the space described is represented by the area of a triangle or quadrilateral. Only when this preliminary work has been accomplished should an exercise such as the following be given :—Draw a pair of rectangular axes. Let 1 inch on the x-axis represent 1 second, and 1 inch on the y-axis a velocity of 1 foot per second. Join the points (0, 2) and (4, $3\frac{1}{2}$). Read from the velocity time diagram so obtained the following information :—The velocity at the end of 5 seconds; the time at which the velocity was 5 feet per second; the acceleration; the space described in 3·8 seconds; the space described in the 4th second, etc.

With regard to the parallelogram of accelerations, the text-book leaves the young student in a state of vague bewilderment over this theorem. Here again I would suggest that the pupil ought to work out such an exercise as the following :—Draw a pair of rectangular axes XOX′ and YOY′. Let 1 inch on each axis represent a space of 1 foot. Suppose a point to move from rest at O subject to two simultaneous uniform accelerations along the axes, mark the positions of the point at the end of 1, 2, 3, 4, and 5 seconds. Join O to the last position of the point, *i.e.*, at the end of the 5th second. Do the other positions lie on this line?

The pupil will in this way convince himself that a body may have two component accelerations, and that the resultant acceleration is obtained by the Parallelogram Law.

The association of this law with the dynamical equation $f=ma$ makes the derivation of the parallelogram of forces theorem comparatively simple and straightforward, but if a boy feels that he does not grasp the parallelogram of accelerations he may suspect that he is being hoodwinked into a belief in the truth of the fundamental theorem of statics, unless he depends upon a purely experimental proof, in which event he must be prepared to agree that two forces may be represented by a single resultant, and that hanging weights over a pulley does not alter the force exerted on the knot.

W. ANDERSON.

Quadratic Equations.—The following is a scheme of classroom work designed to introduce a pupil to quadratic equations, graphically and algebraically.

Newton's Chord Rule is applied in (3) to calculate by Arithmetic close approximations to the roots of an equation. *One graph suffices* *or any number of equations, as in (2), (5) and (7).* Both the method of completing the square and the method of factors are required as algebraic equivalents, as in (4), (6), (7). The general rules for solving quadratics sum up the lessons.

1. Draw a graph of $y = x^2 - 2x - 1$, between $x = -1$ and $x = 3$, taking one inch as scale-unit on both axes.

2. Note that the graph cuts the axis of x where $x = 2{\cdot}4$ and $-0{\cdot}4$, approx.; so that $x = 2{\cdot}4$ or $-0{\cdot}4$ is the solution of the equation $x^2 - 2x - 1 = 0$.

3. When $x = 2{\cdot}4$, $x^2 - 2x - 1 = 5{\cdot}76 - 4{\cdot}8 - 1 = -0{\cdot}04$.

The graph shows that the correct value of x is greater than $2{\cdot}4$.

When $x = 2{\cdot}5$, $x^2 - 2x - 1 = 6{\cdot}25 - 5 - 1 = +0{\cdot}25$.

The graph shows that the correct value of x is less than $2{\cdot}5$.

Assuming that the graph is a straight line between $x = 2{\cdot}4$ and $x = 2{\cdot}5$, we have the following problem in simple proportion.

An increase of $0{\cdot}1$ in the value of x gives an increase of $0{\cdot}29$ in the value of $x^2 - 2x - 1$, what increase in the value of x gives an increase of $0{\cdot}04$ in the value of $x^2 - 2x - 1$?

Required increase in the value of $x = \dfrac{0{\cdot}1 \times 0{\cdot}04}{0{\cdot}29} = {\cdot}014$, to the third decimal place.

When $x = 2{\cdot}414$, $x^2 - 2x - 1 = 5{\cdot}827 - 4{\cdot}828 - 1 = -0{\cdot}001$.

When $x = 2{\cdot}415$, $x^2 - 2x - 1 = 5{\cdot}832 - 4{\cdot}830 - 1 = +0{\cdot}002$.

$\therefore$ $x = 2{\cdot}414$ is a solution correct to the third decimal place.

The case of $x = -0{\cdot}4$ is then dealt with in the same way.

4. Algebraical calculation of roots of $x^2 - 2x - 1 = 0$. We have $x^2 - 2x = 1$, *i.e.*, $(x-1)^2 = 2$, *i.e.*, $x = 1 \pm \sqrt{2} = 2{\cdot}414$ or $-0{\cdot}414$ to the third decimal place.

The use of a table of square roots should be allowed to begin with.

5. *From the same graph* find the roots of the equations

$$2x^2 - 4x - 3 = 0,\ 3x^2 - 6x + 2 = 0,\ 5x^2 - 10x - 4 = 0, \text{ etc., etc.}$$

Thus write $2x^2 - 4x - 3 = 0$ as $x^2 - 2x - 1 = 1/2$, and read off the abscissae of the intersections of the graph and $y = 1/2$. Then proceed as in (3).

6. Algebraic calculations of the roots of $2x^2 - 4x - 3 = 0$, etc. **Thus**: $x^2 - 2x - 3/2 = 0$, $\therefore$ $x^2 - 2x = 3/2$,

$$\therefore (x-1)^2 = 5/2,\ \therefore x = 1 \pm \sqrt{2{\cdot}5} = 2{\cdot}58 \text{ or } -0{\cdot}58.$$

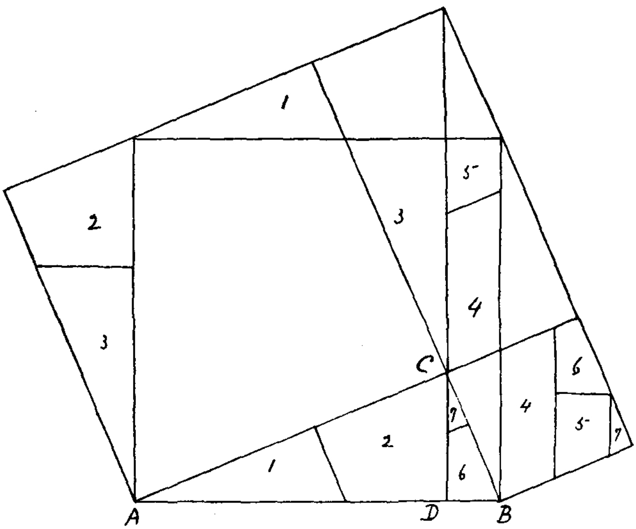
7. Graphical solution of $x^2 - 2x = 0$, $4x^2 - 8x + 3 = 0$, $x^2 - 2x + 1 = 0$ and other special cases, as in (2) and (3), *still from the same graph.*

Algebraical solution of these equations by using factors.

8. Rules for the calculation, by Algebra, of the roots of quadratic equations.

P. PINKERTON.

The Theorem of Pythagoras.—The parts of the figures are numbered to indicate a proof of the theorem by dissection. The congruence of parts on which the same number is marked can be demonstrated by geometry. The steps of the proof are in order (i), (ii), (iii). Sufficient importance is, perhaps, not attached to (i) and (ii) at this stage; for they give (1) the construction of a square equal to a given rectangle (2) the graphical construction of $\sqrt{2}$, $\sqrt{2}+1$, etc.



(i) $AC^2 = AB \cdot AD$; (ii) $BC^2 = AB \cdot BD$; (iii) $AC^2 + BC^2 = AB^2$

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