PART 4

SOLAR CYCLE, DYNAMO AND TRANSPORT PROCESSES

THE SOLAR DYNAMO

K.-H. RÄDLER
Sternwarte Babelsberg
DDR-1591 Potsdam
GDR

ABSTRACT. The phenomena of solar activity are connected with a general magnetic field of-the Sun which is due to a dynamo process essentially determined by the α-effect and the differential rotation in the convection zone. A few observational facts are summarized which are important for modelling this process. The basic ideas of the solar dynamo theory, with emphasis on the mean-field approach, are explained, and a critical review of the dynamo models investigated so far is given. Although several models reflect a number of essential features of the solar magnetic cycle there are many open questions. Part of them result from lack of knowledge of the structure of the convective motions and the differential rotation. Other questions concern, for example, details of the connection of the α-effect and related effects with the convective motions, or the way in which the behaviour of the dynamo is influenced by the back-reaction of the magnetic field on the motions.

1. Introduction

It is widely believed that the phenomena of solar activity are closely coupled with a general magnetic field of the Sun, and that the 11 years activity cycle is connected with a magnetic cycle. Let us first give a crude idea of the general magnetic field of the Sun. Its dominant constituent is a toroidal, that is, belt-shaped field confined to the convection zone. It is in the main symmetric about the rotation axis and possesses opposite orientations in the two hemispheres or, in other words, shows antisymmetry about the equatorial plane. In addition to the toroidal field there is a much weaker poloidal field, which penetrates not only the convection zone but continues in the atmosphere. To have a very simple picture it may be assumed to be again symmetric about the rotation axis and to show antisymmetry about the equatorail plane as, for example, dipole or octupole fields do; this assumption ignores, of course, any sectorial structure of the general magnetic field. The magnetic cycle occurs as an interplay of the toroidal and the poloidal field, which change their magnitudes and their orientations with a period of 2 x 11 years, that is, 22 years.

The general magnetic field of the Sun is usually attributed to dynamo processes, that is, to the interaction of fluid motions and magnetic fields in the convection zone. This idea was already proposed by Larmor (1919). Its elaboration, however, proved to very complicated. A part of the difficulties, as is attested by Cowling's theorem (1934), is due to the fact that a dynamo cannot work with completely axisymmetric magnetic fields.

G. Berthomieu and M. Cribier (eds.), Inside the Sun, 385–402.

It is easy to understand that, as a consequence of the differential rotation of the convection zone, a toroidal magnetic field is generated from a given poloidal field. The crucial question is the regeneration of the poloidal from the toroidal field. Parker (1955) elaborated the idea that the regeneration is achieved by the cyclonic motions in the convection zone. Within the framework of mean-field electrodynamics established by Steenbeck, Krause and Rädler (1966) the influence of small-scale motions on large-scale magnetic fields has been systematically investigated. Within this scope the effect of cyclonic motions occurs as a contribution to the α-effect, that is, to a large-scale electromotive force caused by small-scale helical motions, which possesses components parallel or antiparallel to the large scale magnetic field. The α-effect is not only able to generate a poloidal from a toroidal field but also a toroidal from a poloidal field.

The α-effect and the differential rotation are considered as the basic elements of the solar dynamo. The resulting scheme of couplings between the toroidal and poloidal magnetic fields includes the possibilities of the $\alpha\omega$ and the α^2-dynamos, which correspond to the cases where the α-effect or the differential rotation is negligible in the generation of the toroidal field, respectively. There are good reasons to assume that the solar dynamo is of $\alpha\omega$-type rather than α^2-type.

In the following we first summarize a few characteristics of the solar convection zone and some observational results on solar magnetic fields. We then turn to kinematic solar dynamo models, in which the back-reaction of the magnetic field on the fluid motion is ignored,explain the basic ideas of mean-field electrodynamics and discuss some general aspects of solar mean-field dynamo models as well as some problems that occur in this context. Finally we consider dynamo models which include the back-reaction of the magnetic field, discuss the extension of the mean-field concept to such models and mention a few attempts to gain understanding of the complex nonlinear behaviour of dynamos.

Being aware that our paper covers only a few aspects of the solar dynamo problem we want to point out other review articles, e.g., by Parker (1979), Stix (1981), Belvedere (1983), Ruzmaikin (1985) or Gilman (1986).

2. Convection zone characteristics

2.1. GENERAL

Unfortunately only the upper boundary of the solar convection zone is more less accessible to direct observations. Most of our knowledge on the deeper layers originates from stellar structure calculations, mainly based on mixing length theory. According to such calculations it seems reasonable to assume that the convection zone extends over the outer 30% of the solar radius.

For the solar dynamo process the convective motions and the differential rotation of the convection zone are of particular interest. We summarize here a few relevant findings. Recent reviews of these subjects are, e.g., by Gilman (1986) and Stix (1989).

2.2. THE CONVECTIVE MOTIONS

There are two clearly defined patterns of convective motion visible at the solar surface. They correspond to granulation and supergranulation. For the granulation the typical length scales are about 1 to 2.10^3 km, the time scales about 5 min, and the velocities about 1 km/s. For the supergranulation the corresponding values are 3.10^4 km, 20 h, and 0.3 to 0.5 km/s.

Moreover, there is some evidence of another pattern, called mesogranulation, with a typical length scale between those of granulation and supergranulation, namely about 5 to 10.10^3 km, a time scale of about 2 h, and rather low velocities of about 60 m/s.

We note that there have been many attempts to identify much larger structures like "giant cells" but their existence is still questionable. Recently an interpretation of observational results by "toroidal convection rolls" was proposed (Ribes et al. 1985, Ribes and Laclare 1988).

The convective motions in deeper layers are a matter of theoretical investigation only, and there are still considerable uncertainties.

2.3. THE DIFFERENTIAL ROTATION

As can be seen from the motion of several tracers, sunspots for example, the equatorial regions of the solar surface rotate faster than the polar regions. The angular velocity Ω of the surface, as inferred from Doppler shift measurements, can be represented by

$$\Omega = A - B \cos^2\theta - C \cos^4\theta \qquad (2.1)$$

where A, B, C are positive coefficients, which slightly vary with the phase of the cycle, and θ is the colatitude.

Roughly, we have $B/A \sim 0.1$ and $C/A \sim 0.2$, which implies that the angular velocity near the poles is by about 30% lower than at the equator. The dependence of A, B and C on the phase corresponds to torsional oscillations.

The angular velocity inside the convection zone has so far been only a matter of theoretical investigations. Although at least a part of the theories is well elaborated they contain assumptions that remain to be checked. The results are controversal even with respect to the sign of the radial derivative of Ω. Recently some conclusions concerning the angular velocity inside the convection zone and below could be drawn from helioseismological observations. These observations seem to rule out the concept in which Ω is constant on cylindrical surfaces. They are, however, compatible with the assumption that Ω shows no radial variation inside the convection zone but, within a small layer at its bottom or below, turns into a value corresponding to a rigid body rotation. In this layer there inevitably exist strong radial derivatives of Ω which change sign in dependence on the latitude (see Gilman et al. 1989).

3. Observational results on magnetic fields

3.1. GENERAL

Let us proceed now to observational results which provide us with some information about the toroidal and poloidal constituents of the general magnetic field of the Sun. We shall explain a few important aspects and refer again to the comprehensive presentations by Gilman (1986) and Stix (1989).

3.2. MAGNETIC FLUX TUBES

We first mention the remarkable discovery that the magnetic flux at the solar surface, and presumably also in deeper layers, is not more or less homogeneously distributed over the solar plasma but concentrated in very thin flux tubes (Stenflo 1973). These flux tubes, whose diameters are small compared with the length scales of granulation, can generally not be resolved by observing the Zeeman splitting of a single line. Their properties have been concluded by measuring ratios of Zeeman splittings of neighbouring lines.

Typical diameters of these flux tubes are 100 to 300 km, typical life times 5 min to 2 h, and typical flux densities 2000 gauss. There are estimates according to which 90% or more of the total flux emerging from the Sun is contained in these flux tubes.

3.3. THE TOROIDAL MAGNETIC FIELD

Essential conclusions concerning the toroidal magnetic field can be drawn from the distribution of sunspots on the solar surface. Sunspots are assumed to occur if the magnetic flux concentration immediately under the solar surface, which is mainly determined by the toroidal field, exceeds a certain bound. They are formed with flux tubes which, as a result of magnetic bouyancy, emerge at the surface. The diameters of sunspots are comparable to the length scale of the supergranulation, that is, 3.10^4 km, the life times are about 1 or 2 months, and the magnetic flux densities about 2000 gauss.

The distribution of the sunspots with respect to latitude and time is represented by the Maunder butterfly diagram. It shows that the occurrence of sunspots is restricted to lower latitudes up to about ± 40 degrees, with a maximum at ± (15...20) degrees. In the course of an 11 years activity cycle sunspots occur at first mainly at the highest of these latitudes but later at lower latitudes. This indicates that the belts of maximum toroidal field are in latitudes lower than ± 40 degrees and show an equatorward migration.

The magnetic polarities of sunspots in bipolar groups follow Hale's rules. The leading and the following spots show opposite magnetic polarities. At a given time, the polarities of, for example, the leading spots coincide in one hemisphere but differ from those in the other hemisphere. In a given hemisphere, these polarities change from one 11 years activity cycle to the next. This indicates that the toroidal field belts in the two hemispheres have opposite orientation and that the orientations change with a 22 years period, which defines the magnetic cycle.

The sunspot activity can be measured, for example, by the Wolf relative sunspot number. This number shows no completely periodic time dependence. There are significant differences between the cycles and typically stochastic features. In particular, there are long-term variations clearly indicated by the grand minima as, for example, the Maunder and Spörer minima in the 17th and 15 century. These variations of the sunspot activity presumably indicate variations of the underlying magnetic field.

The sunspots are not the only indicators of the toroidal magnetic field. There are, for instance, the ephemeral active regions, which are small bipolar regions with life times and magnetic fluxes much smaller than those of sunspots, and which occur also in higher latitudes, namely up to ± (50...60) degrees.

3.4. THE POLOIDAL MAGNETIC FIELD

The poloidal magnetic field, by contrast with the toroidal field, is very hard to observe, for its flux density is 1 to 2 gauss only.

One interesting feature of the poloidal field known from observation is the polarity reversals at the poles. They have been observed directly since 1957/58. There is, in addition, some indirect evidence of such reversals. They can be concluded from the motion of lines of zero radial field towards the poles. These lines are the places were prominences occur, and systematic observations of prominences are available since 1870.

A simple relation has been found between the polarities of the poloidal field at the poles and the orientations of the toroidal field in the latitudes of maximum sunspot activity during the reversal. If, at a pole, negative polarity changes into positive polarity, the toroidal field in the same hemisphere points eastward, and the same applies with opposite field directions. Let us denote the magnetic flux density of the general field by $\overline{\mathbf{B}}$ and refer to spherical coordinates r, θ, ø, corresponding to radius, colatitude and longitude. Then we have during a reversal

$$(\partial B_r / \partial t)_{pole} \,.\, (B_\phi)_{maximum} > 0 \qquad (3.1)$$

Direct measurements of the poloidal magnetic field at the whole solar surface have been carried out at the Mount Wilson and Kitt Peak observatories since 1959. The results revealed a phase relation between the poloidal and the toroidal magnetic field in the latitudes of maximum sunspot activity. The maximum of the radial field component precedes that of the toroidal field by a phase angle of (0.85...1) π. We therefore have almost always

$$(B_r \,.\, B_\phi)_{maximum} < 0 \qquad (3.2)$$

The observational material on the poloidal field gained since 1959 has been used for an analysis of the space and time structure of this field (Stenflo and Vogel 1986, Stenflo and Güdel 1987). In this analysis the radial field component was represented by

$$B_r(\theta,\varphi,t) = \sum_{l \geq 0} \sum_{|m| \leq 1} \int_{-\infty}^{\infty} \frac{d\omega}{2\Pi} c_l^m(\omega)\, Y_l^m(\theta,\varphi)\, exp(im\varphi) \qquad (3.3)$$

where the $Y_l^m(\theta,\varphi)$ are spherical harmonics, and ω the frequency that occurs in the Fourier analysis of the time dependance. Conclusions were mainly drawn from the power spectrum of the coefficients $c_1^m(\omega)$.

Let us first consider the axisymmetric part of B_r, given by the contributions with m=0. For this part there is a striking difference between the contributions with odd or even 1, which correspond to antisymmetry or symmetry about the equatorial plane, respectively. The contributions with odd 1 are generally larger compared with those with even 1. In particular, the contributions with 1=5, 7, and 9 are remarkably large. For odd 1 there is a clearly dominant period near 22 years. For even 1 this period also appears but is by no means dominant compared with some much shorter periods.

Since the differential rotation renders the definition of the longitude coordinate system difficult the situation with the non-axisymmetric part of B_r, that is, with the contributions with m≠0, is rather complex. These contributions are related to the sectorial structure of the solar magnetic field. No noticeable difference between odd and even 1 has been found. The same applies to odd or even 1-m, which correspond to antisymmetry or symmetry about the equatorial plane, respectively.

4. Cinematic dynamo models

4.1. BASIC EQUATION

When studying the dynamo process in the Sun theoretically, we consider the solar plasma simply as an electrically conducting fluid and use Maxwell's equations in the magnetohydrodynamic approximation. The magnetic flux density B is then governed by the equations

$$curl\,(\eta curl\, B - u\times B) + \frac{\partial B}{\partial t} = 0 \qquad (4.1a)$$

$$div\, B = 0 \qquad (4.1b)$$

where ε is the magnetic diffusivity and u the velocity of the fluid motion. At first we consider the motion to be given, that is, restrict ourselves to the kinematic aspect of the dynamo problem.

4.2. MEAN-FIELD ELECTRODYNAMICS

As explained above, the solar magnetic field as well as the motions of the convective zone show complex structures in space and time, which can hardly be taken into account in detail. It proved useful to discuss the dynamo process in terms of mean-field electrodynamics. We briefly explain some basic ideas; for more details see, e.g., Krause and Rädler (1980) or Rädler (1980).

Within this framework each field quantity F, like $\overline{\mathbf{B}}$ or $\overline{\mathbf{u}}$, is understood as a superposition of a mean part, $\overline{F}$, showing more or less smooth dependence on the space and time coordinates, and a fluctuating part, **F'**. The mean part $\overline{F}$ is defined as a proper average of F, which may be over space or time, or may also be an ensemble average. It has to be required that certain averaging rules, the Reynolds rules, apply. This, in turn, generally implies requirements concerning the length or time scales of $\overline{F}$.

From equations (4.1) we obtain equations for the mean magnetic flux density $\overline{\mathbf{B}}$,

$$curl\,(\eta curl\, B - \overline{u}\times\overline{B} - \mathcal{E}) + \frac{\partial \overline{B}}{\partial t} = 0 \qquad (4.2a)$$

$$\mathrm{div}\, \overline{\mathbf{B}} = 0 \qquad (4.2b)$$

where $\overline{\mathbf{u}}$ is the velocity of the mean motion, and $\mathcal{E}$ is a mean electromotive force caused by the fluctuating parts of motion and magnetic field,

$$\mathcal{E} = \overline{\mathrm{u}'\times\mathrm{B}'} \qquad (4.3)$$

This electromotive force $\mathcal{E}$ at a given point in space and time is determined by $\overline{\mathbf{u}}$, u' and $\overline{\mathbf{B}}$ in a certain neighborhood of this point, and it depends linearly on $\overline{\mathbf{B}}$. Provided the variation of $\overline{\mathbf{B}}$ in this neighbourhood is sufficiently weak, that is, the length and time scales of $\overline{\mathbf{B}}$ are sufficiently large, $\mathcal{E}$ can, relative to Cartesian coordinates, be represented in the form

$$\mathcal{E}_i = a_{ij}\overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k} \qquad (4.4)$$

https://doi.org/10.1017/S0252921100068081 Published online by Cambridge University Press

The tensorial coefficients a_{ij} and b_{ijk} are determined by $\overline{\mathbf{u}}$ and u' but do no longer depend on $\overline{\mathbf{B}}$.

In the very simple case in which u is equal to zero and u' corresponds to homogeneous isotropic turbulence, ε takes the form

$$\mathscr{E} = \alpha\overline{B} - \beta curl\,\overline{B} \qquad (4.5)$$

with two constants α and β which are determined by u'. The term $\alpha\overline{\mathbf{B}}$ describes the α-effect, that is, an electromotive force parallel or antiparallel to the mean magnetic field. The coefficient α and thus the α-effect are, however, only non-zero if the turbulence lacks mirror-symmetry. For turbulences on rotating bodies the mirror-symmetry is generally violated because of the action of Coriolis forces. The deviation from mirror-symmetry can, for example, be indicated by a non-zero helicity of the motions. The term $-\beta curl\overline{\mathbf{B}}$ gives rise to introducing a "turbulent" electric conductivity or a corresponding"turbulent" magnetic diffusivity η_T defined by $\eta_T = \eta + \beta$.

In (4.2a), this contribution to η can be disregarded if η is replaced by $\eta._T$

Returning to the general case we note that (4.4) can be written in the form

$$\mathscr{E} = -\alpha\overline{B} - \beta curl\,\overline{B} - \delta\times curl\,\overline{B} - \ldots \qquad (4.6)$$

Now α and β are symmetric tensors, γ and δ vectors, all depending on $\overline{\mathbf{u}}$ and u'. The term $-\alpha\overline{\mathbf{B}}$ describes an anisotropic α–effect, and $-\gamma x\overline{\mathbf{B}}$ a transport of mean magnetic flux different from that by a mean motion, covering also effects discussed as "turbulent diamagnetism" or "pumping" of magnetic flux. The term $-\beta.curl\overline{\mathbf{B}}$ gives rise to introduce an anisotropic "turbulent" electric conductivity or corresponding magnetic diffusivity, and $-\delta x curl\overline{\mathbf{B}}$ covers in particular the "ωxj-effect", which, like the α-effect, in combination with differential rotation allows dynamo action. We note that $curl\overline{\mathbf{B}}$ is connected with the antisymmetric part of the tensor $\partial\overline{\mathbf{B}}_i/\partial x_j$. For the sake of simplicity the terms connected with the symmetric part of $\partial\overline{\mathbf{B}}_i/\partial x_j$ are not given explicitely in (4.6).

An important problem in the elaboration of mean-field electrodynamics consists in the calculation of the electromotive force $\mathscr{E}$, or of the quantities α, β,... for given $\overline{\mathbf{u}}$ and u'. Most of the results available have been derived in the second-order correlation approximation, often called "first-order smoothing", which neglects all higher-order correlations of u'.

4.3. TRADITIONAL SPHERICAL KINEMATIC MEAN-FIELD DYNAMO MODELS

On the basis explained here a number of spherical mean-field dynamo models have been elaborated. In all cases it is assumed that the mean magnetic flux density $\overline{\mathbf{B}}$ in a rotating spherical fluid body is governed by equations (4.2) and (4.6) with more or less simple assumptions on $\overline{\mathbf{u}}$, α, β,...As a rule, the surroundings of the body are considered to be free space, and it is required that $\overline{\mathbf{B}}$ inside the body fits continuously to an irrotational solenoidal field in this external region.

In almost all cases it is assumed that $\overline{\mathbf{u}}$, u' and η are symmetric about the equatorial plane and the rotation axis, and steady. For $\overline{\mathbf{u}}$ and η this should be understood in the usual sense, for u' in the sense of the invariance of all averaged quantities under reflexions of the u'-field about the equatorial plane, under rotations about the rotation axis, and under time shift. If $\mathscr{E}$ contains, for example, contributions like $\alpha\overline{\mathbf{B}}$ or $\beta\mathrm{curl}\,\overline{\mathbf{B}}$, then, owing to these assumptions, α is antisymmetric and β symmetric about the equatorial plane, and both are symmetric about the rotation axis and steady.

These assumptions further imply that the solutions of the equations governing B have the form of the real part of

$$\hat{B}exp(im\varphi + (\lambda + i\omega)t) \tag{4.7}$$

or are superpositions of such solutions. $\hat{\mathbf{B}}$ is a complex field which is either antisymmetric or symmetric about the equatorial plane, symmetric about the rotation axis and steady, m is a non-negative integer, φ again the longitudinal coordinate, and λ and ω are real constants. As usual the symmetry of such solutions is characterized by Am or Sm, where A or S stand for antisymmetry or symmetry about the equatorial plane and m is the integer introduced above describing the longitudinal variation. Simple examples of A0 or S1-symmetry are the fields of central dipoles parallel or perpendicular to the rotation axis, and an example of S0-symmetry is the field of a central quadrupole symmetric about that axis. Depending on kind and intensity of the induction effects the solutions in general exponentially grow or decay; λ is the growth rate. Of course, the marginal case deserves special interest, in which the solutions neither grow nor decay, that is, $\lambda = 0$. In this case they may be steady or oscillatory, where ω defines the frequency. For axisymmetric solutions, m=0, oscillatory behaviour implies real changes of the field configuration in the course of a period, for non-axisymmetric solutions, m≠0, simply a rotation of the field configuration.

In the simplest models α-effect and differential rotation are taken into account in the form

$$\mathscr{E} = \alpha\overline{B}, \quad \alpha = -\tilde{\alpha}(r)\cos\theta \tag{4.8a,b}$$

$$\overline{u} = \Omega \times r, \quad \Omega = \Omega(r)\,\hat{z} \tag{4.8c,d}$$

where θ is again the colatitude, r the radius vector and z the unit vector parallel to the rotation axis. The ratio of the two induction effects can be characterized by

$$q = \left|\frac{\alpha_0}{\Delta\Omega\, l}\right| \tag{4.9}$$

where α_0 and ΔΩ are typical values of α and of the variation of Ω, and l is the thickness of the layer where the dynamo process takes place. Roughly speaking, the limits $q \ll 1$ and $\theta \gg 1$ correspond to the αω and α^2-regimes of a dynamo, in which α-effect or differential rotation, respectively, no longer contribute essentially to the generation of the toroidal field from the poloidal field.

Using those simple assumptions on $\mathscr{E}$ and $\overline{\mathbf{u}}$ Steenbeck and Krause (1969a,b) elaborated $\mathscr{E}$ model of an αω-dynamo, which reflects already some essential features of the solar magnetic fields, and also a model of an α^2-dynamo, which was discussed in view of planetary magnetic fields. Later on a number of other models of the same kind, or with somewhat more general assumptions concerning $\mathscr{E}$ and $\overline{\mathbf{u}}$ have been investigated, e.g. by Deinzer and Stix (1971), Krause (1971), Stix (1971, 1973, 1976a,b), Levy (1972), Roberts (1972), Roberts and Stix (1972), Köhler (1973), Deinzer, von Kusserow and Stix (1974), Jepps (1975), Rädler (1975, 1986a), Yoshimura (1975b), Ivanova and Ruzmaikin (1977), Busse (1979), Busse and Miin (1979), Belvedere et al. (1980a,b) Rüdiger (1980), Weisshaar (1982), Yoshimura et al. (1984a,b,c), Schmitt (1987), Brandenburg (1988), Brandenburg and Tuominen (1988) and Brandenburg et al. (1989e).

Let us briefly summarize a few features of $\alpha\omega$ and α^2-dynamos revealed by those investigations. In $\alpha\omega$-dynamos the generation of axisymmetric magnetic fields is in general favoured over that of non-axisymmetric fields (see also section 4.5). The toroidal part of an axisymmetric field is much stronger than its poloidal part. In the marginal case both steady and oscillatory axisymmetric fields proved to be possible. In α^2-dynamos, however, there is no such discrimination between the two field types. Depending on, e.g., the anisotropy of the α-effect axisymmetric or non-axisymmetric fields can be favoured. The toroidal and poloidal parts of a field have the same order of magnitude. Only in very exceptional cases oscillatory axisymmetric fields have been found (Rädler and Bräuer 1987).

We note that, in addition to the $\alpha\omega$ and α^2-mechanisms other dynamo mechanisms have been investigated, which are based on a combination of induction effects covered by the terms $-\beta.\mathrm{curl}\overline{\mathbf{B}}$ or $-\delta\mathrm{xcurl}\overline{\mathbf{B}}$ in (4.6), e.g., the "ωxj-effect", and differential rotation (Rädler 1969, 1976, 1980, 1986a, Stix 1976b). They are presumably less effective than the $\alpha\omega$ and α^2-mechanisms and therefore ignored in the following.

4.4. CONSTRAINTS ON SOLAR DYNAMO MODELS AND SOME CONCLUSIONS

Let us now list a few observational constraints which should be met by kinematic mean-field models of the solar dynamo.

(i) The magnetic field possesses mainly A0-symmetry. Its toroidal part is much stronger than its poloidal part.

(ii) The magnetic field is oscillatory, with a period of about 22 years.

(iii) The dependence of the toroidal magnetic field on latitude and time allow us to reproduce the butterfly diagram; in particular, the maxima of this field occur in latitudes lower than ±40 degrees and migrate equatorward.

(iv) The toroidal field in the latitudes of maximum activity and the poloidal field near the poles during its reversal satisfy the relation (3.1).

(v) The toroidal and the poloidal field in the latitudes of maximum activity satisfy the phase relation (3.2).

(vi) If the radial field component is represented in the form (3.3) the coefficient c_1^m are close to those derived from observation.

Of course, phenomena like the long-term variation of the sunspot activity cannot be comprehended by kinematic models. In a more general context observational constraints have recently been compiled by Tuominen et al. (1988).

The constraints (i) and (ii) are met by a dynamo working in the ω-regime rather than the α^2-regime. That is the generation of the toroidal field is mainly due to differential rotation.

In our further discussion of the constraints listed above we assume an $\alpha\omega$-dynamo for which $\mathscr{E}$ and $\overline{\mathbf{u}}$ are defined by (4.8) and α and $d\Omega/dr$ do not change their signs all through the fluid body. In this case some remarkable conclusions can be drawn. Of course, these simple assumptions are presumably rather unrealistic but the conclusions can help us to understand more realistic models. We briefly return to the constraints (i) and (ii) and note that under the assumptions mentioned a preferrence of fields with A0-symetry and oscillatory behaviour has been observed so far only if the signs of a in the northern hemisphere and of $d\Omega/dr$ are opposite. As far as constraint (iii) is concerned we consider only the migration of the maxima of the toroidal field. According to general results on dynamo waves (Yoshimura 1975a,1976) an equatorward migration occurs just under the same condition concerning the signs of α and $d\Omega/dr$. Whereas constraint (iv) does not lead us to a simple conclusion, constraint (v) can only be fulfilled if α is positive in the northern hemisphere (Stix 1976b). Since, as inferred from (i) and (ii), and also from (iii), α and $d\Omega/dr$ have there opposite signs, $d\Omega/dr$ should be negative, that is, the angular valocity increases inward. The constraint (vi) has been considered so far only for one model (Brandenburg 1988), and we see no simple conclusions of general nature.

4.5. SOME REMARKS CONCERNING THE FURTHER ELABORATION OF KINEMATIC MEAN-FIELD MODELS

We have seen that the mean-field concept provides us with some basis for the understanding of the solar dynamo process, and very simple models yielded some promissing results. Further elaboration of mean-field models of the solar dynamo is however hampered by several difficulties. Part of them result from unsolved problems of the mean-field theory, others from lack of knowledge on convective motions, differential rotation, etc... There is at the moment no really comprehensive mean-field model of the solar dynamo. In the following we shall explain some of the problems that we are faced with in this context and some efforts to improve models.

Let us first recall that the mean fields have to be defined so that the Reynolds rules apply. For many purposes it seems reasonable to define them by a time average over 1 or 2 years (Stix 1976a, 1981). One should bear in mind that then the Reynolds rules hold only in some approximation. With averages over the longitude or stastitical averages these rules apply exactly but some other difficulties occur (e.g., Rädler 1981a). The finding that the magnetic flux is concentrated in thin tubes implies no objection to the applicability of the mean-fields more difficult.

While the dependance of $\mathcal{E}$ on $\overline{\mathbf{B}}$ given by (4.4) or (4.6) seems to be justified to a certain extend (Rädler 1981a), there is considerable uncertainty as to how quantities like α,β... depend on the motions. As mentioned above, most of the results available were derived in the second-order correlation approximation, which is not well justified for the solar convection zone. A recent investigation (Nicklaus and Stix 1988) demonstrates that corrections by fourth order correlations may change the results considerably; even the sign of the scalar α occuring with isotropic turbulence can change if higher approximations are used. Despite several results on the α-effect and related effects under more realistic conditions (e.g., Walder et al. 1980, Stix 1983, Schmitt 1985, Durney 1988) are further efforts necessary.

Some discussions on the solar dynamo are based on the idea that the α-effect can simply be described by a scalar α which is essentially equal to $-(1/3)\overline{\mathbf{u}' \, \mathbf{curl}(\mathbf{u}')}\,\tau$, where $\overline{\mathbf{u}' \, \mathbf{curl}(\mathbf{u}')}$ is the helicity of the fluctuating motions and τ some turnover time. This idea implies, apart from the second-order correlation approximation and the restriction to the high-conductivity limit, the assumption of the isotropy of fluctuating motions and is thus rather questionable. In spherical dynamo models the generation of an axisymmetric toroidal field depends on the component a_{00} of the tensor α (Rädler 1981b). Even if we

https://doi.org/10.1017/S0252921100068081 Published online by Cambridge University Press

accept the second-order correlation approximation and the high conductivity limit it is not a_{00} but (1/3) trace (α) which is equal to $(1/3)\overline{\mathbf{u}' \,\mathbf{curl}(\mathbf{u}')}\,\tau$, and these quantities coincide only in the isotopic case. In general, a_{00} may even vanish for non-zero trace (α).

The magnetic flux which penetrates the plasma of the convection zone contributes to its buoyancy. The motions due to this magnetic buoyancy reduce the mean flux in the convection zone (e.g., Krivodubski 1984). With this mind, in several cases on the right-hand side of the induction equation (4.2a) an additional term $-\kappa\overline{\mathbf{B}}$, with some positive coefficient κ, has been introduced (e.g., De Luca and Gilman 1986). We should like to emphasize that this is hardly the correct way to comprehend the effect of magnetic buyancy. According to the derivation of (4.2a) from Maxwwell's equations any effect of motions must be described by $\overline{\mathbf{u}}$ or $\mathcal{E}$.

Apart from the reasons explained above, the lack of knowledge on the convective motions and the differential rotation in deeper layers poses several problems for the construction of realistic solar dynamo models. There have been many attempts to meet observational constraints such as those formulated above, by more complex assumptions on the radial and latitudinal dependence of the α–effect and the differential rotation, and by taking into account anisotropies of the α-effect and related effects (e.g., Krause and Steenbeck 1969a, Stix 1973, 1976, Köhler 1973, Yoshimura 1975, Ivanova and Ruzmaikin 1977, Belvedere et al. 1980a, Yoshimura et al. 1984a,b,c, Schmidt 1987, Brandenburg 1988). We mention in particular a recent model in which in addition to an anisotropic α-effect several related effects are included and an angular velocity inferred from helioseismological measurements is used (Brandenburg and Tuominen 1988). This model reflects rather well the latitudinal dependence of the toroidal magnetic field as derived from observations.

It has been argued that the flux losses in the convection zone due to magnetic buoyancy do not allow a generation process of magnetic fields in this zone. For this and other reasons, models have been proposed in which this process is mainly localized at the bottom of the convective zone or in the overshoot layer below it (e.g., Glatzmaier 1985. De Luca and Gilman 1986, Gilman et al. 1989).

In several models of $\alpha\omega$-dynamos in addition to axisymmetric also non-axisymmetric magnetic fields have been considered (Krause 1971, Stix 1971, Roberts and Stix 1972, Ivanova and Ruzmaikin 1986a, Ruzmaikin et al. 1988), which deserve some interest in view of the sectorial structure of the solar magnetic field. Under certain conditions comparable excitation conditions for axisymmetric and non-axisymmetric fields have been found. For general reasons this is only to be expected if the rotational shear is sufficiently small (Rädler 1986a,b). It turned out that some of the models mentioned (Krause 1971, Stix 1971, Ivanova and Ruzmaikin 1985, Ruzmaikin et al. 1988), which are in conflict to that, have to be revised (Rädler et al. 1989).

5. Dynamo models including the back-reaction of the magnetic field on the motion

5.1. BASIC EQUATIONS

So far we discussed the dynamo process assuming that the fluid motion is given. In particular, we ignored any back-reaction of the magnetic field on the motion. However, it is the back-reaction that determines the magnitude to which the magnetic field grows and it can influence the geometrical structure and time behaviour of the field considerably. Changing our view we now consider the motion no longer given and assume that the velocity $\overline{\mathbf{u}}$ obeys the Navier-Stokes equation and the mass balance,

$$\rho(\frac{\partial u}{\partial t} + (u \bullet \nabla)u = -\nabla p + F + \frac{1}{\mu} curl(B \times B) \tag{5.1a}$$

$$\frac{\partial \rho}{\partial t} + div(\rho u) = 0 \tag{5.1b}$$

Here ρ is the mass density, p the pressure, and F stands for viscous or any other but electromagnetic forces; if F only means viscous forces we have, in Cartesian coordinates,

$$F_i = \frac{\partial \tau_{ij}}{\partial x_j} \tag{5.2}$$

with some stress tensor τ_{ij}. The term $(1/\mu)\, curl \overline{\mathbf{B}} x \overline{\mathbf{B}}$ describes the Lorentz force, where μ is the permeability of free space. For compressible fluids, equations (5.1) have to be supplemented by the equation of state and the energy balance, which we do not write down here.

Compared to the kinematic dynamo problem posed by equations (4.1), the full dynamo problem defined by equations (4.1), (5.1) and some supplementary relations is much more complex. It is no longer a linear but an involved nonlinear problem.

5.2. MEAN-FIELD MAGNETOHYDRODYNAMICS

The mean-field concept introduced above with the kinematic dynamo problem can readily be extended to the full dynamo problem considered now; see, e.g., Rädler (1976). The equations (4.2) and (4.3) have then to be supplemented by corresponding equations derived from (5.1). We shall demonstrate here only the basic idea. To this end we restrict ourselves to incompressible fluids. From (5.1) we then obtain

$$\rho(\frac{\partial \overline{u}}{\partial t} + (\overline{u} \bullet \nabla)\overline{u} = -\nabla \overline{p} + \overline{F} + \frac{1}{\mu} curl(\overline{B} \times \overline{B}) + \mathscr{F} \tag{5.3a}$$

$$div(\rho \overline{u}) = 0 \tag{5.3b}$$

where $\mathscr{F}$ is a mean force resulting from fluctuating motions and magnetic fields,

$$\mathscr{F} = -\rho\ \overline{(u' \bullet \nabla)u'} + \frac{1}{\mu} curl(\overline{B' \times B'}) \tag{5.4}$$

In the case of vanishing magnetic field these equations coincide with the Reynolds equations of hydrodynamic turbulence theory. We note that F and f appear only as their sum, and that this can be represented in the form

$$\overline{F_i} + \overline{\mathscr{F}_i} = \frac{\partial\left(\overline{\tau_{ij}} + \tau^R_{ij} + \tau^M_{ij}\right)}{\partial x_j} \tag{5.5a}$$

$$\tau^R_{ij} = -\rho \overline{u'_i u'_j} \tag{5.5b}$$

$$\tau^M_{ij} = (\frac{1}{\mu})(\overline{B'_i B'_j}) - (\frac{1}{2})\, B'^2 \delta_{ij} \tag{5.5c}$$

where τ_{ij}^R is the Reynolds stress tensor and τ_{ij}^M the Maxwell stress tensor resulting from the fluctuating magnetic field.

In the same sense in which $\mathcal{E}$ was considered a quantity depending on $\overline{\mathbf{B}}$ and its derivatives, $\mathcal{E}$ and $\mathcal{F}$ are now to be understood as quantities depending on $\overline{\mathbf{B}}$, $\overline{\mathbf{u}}$ and their derivatives. We do not want to discuss the general aspects of such relations. We only mention that, for example, the dependence of ε on B is now no longer linear. To explain this in some more detail we return to the simple case considered above in which $\overline{\mathbf{u}}$ is equal to zero, $\overline{\mathbf{u}}'$ corresponds to a homogeneous isotropic turbulence, and $\mathcal{E}$ then is given by (4.5). Consider this turbulence to be due to any forces independent of the magnetic field but admit, in addition, Lorentz forces. The latter will influence the intensity of the turbulence and disturb its isotropy. Under these circumstances $\mathcal{E}$ again contains a term $\alpha\overline{\mathbf{B}}$ but α now depends on $\overline{\mathbf{B}}$. It was shown that, under certain assumptions, α decreases with growing $|\overline{\mathbf{B}}|$ (Rüdiger 1974a).

As already mentioned, in the case of vanishing magnetic field our equations reduce to Reynolds equations. we note that on this basis a comprehensive theory of the differential rotation has been developed (Rüdiger 1989).

5.3. NONLINEAR MEAN-FIELD DYNAMO MODELS

When starting from kinematic mean-field dynamo models and introducing the back-reaction of the magnetic field on the motion we have to consider a variety of new effects. All the coefficients defining the mean electromotive force $\mathcal{E}$, for example $\alpha,\beta,$...occuring in (4.6), depend now on $\overline{\mathbf{B}}$. This concerns not only their magnitudes but also their tensorial structures. In addition, the velocity $\overline{\mathbf{u}}$ of the mean motion is influenced by $\overline{\mathbf{B}}$. This may happen in a direct way via the Lorentz force $(1/\mu)\,\mathrm{curl}\overline{\mathbf{B}}\times\overline{\mathbf{B}}$ or in an indirect way via the force $\mathcal{F}$, that is, via the Reynolds stresses and via the Maxwell stresses of the fluctuating magnetic field. The mathematical problem posed by a kinematic dynamo model is linear in $\overline{\mathbf{B}}$. All the effects mentioned here disturb this linearity.

Several mean-field dynamo models have been investigated which deviate from kinematic models with isotopic α-effect only in so far as α now depends on $\overline{\mathbf{B}}$ and decays with growing $|\overline{\mathbf{B}}|$, sometimes referrred to as "α-quenching", e.g., by Stix (1972), Rüdiger (1974b), Jepps (1975), Ivanova and Ruzmaikein (1977), Kleeorin and Ruzmaikin (1984), Krause and Meinel (1988), Brandenburg et al. (1988a,b,d) and Rädler and Wiedemann (1989). In $\alpha\omega$-models of that kind which, in its zero-field limit, show the symmetries discussed above about equatorial plane and rotation axis, axisymmetric oscillatory magnetic fields have been found which show no longer pure A0 or S0-symmetry but contain both A0 and S0-parts (Brandenburg et al. 1989a,b,d). The magnitude of these fields and its composition of A0 and S0-parts vary, apart from the fundamental oscillation, also with a longer period. This is, of course, of interest in view of the long-term variation of the solar activity. However, in these models the possibility of non-axisymmetric fields has not been considered at all. As we know from a^2-models the stability of an axisymmetric field may well be disturbed by non-axisymmetric fields (Rädler and Wiedeman 1989).

In some other models the back-reaction of the magnetic field on the mean motions via Lorentz-forces of the mean field was studied, e.g., by Malkus and Proctor (1975), De Luca and Gilman (1986), Brandenburg, et al. (1989a,b,d) and Belvedere and Proctor (1989). A reduction of both the α-effect and the differential rotation by the back-reaction of the magnetic field was taken into account by Yoshimura (1978a,b).

We further refer to a model by Brandenburg et al. (1989c) in which, starting from assumptionss concerning the turbulent motions in the solar convection zone, both the magnetic field and the differential rotation are computed, taking into account the two types of back-reaction considered above of the magnetic field on the motions.

Incidentally, for some mean-field dynamo models the Lorentz-forces of the mean fields have been computed and discussed in view of the torsional oscillations (Yoshimura 1981, Rüdiger et al. 1986).

5.4. OTHER NONLINEAR DYNAMO MODELS

Interesting studies of the solar dynamo problem beyond the mean-field approach have been done by Gilman and Miller (1981) and by Gilman (1983). For a spherical shell modelling the solar convection zone the full set of the magnetohydrodynamic equations governing $\overline{\mathbf{B}}$ and $\overline{\mathbf{u}}$ in the Bussinesq approximation has been integrated numerically. The model reflects several features of solar magnetic phenomena. In particular, it shows a magnetic cycle. However , the period is too short for describing the solar cycle, and the equartorward migration of the toroidal field belts as indicated by sunspots is not reproduced. It is not clear whether this is to be ascribed to the Boussinesq approximation, to the unsufficient consideration of the interaction between magnetic field and motion in small scales, or to other reasons. Similar results have been obtained with somewhat different assumptions by Glatzmaier (1985).

There are also a few attempts to mimic features of the solar dynamo by a low-order set of non-linear ordinary differential equations. Ruzmaikin (1985) proposed a set of three first-order equations which form a Lorentz system and tried to find so some explanation of the stochastic nature of the solar activity. Preliminary results of an attractor analysis of the Wolf relative sunspot numbers obtained by Kurths (1987) indeed suggest that the underlying process can be described by a small number of equations, the number being estimated between 3 and 7. In this context an investigation by Weiss (1985) on the chaotic behavior of dynamos, in which a system of seven first-order ordinary differential equations is considered, deserves some interest.

I would like to thank my colleagues Drs. G. Rüdiger and J. Staude (Potsdam) as well as Dr. A. Brandenburg (Helsinki) for many important suggestions.

References

Belvedere, G. (1983) 'Dynamo theory in the Sun and stars', in P.B. Byrne and M. Rodonõ (eds.), Activity in Red-Dwarf Stars, D. Reidel Publishing Co., Dordrecht, pp. 579-599.

Belvedere, G., Padernõ, L. and Stix, M. (1980a) 'Dynamo action of a mean flow caused by latitude-dependent heat transport', Astron. Astrophys. 86, 40-45.

Belvedere, G., Padernõ, L. and Stix M. (1980b) 'Magnetic cycles of lower main sequence stars', Astron. Astrophys. 91, 328-330.

Belvedere, G. and Proctor, M.R.E. (1989) 'Nonlinear dynamo modes and timescales of stellar activity', submitted to Proceedings IAU-Symp. 138.

Brandenburg, A. (1988) 'kinematic dynamo theory and the solar activity cycle', Licenciate thesis, University of Helsinki.

Brandenburg, A., Krause, F., Meinel, R., Moss, D. and Tuominen, I. (1989a) 'The stability of nonlinear dynamos and the limited role of kinematic growth rates', Astron. Astrophys. 213, 411-422.

Brandenburg, A., Krause, F., and Tuominen, I. (1989b) 'Parity selection in nonlinear dynamos', in M. Meneguzzi et al. (eds.), Turbulence and Nonlinear Dynamics in MHD Flows, Elsevier Science Publishers, North Holland.

Brandenburg, A., Moss, D., Rüdiger, G. and Tuominen. I. (1989c) 'The nonlinear solar dynamo and differential rotation: A Taylor number puzzle?', submitted to Solar Physics.

Brandenburg, A., Moss, D. and Tuominen, I. (1989d) 'On the nonlinear stability of dynamo models', Geophys. Astrophys. Fluid Dyn., in press.

Brandenburg, A. and Tuominen, I. (1988) 'Variation of magnetic fields and flows during the solar cycle', Adv. Space Res. 8, No 7, (7)185 - (7)189.

Brandenburg, A., Tuominen, I. and Rädler, K.-H. (1989e) 'On the generation of non-axisymmetric magnetic fields in mean-field dynamos', Geophys. Astrophys. Fluid Dyn., in press.

Busse, F.H. (1979) 'Some new results on spherical dynamos', Physics Earth Planet. Inter. 20, 152-157.

Busse, F.H. and Miin, S.W. (1979) 'Spherical dynamos with anisotropic α-effect', Geophys. Astrophys. Fluid Dyn. 14, 167-181.

Cowling, T.G. (1934) 'The magnetic fields of sunspots', Mon. Not. Roy. Astr. Soc. 94, 39-48.

Deinzer, W. and Stix, M. (1971) 'On the eigenvalues of Krause-Steenbeck's solar dynamo', Astron. Astrophys. 12, 111-119.

Deinzer, W., von Kusserow, H.U. and Stix, M. (1974) 'Steady and oscillatory $a\omega$-dynamos', Astron. Astrophys. 36, 69-78.

Deluca, E.E. and Gilman, P.A. (1986) 'Dynamo theory for the interface between convection zone and the radiative interior of a star. Part I. Model equations and exact solutions', Geophys. Astrophys. Fluid Dyn. 37, 85-127.

Durney, B.R. (1988) 'On a simple dynamo model and the anisotropic α–effect', Astron. Astrophys. 191, 374.

Gilman, P.A. (1983) 'Dynamically consistent nonlinear dynamos driven by convection in a rotating spherical shell. II. Dynamos with cycles and strong feedbacks', Astrophys. J. Suppl. 53, 243-268.

Gilman, P.A. (1986) 'The solar dynamo: observations and theories of solar convection, global circulation, and magnetic fields', in P.A. Sturrock et al. (eds.), Physics of the Sun, D. Reidel Publishing Co., Dordrecht, pp. 95-160.

Gilman, P.A. and Miller, J. (1981) 'Dynamically consistent nonlinear dynamos driven by convection in a rotating spherical shell', Astrophys. J. Suppl. 46, 211-238.

Gilman, P.A., Morrow, C.A. and Deluca, E.E. (1989) 'Angular momentum transport and dynamo action in the Sun: Implications of recent oscillation measurements', Astrophys. J. 338, 528-537.

Glatzmaier, G.A. (1985) 'Numerical simulations of stellar convective dynamos. II. Field propagation in the convection zone', Astrophys. J. 291, 300-307.

Ivanova, T.S. and Ruzmaikin, A.A. (1975) 'A magnetohydrodynamic dynamo model of the solar cycle', Sov. Astron. 20, 227-234.

Ivanova, T.S. and Ruzmaikin, A.A. (1977) 'A nonlinear MHD-model of the dynamo of the Sun', Astron. Zh. (USSR) 54, 846-858 (in Russian).

Ivanova, T.S. and Ruzmaikin, A.A. (1985) 'Three-dimensional model for the generation of the mean solar magnetic field', Astron. Nachr. 306, 177-186.

Jepps, S.A. (1975) 'Numerical models of hydromagnetic dynamos', J. Fluid Mech. 67, 629-646.

Kleeorin, N.I. and Ruzmaikin, A.A. (1984) 'Mean-field dynamo with cubic non-linearity', Astron. Nachr. 305, 265-275.

Köhler, H. (1973) 'The solar dynamo and estimates of the magnetic diffusivity and the α-effect', Astron. Astrophys. 25, 467-476.

Krause, F. (1971) 'Zur Dynamotheorie magnetischer Sterne: Der 'symmetrische Rotator' als Alternative zum 'schiefen Rotator'', Astron. Nachr. 293, 187-193.

Krause, F. and Meinel, R. (1988) 'Stability of simple nonlinear α^2-dynamos', Geophys. Astrophys. Fluid dyn. 43, 95-117.

Krause, F. and Rädler, K.-H. (1980) 'Mean-Field Magnetohydrodynamics and Dynamo Theory', Akademie-Verlag, Berlin and Pergamon Press, Oxford.

Krivodubski, V.N. (1984) 'Magnetic field transfer in the turbulent solar envelope', Sov. Astron. 28, 205-211.

Kurths, J. (1987) 'An attractor analysis of the sunspot relative number', Preprint PRE-ZIAP (Potsdam) 87-02.

Larmor, J. (1919) 'How could a rotating body such as the Sun become a magnet?' Rep. Brit. Assoc. adv. Sc. 1919, 159-160.

Levy, E.H. (1972) 'Effectiveness of cyclonic convertion for producing the geomagnetic field', Astrophys. J. 171, 621-633.

Malkus, W.V.R. and Proctor, M.R.E. (1975) 'The macrodynamics of α-effect dynamos in rotating fluids', J. Fluid Mech. 67, 417-444.

Nicklaus, B. and Stix, M. (1988) 'Corrections to first order smoothing in mean-field electrodynamics', Geophys. Astrophys. Fluid Dyn. 43, 149-166.

Parker, E.N. (1955) 'Hydromagnetic dynamo models', Astrophys. J. 122, 293-314.

Parker, E.N. (1979) 'Cosmical Magnetic fields', Clarendon Press, Oxford.

Rädler, K.-H. (1969) 'über eine neue Möglichkeit eines Dynamomechanismus in turbulenten leitenden Medien', Mber. Dtsch. Akad. Wiss. Berlin 11, 194-201.

Rädler, K.-H. (1975) 'Some new results on the generation of magnetic fields by dynamo action', Mem. Soc. Roy. Sc. Liege VIII, 109-116.

Rädler, K.-H. (1976) 'Mean-field magnetohydrodynamics as a basis of solar dynamo theory', in B. Bumba and J. Kleczek (eds.), Basic Mechanisms of Solar Activity, D. Reidel Publishing Co., Dordrecht, pp. 323-344.

Rädler, K.-H. (1980) 'Mean-field approach to spherical dinamo models', Astron. Nachr. 301, 101-129.

Rädler, K.-H. (1981a) 'On the mean-field approach to spherical dynamo models', in A.M. Soward (ed.), Stellar and Planetary Magnetism, Gordon and Breach Publishers, New York, pp. 17-36.

Rädler, K.-H. (1981b) 'Remarks on the α-effect and dynamo action in spherical models', in A.M Soward (ed.), Stellar and Planetary Magnetism, Gordon and Breach Publishers, New York, pp. 37-48.

Rädler, K.-H. (1986a) 'Investigations of spherical kinematic mean-field dynamo models', Astron. Nachr. 307, 89-113.

Rädler, K.-H. (1986b) 'On the effect of differential rotation on axisymmetric and non-axisymmetric magnetic fields of cosmical bodies', Plasma-Astrophysics, ESA SP-251, 569-574.

Rädler, K. -H. and Bräuer, H.-J. (1987) 'On the oscillatory behaviour of kinematic mean-field dynamos', Astron. Nachr. 308, 101-109.

Rädler, K.-H., Brandenburg, A. and Tuominen, I. (1989) 'On the non-axisymmetric magnetic-field modes of the solar dynamo', Poster IAU-Colloquium No 121, to be submitted to Solar Physics.

Rädler, K.-H. and Wiedemann, E. (1989) 'Numerical experiments with a simple nonlinear mean-field dynamo model',Geophys. Astrophys. fluid Dyn., in press.

Ribes, E., Mein, P. and Manganey, A. (1985) 'A large scale meridional circulation in the convective zone', Nature 318, 170-171.

Ribes, E. and Laclare, F. (1988) 'Toroidal convection rolls in the Sun', Geophys. Astrophys. Fluid Dyn. 41, 171-180.

Roberts, P.H. (1972) 'Kinematic dynamo models', Phil. Trans. Roy. Soc. A 272, 663-703.

Roberts, P.H. and Stix, M. (1972) 'α-effect dynamos, by the Bullard-Gellman formalism', Astron. Astrophys. 18, 453-466.

Rüdiger, G. (1974a) 'The influence of a uniform magnetic field of arbitrary strength on turbulence', Astron. Nachr. 295, 275-283.

Rüdiger, G. (1974b) 'Behandlung eines einfachen hydromagnetischen Dynamos mit Hilfe der Gitterpunktmethode', Pub. Astrophys. Obs. Potsdam 32, 25-29.

Rüdiger,G. (1980) 'Rapidly rotating α^2-dynamo models', Astron. Nachr. 301, 181-187.

Rüdiger, G. (1989) 'Differential Rotation and Stellar Convection', Akademie-Verlag, Berlin and Gordon and Breach Science Publishers, New York.

Rüdiger, G. Tuominen, I., Krause, F. and Virtanen, H. (1986) 'Dynamo generated flows in the Sun', Astron. Astrophys. 166, 306-318.

Ruzmaikin, A.A. (1985) 'The solar dynamo', Solar Physics 100, 125-140.

Ruzmaikin, A.A., Sokoloff, D.D. and Starchenko, S.V. (1988) 'Excitation of non-axially symmetric modes of the Sun's magnetic field', Solar Phys. 115, 5-15.

Schmitt, D. (1985) 'Dynamowirkung magnetischer Wellen', Thesis, Univ. Göttingen.

Schmitt, D. (1987) 'An α-dynamo with an α-effect due to magnetostrophic waves', Astron. Astrophys. 174, 281-287.

Steenbeck, M. and Krause, F. (1969a) 'Zur Dynamotheorie stellarer und planetarer Magnetfelder. I. Berechnung sonnenähnlicher Wechselfeldgeneratoren', Astron. Nachr. 291, 49-84.

Steenbeck, M. and Krause, F. (1969b) 'Zur Dynamotheorie stellarer und planetarer Magnetfelder. II. Berechnung planetenähnlicher Gleichfeldgeneratoren', Astron. Nachr. 291, 271-286.

Steenbeck, M., Krause, F. and Rädler, K.-H. (1966) 'Berechnung der mittleren Lorentz-Feldstärken vxB für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflubter Bewegung', Z. Naturforsch. 21a, 369-376.

Stenflo, J.O. (1973) 'Magnetic-field structure of the photospheric network', Solar Physics 32, 41-63.

Stenflo, J.O. and Vogel, M. (1986) 'Global resonances in the evolution of solar magnetic fields', Nature 319, 285.

Stenflo, J.O. and Güdel, M. (1987) 'Evolution of solar magnetic fields: Modal stucture', Astron. Astrophys. 191, 137.

Stix, M. (1971) 'A non-axisymmetric α-effect dynamo', Astron. Astrophys. 13, 203-208.

Stix, M. (1972) 'non-linear dynamo waves', Astron. Astrophys. 20, 9-12.

Stix, M. (1973) 'Spherical α-dynamos, by a variational method', Astron. Astrophys. 24, 275-281.

Stix, M. (1976a) 'Dynamo theory and the solar cycle', in V. Bumba and J. Kleczek (eds.), Basic Mechanisms of Solar Activity, D. Reidel Publishing Co., Dordrecht, pp. 367-388.

Stix, M. (1976b) 'Differential rotation and the solar dynamo', Astron. Astrophys. 47, 243-254.

Stix, M. (1981) 'Theory of the solar cycle', Solar Physics 74, 79-101.

Stix, M. (1983) 'Helicity and a-effect of simple convection cells', Astron. Astrophys. 118, 363-364.

Stix, M. (1989) 'The Sun', Springer-Verlag Berlin.

Tuominen, I., Rüdiger, G. and Brandenburg, A. (1988) Observational constraints for solar-type dynamos', in O. Havens et al. (eds.), Activity in Cool Star Envelopes, Kluwer Academic Publishers, London, pp. 13-20.

Walder, M., Deinzer, W. and Stix, M. (1980) 'Dynamo action associated with random waves in a rotating stratified fluid', J. Fluid Mech. 96, 207-222.

Weiss, N.O. (1985) 'Chaotic behaviour in stellar dynamos', Journal of Statistical Physics 39, 477-491.

Weisshaar, E. (1982) 'A numerical study of α^2-dynamos with anisotropic α-effect', Geophys. Astrophys. Fluid dyn. 21, 285.

Yoshimura, H. (1975a) 'Solar-cycle dynamo wave propagation', Astrophys. J. 201, 740-748.

Yoshimura, H. (1975b) 'A model of the solar cycle driven by the dynamo action of the global convection in the solar convection zone', Astrophys. J. Suppl. 29, 467-494.

Yoshimura, H. (1976) 'Phase relation between the poloidal and toroidal solar-cycle general magnetic fields and location of the origin of the surface magnetic fields', Solar Physics 50, 3-23.

Yoshimura, H. (1978a) 'Nonlinear astrophysical dynamos: The solar cycle as the non-linear oscillation of the general magnetic field driven by the non-linear dynamo and the associated modulation of the differential-rotation-global-convection system', Astrophys. J. 220, 692-711.

Yoshimura, H. (1978b) 'Nonlinear astrophysical dynamos: multiple-period dynamo wave oscillations and long-term modulations of the 22 years solar cycle', Astrophys. J. 226, 706-719.

Yoshimura, H. (1981) 'Solar cycle Lorentz force waves and the torsional oscillations of the Sun', Astrophys. J. 247, 1102-1112.

Yoshimura, H., Wang, Z. and Wu, F. (1984a) 'Linear astrophysical dynamos in rotating spheres: Differential rotation, anisotropic turbulent magnetic diffusivity, and solar-stellar cycle magnetic parity', Astrophys. J. 280, 865-872.

Yoshimura, H., Wang, Z. and Wu, F. (1984b) 'Linear astrophysical dynamos in rotating spheres: Mode transition between steady and oscillatory dynamos as a function of the dynamo strength and anisotropic turbulent magnetic diffusivity', Astrophys. J. 283, 870-878.

Yoshimura, H., Wu, F. and Wang, Z. (1984c) 'Linear astrophysical dynamos in rotating spheres: Solar and stellar cycle north-south hemisphere parity selection mechanism and turbulent magnetic diffusivity', Astrophys. J. 285, 325-338.