are all similar to the whole triangle. Now PR and QR are equally inclined to AD (a pedal property), and N is the image of Q in AD. $\therefore$ PR produced passes through N; similarly LM produced passes through Q. Let these lines cut BC in S and S'.

Again $PN \parallel DB$ since alternate angles BDR, DRP are corresponding angles of the similar triangles BDA, DRP; similarly $LQ \parallel DC$.

But $BS : SC = DP : PC$ (BD parallel to SP)

$= BL : LD$ (complete similarity of the figures)

$= BS' : S'C$ (LS' parallel to DC)

$\therefore$ S and S' coincide.

Now area of triangle NBS = area of triangle NDS (NS parallel to BD)

,, ,, QCS = ,, ,, QDS (QS ,, DC).

To the sum of these areas add area $ANSQ$.

$\therefore$ in area, triangle ABC = kite $ANDQ$ = twice triangle AND.

G. D. C. Stokes.

Editor's Note.—Mr John T. Brown suggests the following neat method of proving that the triangle ABC is twice the triangle AND:

Suppose DN produced its own length to E.

Then the angles EAD, BAC are equal,

$$\text{and } AE \,.\, AD = AD^2 = AB \,.\, AC.$$

Hence, by Euc. VI, 15, the triangles AED, ABC are equal; *i.e.* twice triangle AND = triangle ABC.

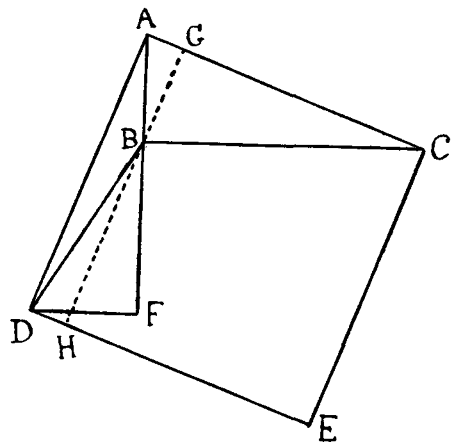
W. A.

A Proof of the Theorem of Pythagoras.

The triangle ABC has a right angle at B. On AC, on the same side as B, describe a square $ADEC$. Draw DF perpendicular to AB or AB produced.

The triangles ABC and DFA are congruent, having sides CA and AD equal, and the corresponding angles equal. Hence DF is equal to AB.

Join DB: then the triangle ADB is equal to half the rectangle with AB as base and FD as altitude; that is, to half the square on AB.



Now through B draw GBH parallel to AD to meet AC in G and DE in H. Then the rectangle $ADHG$, being equal to twice the triangle ADB, is equal to the square on AB.

Similarly the rectangle $CGHE$ is equal to the square on BC. Thus, on adding, we find that the square on AC is equal to the sum of the squares on AB and BC.

T. M. MacRobert.

Note on a Vanishing Determinant.

1. In one of Sir Thomas Muir's more recent historical papers on the theory of determinants (*Proc. R. S. Edin.*, XLIII, 1922, p. 129), is included a result due to V. Jung, commented on as being "verified in an unsuggestive way for the first three cases." The theorem is that the determinant

$$\begin{vmatrix} 1 & 1^2 & \dots & 1^n & 1^{n+1} \\ 2 & 2^2 & \dots & 2^n & 2^{n+1} \\ \dots & \dots & \dots & \dots & \dots \\ n & n^2 & \dots & n^n & n^{n+1} \\ \frac{n}{2} & \frac{n^2}{3} & \dots & \frac{n^n}{n+1} & \frac{n^{n+1}}{n+2} \end{vmatrix}$$

vanishes when n is even. Below we give a simple proof.

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