

Andrew Ranicki

This special edition honours the life and work of Professor Andrew Ranicki, who worked at the University of Edinburgh from 1982 until his death in 2018. From 1995, he was the *Professor of Algebraic Surgery* in the School of Mathematics, and from 1992 he was a Fellow of the Royal Society of Edinburgh. From 2008 to 2017, he served as an editor of the society's journal *Proceedings of the Royal Society of Edinburgh, Section A: Mathematics*, in which this edition appears. For four of those years Andrew was editor-in-chief, and was instrumental in modernizing the journal's editorial processes and broadening its mathematical scope. It is particularly fitting that this edition be published here.

Andrew obtained his BA in 1969 and PhD in 1972, both from Cambridge. He spent a further 5 years at Cambridge as a postdoc from 1972 to 1977. He was an assistant professor in Princeton from 1977 to 1982, before moving to Edinburgh in 1982. Andrew was awarded the Trinity College Yeats Prize in 1970, the Cambridge University Smith Prize in 1972, and from the London Mathematical Society, both the Junior Whitehead Prize in 1983 and the Senior Berwick Prize in 1994.

Andrew and his wife Ida's legendary hospitality illuminated Scottish topology, and attracted a range of prestigious visitors to Edinburgh, who enjoyed the food, the garden, the pictures, the infectious laugh, the whiskey, and above all the company.

Andrew was an attentive and warm advisor for 12 PhD students, whom he supported far beyond the norm, often welcoming them into his home and facilitating their introduction into the mathematical world through his network of contacts.

Professionally, Andrew was a huge contributor to the community. He was an editor at multiple journals, wrote seven books, edited 12 collections of articles, and organized numerous conferences and workshops. His website remains a resource for the study of algebraic and geometry topology. He served for many years as Director of the Graduate School of the Edinburgh University School of Mathematics.

The articles appearing in this special edition were solicited to reflect Andrew's wide interests in topology and algebra, and to showcase the enduring influence of his work. Andrew loved surgery theory, which is the primary tool for classifying manifolds, particularly those of dimension five and higher, the so-called high dimensions. He was also fascinated by knot theory, again particularly in high dimensions. Among topologists, Andrew was famous for his algebraic skills. He possessed an uncanny ability to discover the right algebraic formulae that both captured the necessary geometric phenomena and led to a clearer understanding of a problem.

Andrew's PhD in the early 1970s was supervised by Frank Adams, one of the leaders in British topology at that time, and unofficially by Andrew Casson, who did not himself have a PhD (but whose contributions have been worth many PhDs).

By James F. Davis and Mark Powell.

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Andrew studied the surgery theory of Browder, Novikov, Sullivan, and Wall. In his early work he developed the *algebraic theory of surgery*, after which his professorship was later named.

In surgery theory, one starts with a finite CW-complex X that looks to the eyes of global algebraic topology like an n -dimensional manifold; in other words, X satisfies Poincaré duality. Such a complex is called a *Poincaré complex*. The driving question in manifold classification is: does there exist an n -manifold homotopy equivalent to X , and if so how many are there up to homeomorphism/diffeomorphism? Surgery theory offers a three-step approach to answer this, when the dimension $n \geq 5$. A Poincaré complex X has a canonical spherical fibration called the Spivak bundle, and step one is to see if it reduces to a \mathbb{R}^n -bundle. If so, one can apply transversality to approximate X by a *degree one normal map* $f: M \rightarrow X$. Bordism classes of degree one normal maps over X are denoted $\mathcal{N}(X)$. The second step is to perform surgery on M below the middle dimension to make f into an $[n/2]$ -connected map $f': M' \rightarrow X$. One obtains an element in the L -group $L_n(\mathbb{Z}[\pi_1(X)])$, which is the remaining obstruction to doing surgery in the middle dimension. This gives rise to the *surgery obstruction map*:

$$\sigma: \mathcal{N}(X) \rightarrow L_n(\mathbb{Z}[\pi_1(X)]).$$

If the surgery obstruction $\sigma(f')$ vanishes, then one can proceed with the third step and surger M' along $[n/2]$ -spheres to obtain a homotopy equivalence $f'': M'' \rightarrow X$, giving a positive answer to the existence part of the manifold classification question above.

Through the s -cobordism theorem, uniqueness is a relative form of existence, and so the three-stage obstruction theory just outlined can in principle classify the homeomorphism classes in a fixed homotopy type too.

We are now in a position to indicate Andrew's early contributions to surgery theory: (1) the L -theory of Laurent polynomial rings; (2) the notion of an algebraic Poincaré complex; and (3) the algebraic bordism exact sequences. All three have been fundamental tools for surgeons ever since.

Shaneson and Novikov computed $L_n(\mathbb{Z}[\pi_1(X \times S^1)])$ using geometric tools and vocabulary, while Andrew redid these fundamental computations purely algebraically, computing $L_n(R[x, x^{-1}])$ for a ring with involution R . This was done in his papers *Algebraic L-theory: I, II, III, and IV*.

Motivated by Steenrod's earlier work on cohomology operations, Andrew algebraized the notion of a Poincaré complex, defining a symmetric algebraic Poincaré complex. Not only did this elucidate the key algebraic properties of a manifold, it led to the notion of a quadratic Poincaré complex, which allowed his formulation of the instant surgery obstruction, combining steps two and three of the surgery programme above. The definition and theory of algebraic Poincaré complexes were developed in his papers *The algebraic theory of surgery: I and II*.

Once he had algebraic versions of Poincaré duality, he could define algebraic bordism groups. This was incredibly important. Wall's definition of his L -groups depended on the congruence class of n modulo 4, but Andrew's bordism definition of the L -groups was independent of the dimension. Also, given the parallel work of Bass, Quillen, and Waldhausen in algebraic K -theory, and geometric work of Shaneson, Novikov, and Cappell in algebraic L -theory, it was clear that exact sequences

were needed as a foundational computational and geometric tool. Andrew's bordism theory provided the requisite exact sequences of pairs and maps. The modestly, but aptly, named book *Exact sequences in the algebraic theory of surgery* has been a vital reference ever since its publication.

Not content with combining two out of three steps in the surgery programme, Andrew studied Quinn's surgery spectra \mathbb{L} , and, by translating them into algebra, formulated the *algebraic surgery exact sequence* and the *total surgery obstruction*. The total surgery obstruction gives a single obstruction that encapsulates the entire surgery programme, and was the topic of his book *Algebraic L-theory and topological manifolds*.

One of the key difficulties in applying the surgery programme to classify manifolds is to evaluate the surgery obstruction map σ . As part of his work on the total surgery obstruction, Andrew refined Quinn's geometric *assembly maps* into algebraic versions. As a result the map σ factors into a composition of two maps: the first compares the \mathbb{L} -homology of X and its classifying space $B\pi_1(X)$, and is in principle computable using spectral sequences. The second map is Andrew's algebraic assembly map. This led to a revolution in our understanding of σ , through subsequent work by many others on *Isomorphism Conjectures in K- and L-theory*, in particular the Farrell–Jones conjectures. These conjectures state that a suitable refinement of the assembly map is an isomorphism, and have been proven for many fundamental groups. Armed with this knowledge, in fortuitous cases one has a chance of calculating the map σ and explicitly classifying manifold structures on a given Poincaré complex X .

Andrew's goal was usually to convert topology into algebra, however his formalism of symmetric and quadratic complexes has recently been re-purposed in the opposite direction, in the drive to 'spacify' algebraic K and L -theory into the language of the ∞ -category of spectra. He showed great prescience in making definitions that mathematicians would continue to use decades later.

While the algebraic theory of surgery and the total surgery obstruction are his most famous legacies, Andrew had broad interests in algebraic aspects of manifold topology. Other notable output includes his books *High dimensional knot theory* and *Algebraic and Geometric Surgery*, and his results on localization in K - and L -theory, the algebraic theory of torsion, controlled K - and L -theory, multiplicativity of the signature, the advent of algebraic transversality, calculating Nil and UNil groups, and Morse theory for manifolds with boundary.

Andrew was an incredibly social individual, which influenced his approach to mathematics. He loved to collaborate; in fact, he collaborated with 37 mathematicians altogether.

Andrew took particular pride in his students and their achievements, on which of course he had a great influence. They worked on a range of topics in surgery theory and knot theory. The PhD thesis of his first student Desmond Sheiham completely computed the algebraic concordance group of boundary links. Joerg Sixt developed the algebraic theory of modified surgery, and wrote an influential paper with Diarmuid Crowley on classifying manifolds within their stable class. Jeremy Brookman worked on splitting homotopy equivalences along codimension one submanifolds and the UNil groups, and was part of a team with Jim Davis and Qayum Khan that computed the structure set of manifolds homotopy equivalent

to $\mathbb{R}P^n \# \mathbb{R}P^n$. Andrew Korzeniewski worked on absolute Whitehead torsion, and with Ian Hambleton and Andrew proved the multiplicativity of the signature in fibre bundles mod 4. Julia Collins studied knot concordance, and computed the subgroup of the knot concordance group generated by the knots of crossing number at most nine. Mark Powell took a multi-stage obstruction theory for knot concordance, and created a single obstruction using symmetric chain complexes. Mark is currently a professor at the University of Glasgow, and an editor of this special edition. Spiros Adams-Florou related controlled and bounded topology, which are technical ingredients underlying Andrew's construction of the \mathbb{L} -spectra, and applied his results to prove a recognition theorem for homology manifolds in terms of bounded Poincaré duality. Patrick Orson developed the theory of double L -groups, including a localization exact sequence, and applied it to study doubly slice knots. Patrick is now an assistant professor at CalPoly in San Luis Obispo, California. Chris Palmer applied algebraic surgery to trisections of 4-manifolds, the L -theory of matrix rings, and to Seifert matrices of braids. Carmen Rovi revisited the multiplicativity of the signature in fibre bundles, and characterized exactly when one has multiplicativity mod 8. Carmen is now an assistant professor at Loyola University in Chicago. Andrew's last student Supreede Dangsikul worked on Seifert surfaces through the lens of differential geometry. Supreede is now an assistant professor at Chiang Mai University in Thailand.

A testament to the kind of supervisor Andrew was, could be seen in the number of his former students who attended both his 60th birthday and retirement conferences, including several former PhD students who had been working in industry for some time.

At the time of writing, Andrew's personal website is still maintained by the Edinburgh School of Mathematics. It is well worth perusing.

<https://www.maths.ed.ac.uk/~v1ranick/>. A Celebratio Mathematica page about Andrew was commissioned, and can be found at:

https://celebratio.org/Ranicki_A/cover/945/. It contains perspectives on his work, personal reminiscences, and many wonderful photos.