead. This is not the place to describe in any detail what occurred but to understand the later stages of her life it is necessary to say something. Of the three children, Sian, Jonathan and Rachel, the only survivor is Rachel, now a consultant psychiatrist. Each of the three, after boarding school at Malvern, went to Oxford. Sian, an exceptionally gifted girl, developed anorexia while at school. Her mental health deteriorated in her third year at Somerville and she never completely recovered. Eventually she put an end to her life, in 1997. A few years earlier Jonathan, by then making a name for himself in publishing, had taken his own life in a period of depression following the failure of his marriage. The later stages of Mary's career must be seen against this background. It is undoubtedly true that in her professional work, where she was increasingly successful, she found an important refuge from these tragic events.

As the leading pure mathematician in an applied-oriented department, Mary carried more than her full share of responsibilities at the City University. She launched an MSc course, just as she had in Rangoon. She was also a highly successful teacher at undergraduate level. In 1984 she took on the first of a series of PhD students. One of these describes her as 'the best supervisor someone could have'. She served on the University Senate and on several of its key committees. Outside the City University she served on a number of bodies, such as the Court of the University of Wales Institute of Science and Technology. School education was one of her special interests. She was for a time Vice-Chairman of the Governing Body of the International School in Geneva. As a Governor of Lindisfarne College, Wrexham, she played a leading role in its transformation into a co-educational school from one that took only boys.

Mary had realized her ambition of becoming a professor in 1996, having been promoted to a readership in 1983. On his retirement Gerry had received a knighthood for his diplomatic services, so that she became Lady Warner. They had a wide circle of friends at home and overseas, whom they enjoyed entertaining, both at their house in Highgate and their country cottage near Tewkesbury. Although Mary had retired a year early from the City University, and was not in the best of health, she never gave up her professional work. In the last year of her life she attended a conference in Prague, at which one of her research students presented a much admired paper, she was herself working on a paper for a conference to be held in Ohio in August that year, and she was planning to accept an invitation to go to Brazil for six months as a visiting professor. However, this was not to be. She died in her sleep on 1st April 1998 at the age of 65, while visiting friends in Spain, and was buried, alongside the graves of her parents Sydney and Esther, her daughter Sian and son Jonathan, in the graveyard of Kemerton church, at the foot of Bredon Hill.

Mathematical work

Mary Warner's main mathematical work was concerned with indistinguishability and uncertainty. She worked on tolerance spaces, tolerability, haziness and fuzziness. The progress of her thinking in this area is described in the survey article [**24**]. In her own words, she wished 'to make precise the property of imprecision'.

The concept of a tolerance space was introduced by E. C. Zeeman in the context of perception theory. A tolerance space consists of a set X and a symmetric, reflexive relation on X; related points are said to be near or indistinguishable. A tolerance map between tolerance spaces preserves nearness. In [**5**] (see also [**8**]) Warner and A. Muir defined the fine tolerance on the product of tolerance spaces and the coarse tolerance on the set of tolerance maps between tolerance spaces. Using the fine product tolerance, they were able to define tolerance groups which are non-trivial (using the categorical product, tolerance groups are uninteresting). Moreover, if the set of homeomorphisms of a tolerance space is given the coarse function space tolerance, it is a tolerance group. These notions were used to study homogeneity. A homogenous tolerance space X is said to be very homogeneous if for each x and y such that x is near y in X, there is a homeomorphism h of X to itself which is close to the identity in the function space X^X (with the coarse tolerance) such that $h(x) = y$. Their main theorem is that a tolerance space is very homogeneous if and only if it is homeomorphic to the quotient of its group of homeomorphisms by a subgroup which fixes a point.

In a series of papers ([**5**], [**6**], [**7**], [**10**], [**11**], [**12**], [**13**], [**14**], [**16**], [**17**], [**20**], [**22**]), some written jointly with A. Muir, Warner studied automata in which the state space is a tolerance space and the input set acts by tolerance maps. In [**6**], using a homology theory for tolerance spaces outlined in [**5**] and the Hopf trace theorem, she showed that under certain conditions, every input sequence maps to itself a set S of points with the property that all points of S are near a given point. In [**10**] an earlier theorem about homogeneity is exploited to show that every homogenous automaton is isomorphic to a group quotient automaton.

In [**22**], Warner with A. Muir introduced the notion of a lattice-valued relation from a set X to a set Y as a function from the product of X and Y to a lattice. They observed that this concept is a generalization of both tolerance spaces and automata. They gave a natural definition of homogeneity and a classification of homogeneous lattice-valued relations by their isomorphism groups, so unifying earlier work.

Fuzzy sets were introduced in the 1960s. A fuzzy subset of a set X is a function from X to the closed unit interval $I = [0,1]$. A fuzzy subset A of X is said to be crisp if $A(X) \subseteq \{0,1\}$. Unions and intersections of families of fuzzy subsets are defined using suprema and infima. Then a fuzzy topology on a set X is a set of fuzzy subsets of X with the obvious properties. In [**23**], Warner reported difficulties that arise

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concerning fuzzy points, fuzzy set-membership and fuzzy set-complementation: to avoid pathological results, crisp points must be disallowed but then the fuzzy theory would not generalize the classical theory. In [**25**], with A. Muir, she proposed the replacement of the unit interval by a more general lattice in the definition of a fuzzy subset.

Warner now worked on L-fuzziness where L is a frame (see ⟨**1**⟩). Here the L-fuzzy subsets of a set X are the functions from X to L; unions and intersections are defined by joins and meets; an L-fuzzy topology on X is a set of L-fuzzy subsets of X with the standard properties of a topology. She began her study of L-fuzzy topology in [**30**] when she showed that if L is in addition a continuous lattice (see ⟨**2**⟩) and (X, τ) is a topological space, then the set $\omega(\tau)$ of continuous functions from X to L (with the Scott topology) is an L-fuzzy topology. She proposed a criterion for 'good' generalizations of topological properties: a property Q of L-fuzzy topological spaces is a good generalization of a topological property P if an L-fuzzy topological space $(X, \omega(\tau))$ has the property Q if and only if the topological space (X, τ) has the property P. She gave some examples of good generalizations.

In [**31**], Warner showed that an L-fuzzy topology on a set X is itself a frame, so that in particular the set $\mathscr{F}(X)$ of all L-fuzzy subsets of X is a frame. She defined the set of L-fuzzy points of X to be $\mathrm{pt}(\mathscr{F}(X))$. (For a frame L, $\mathrm{pt}(L)$ is the set of all frame morphisms from L to $\{0, 1\}$ and has a natural topology.) She showed that a satisfactory definition of set-membership can be given and that local properties including separation axioms can be suitably defined. This study was continued in joint work with R. G. McLean [**33**]. They showed that if L is a frame and (X, Ω) is an L-fuzzy topological space then there is a frame morphism φ from Ω to the power set of $X \times \mathrm{pt}(L)$ and $\varphi(\Omega)$ is a topology on $X \times \mathrm{pt}(L)$; moreover, if L is spatial (see ⟨**1**⟩) then φ is a frame isomorphism. They showed further that in the spatial case there is a one–one correspondence between the L-fuzzy points of X and the points of $X \times \mathrm{pt}(L)$ which agrees with φ and fuzzy and classical membership in the domain and codomain respectively. They followed up this work in [**35**], showing that if L is a continuous frame, (X, τ) is a topological space and $\omega(\tau)$ is the L-fuzzy topology on X defined by Warner in [**30**], then $\varphi(\omega(\tau))$ is the product topology on $X \times \mathrm{pt}(L)$.

The joint paper [**35**] was the first of a series ([**35**], [**37**], [**39**], [**40**], [**41**]) in which Warner, with various collaborators, studied good (in the sense defined in [**30**]) generalizations of compactness and local compactness. In the survey article [**43**], she discussed the history of fuzzy compactness and described the present position.

The early history of fuzziness was accompanied by much controversy. Mary Warner was one of the people who supplied fuzzy topology with a firm foundation, reviewing the definitions and establishing worthwhile new results that are not just simple generalizations of classical topological properties. Her contribution can be judged by the survey article [**36**]. Also, from the number of papers that she wrote in collaboration with others, it can be seen that she inspired many people to work in this field. Mary Warner was one of the foremost researchers in fuzzy mathematics, highly respected by all her colleagues in the field.

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