From Interacting Binaries to Exoplanets: Essential Modeling Tools
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TODCOR – Two-Dimensional Correlation

Shay Zucker

Department of Geophysics and Planetary Sciences
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, 69978 Tel Aviv, Israel
email: shayz@post.tau.ac.il

Abstract. TODCOR is a TwO-Dimensional CORrelation technique to measure radial velocities of the two components of a spectroscopic binary. Assuming the spectra of the two components are known, the technique correlates an observed binary spectrum against a combination of the two spectra with different shifts. TODCOR measures simultaneously the radial velocities of the two stars by finding the maximum correlation. The main use of the technique has been to turn single-lined binaries into double-lined systems. This helps to explore the binary mass-ratio distribution, especially the low-mass regime, where the secondaries are usually very faint and therefore hard to detect. The technique has been generalized to study multi-order spectra, and also triple- and quadruple-lined systems. It has several applications in studying extrasolar planets and in the future may even help to dynamically measure stellar masses of binaries through relativistc effects.

Keywords. Celestial mechanics, methods: data analysis, techniques: radial velocities, techniques: spectroscopic, binaries: spectroscopic, stars: low mass, brown dwarfs, planetary systems

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1. Introduction

Since the seminal works of Simkin (1974) and Tonry & Davis (1979) the cross-correlation technique to measure astronomical Doppler shifts has become extremely popular. The advent of digitized spectra and computers made it the preferred method. It has been applied in all the astronomical fields that required the measurement of radial velocities from observed spectra, ranging from binary and multiple stellar systems to cosmology. The most prominent impact of radial velocities measured through cross-correlation was the detection of extrasolar planets (e.g., Mayor & Queloz 1995).

The calculation of the cross-correlation function is quite simple. Let $g(n)$ denote the zero-averaged, continuum-subtracted, observed spectrum, whose Doppler shift is to be found. Let $t(n)$ denote the corresponding 'template' spectrum of zero shift. Both the stellar spectrum and the template are assumed to be described as functions of the bin number, n, where $n = A \ln \lambda + B$. Thus, the Doppler shift results in a uniform linear shift of the spectrum (Tonry & Davis 1979). The cross-correlation function is then given by the expression:

$$c(n) = \frac{1}{N\sigma_g\sigma_t} \sum_m g(m)t(m-n), \tag{1.1}$$

where σ_g and σ_t are the RMS values of the corresponding spectra and N is the length of the spectra (for simplicity, I will ignore here the problem of edge effects). The location of the maximum of the function $c(n)$ is used as an estimate of the Doppler shift of the spectrum. Figure 1 presents a sample cross-correlation function with a well-defined peak.

Single-lined spectroscopic binaries (SB1) are characterized by the periodic variation of the star's radial velocity, corresponding to its projected orbital motion. Double-lined spectroscopic binaries (SB2) are characterized by the presence of two stellar spectra,

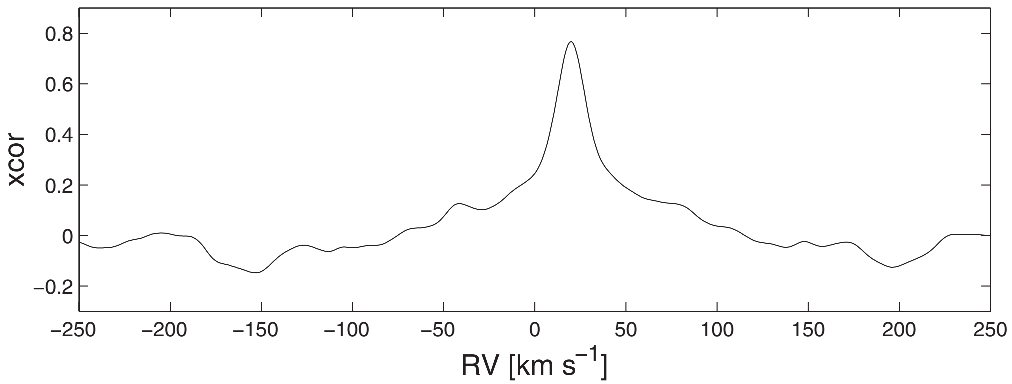

Figure 1. A typical cross-correlation function. The spectrum is a noisy Doppler-shifted synthetic spectrum of a G-star and the template is the original spectrum. The simulated radial velocity is $20\,\mathrm{km\,s^{-1}}$.

which are manifested in the cross-correlation function as two peaks (Figure 2.A). The locations of the two peaks can serve to estimate the velocities of the two components. However, in certain cases, or in certain orbital phases, the peaks might blend, which prevents a simple estimate of the two velocities (Figure 2B). Even when two separate maxima can be detected, the blend can affect their location and introduce systematic bias.

The TwO-Dimensional CORrelation technique – TODCOR, presented by Zucker & Mazeh (1994), offers a way to overcome these difficulties. Some details of the technique are presented in Section 2. Section 3 reviews some current applications of TODCOR and I conclude in Section 4.

2. The Two-Dimensional CORrelaion

The Two-Dimensional Correlaion is a straightforward generalization of the conventional cross-correlation technique. Instead of using a template spectrum – $t(m-n)$, TODCOR uses a combination of two templates, using a different Doppler shift for each template:

$$t_1(m-n_1) + \alpha t_2(m-n_2)$$

where α denotes the relative weight (light ratio) of the two components.

Substituting the above combined template in the expression in Equation 1.1, and after some algebraic manipulation, we obtain:

$$c(n_1,n_2) = \frac{c_1(n_1) + \alpha c_2(n_2)}{\sqrt{1 + 2\alpha c_{12}(n_2-n_1) + \alpha^2}} \ .$$

In cases where the light ratio – α – is not known in advance, the value of α that would give the best results can be searched for. Alternatively, a value of α that would maximize $C(n_1,n_2)$ can be found analytically:

$$\hat{\alpha}(n_1,n_2) = \frac{c_1(n_1)c_{12}(n_2-n_1) - c_2(n_2)}{c_2(n_2)c_{12}(n_2-n_1) - c_1(n_1)} \ .$$

Incorporating this value in the correlation expression, we obtain the symmetric

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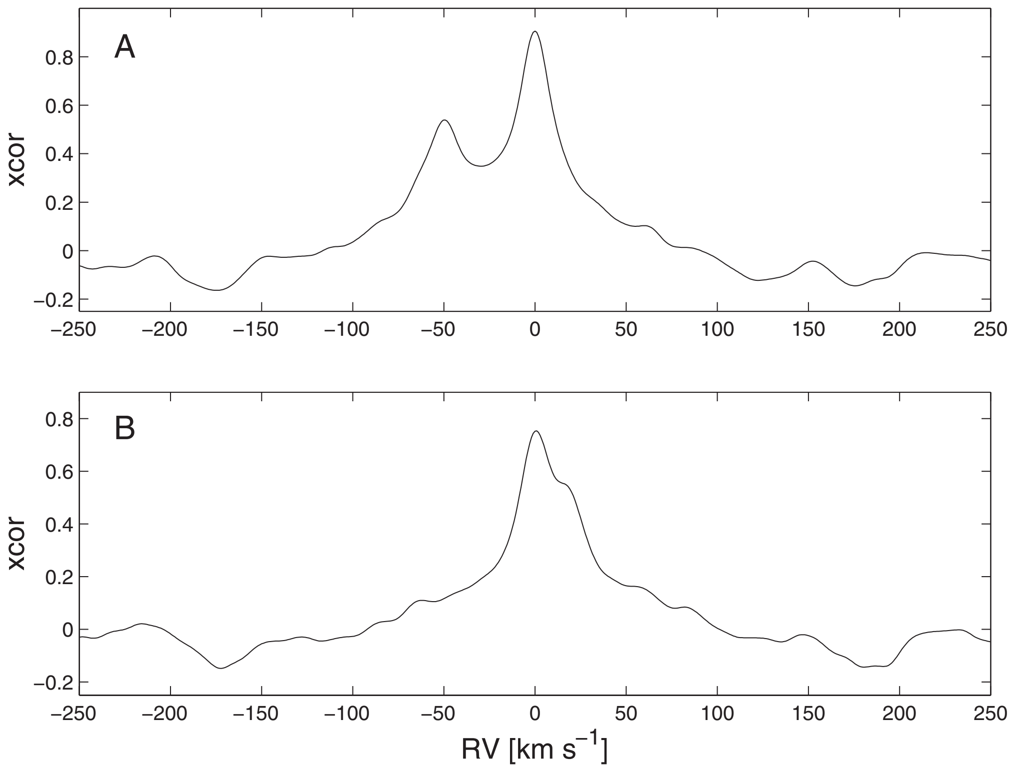

Figure 2. Cross-correlation function for simulated composite spectra. A: the simulated velocities are 0 and $-50\,\mathrm{km\,s^{-1}}$ and two separate peaks can be spotted at these velocities. B: the simulated velocities are 0 and $20\,\mathrm{km\,s^{-1}}$ and the two peaks are severly blended.

expression:

$$R(n_1, n_2) = \sqrt{\frac{c_1^2(n_1) - 2c_1(n_1)c_2(n_2)c_{12}(n_2 - n_1) + c_2^2(n_2)}{1 - c_{12}^2(n_2 - n_1)}}$$

In any case, the correlation is now a function of two variables – the two Doppler shifts n_1 and n_2, or, equivalently, the two radial velocities v_1 and v_2. Figure 3 presents a plot of this two-dimensional function for the same spectrum whose cross-correlation is presented in Figure 2.B. This spectrum was simulated using a primary velocity of 0 and a secondary velocity of $20\,\mathrm{km\,s^{-1}}$. One can see a clear maximum, whose location serves as an estimate of these two velocities.

A closer look at Figure 3 reveals an illuminating topographical structure: the maximum seems to be located at the intersection of two 'ridges'. Each ridge corresponds to 'freezing' one velocity and varying only the other one. Figure 4 shows the function when the secondary velocity is fixed (4.A) and when the primary velocity is fixed (4.B). These two 'cuts' can be regarded as kinds of cross-correlation functions, where we look for the best estimate for one velocity, after already having dealt with the other velocity. Comparing these two plots to Figure 2 highlights the advantage of TODCOR in measuring the two velocities. In these two 'cuts' the corresponding peak (the primary peak in A and the secondary peak in B) is accentuated and the second one is attenuated.

TODCOR is conceptually tailored to analyse composite spectra. The use of two templates allows better chances to detect secondary spectra that are significantly different

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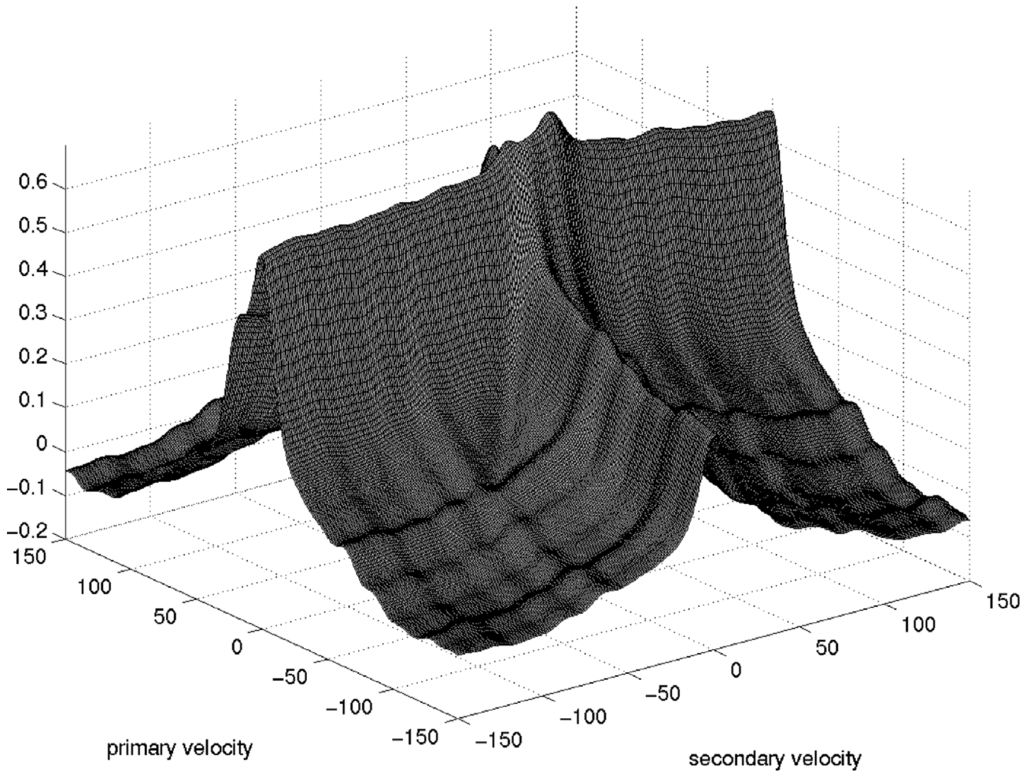

Figure 3. The two-dimensional correlation function for the spectrum whose cross-correlation function is depicted in Figure 2.B. The maximum is very close to the correct velocities – 0 and $20\,\mathrm{km\,s^{-1}}$.

from their primaries, whereas the detection of the secondary peak in cross-correlation relies on some similarity between the two spectra.

Originally, TODCOR was designed to analyse single-order spectra. Nowadays, as the use of Echelle spectrographs is widespread, it became necessary to adapt TODCOR to accept multi-oder spectra as input. Zucker (2003) suggested a statistically optimal formula to combine the correlation function of different orders into a combined function. This practice allows full exploitation of the information in all the orders, thus allowing the detection of very faint secondaries. This led to the complete solution of the system HD 41004, where a brown dwarf orbits the faint companion of a binary system, and a planet orbits the bright companion (Zucker *et al.* 2003).

Another generalization of the original TODCOR approach is its application to analyze 'triple-lined' and 'quadruple-lined' spectra, of triple and quadruple systems. The codes that implement this approach are based on the generalizations of the above formulae. Zucker *et al.* (1995) first developed a TODCOR generalization for spectroscopic triples, and Torres *et al.* (2007) repeated this feat for quadruple stars.

3. Applications

In the years since the introduction of TODCOR, many groups used it in their studies. The most common application is in the study of eclipsing binaries. The common practice is enumeration over a grid of templates. This grid of templates may comprise synthetic

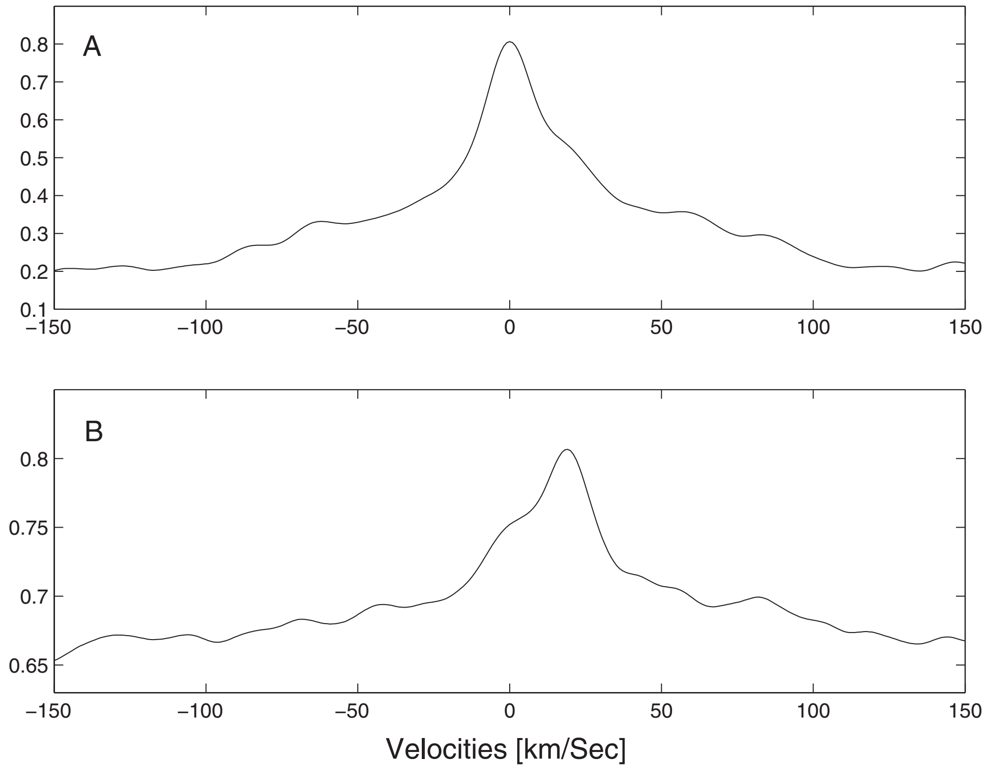

Figure 4. 'Cuts' through the two-dimensional correlation function shown in Figure 3. In panel A the secondary velocity is fixed while in panel B the primary velocity is fixed.

spectra or alternatively high-S/N observed spectra. The criterion for the optimization is usually the value of the correlation at the maximum.

Several studies pointed to the fact that residual systematics may still remain in the velocities TODCOR found. These systematics can be estimated by using the templates to simulate the observed spectra without noise and applying TODCOR on them (e.g., Torres *et al.* 2009).

A common application of TODCOR is in the search of faint companions. Basically, the goal is to turn an SB1 into an SB2. From the orbital elements of an SB1, one can calculate only the so-called 'mass function':

$$f(M_2) = M_1 \frac{(q \sin i)^3}{(1+q)^2},$$

where M_1 and M_2 are the binary component masses, i is the orbital inclination, and q is the mass ratio. Thus, only a lower bound for q can be deduced, by substituting $\sin i = 1$. If even only a few measurements of the secondary velocities are available, we can measure the radial velocitiy semi-amplitudes of both components - K_1 and K_2, whose inverse ratio is the mass ratio:

$$q = \frac{K_1}{K_2} \ .$$

Figure 5 demonstrates this use of TODCOR. This Figure is taken from Simon & Prato (2004), who applied TODCOR to study the PMS binary Haro 1-14c. As the figure demonstrates, they needed only a few (eight) successful measurements of the secondary velocity to constrain K_2. Most of the primary velocities were determined earlier by Reipurth *et al.*

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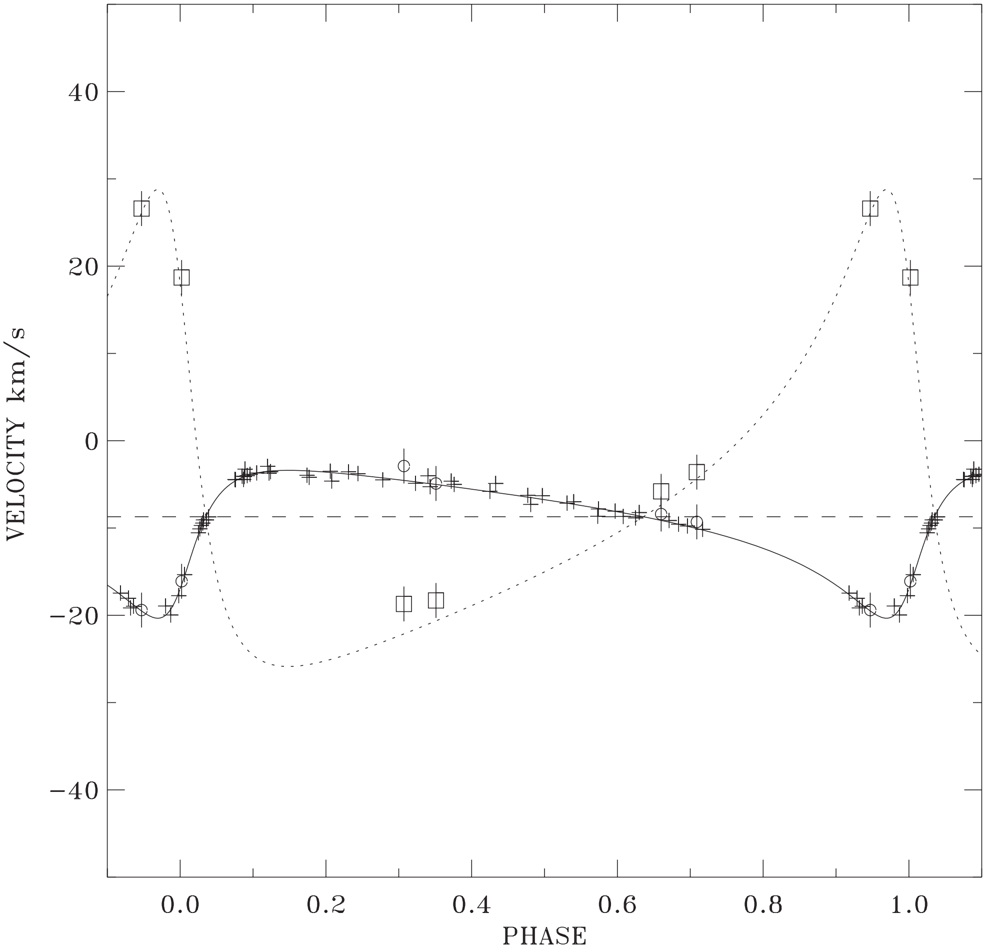

Figure 5. Radial velcity orbital solution of Haro 1-14c. Crosses represent the primary velocities obtained by Reipurth *et al.* (2002). Open symbols represent velocities obtained by Simon & Prato (2004) obtained using TODCOR (image taken from Simon & Prato 2004).

(2002) and served to constrain the other orbital elements. Simon & Prato obtained a mass ratio of $q = 0.310 \pm 0.014$. Schaefer *et al.* (2008) later confirmed this result through interferometry.

The study of extrasolar planets utilizes TODCOR for several purposes. As TODCOR was originally tailored to study binary stars, the most obvious application is the search for planets in spectroscopic binaries. Konacki *et al.* (2009) introduced the TATOOINE survey for that purpose, using TODCOR to get precise radial velocities that may show the signature of a hypothetical planet. TATOOINE uses the Iodine cell technique to calibrate the radial velocities, which required some modification of the original TODCOR approach.

Another use is to exclude the stellar nature of a candidate planet. In principle, the RV motion detected in cross-correlation may be the result of the combined opposite motions of the primary star and a faint companion. Researchers use TODCOR to verify that there's no secondary spectrum detectable in the observed spectra. Konacki *et al.* (2003) used this approach to rule out the stellar nature of the transiting planet OGLE-TR-56.

TODCOR is also being used to identify blend scenarios in candidate extrasolar planetary transits. Thus, for example, Mandushev *et al.* (2005) used TODCOR to prove that what seemed like a transiting brown dwarf around the F star GSC 01944-02289, was actually an eclipsing binary comprising G0V and M3V stars, orbiting the F5V star.

4. Concluding remarks

A few works presented in this meeting use Spectral Disentagling (SD) techniques (Simon & Sturm 1994; Hadrava 1995; Ilijic, Hensberge, & Pavlovski 2002) to analyze SBs. The SD techniqhe was also the topic of the talk by Petr Hadrava. SD aims to solve a different problem than TODCOR: whereas TODCOR assumes the spectra themselves are known or at least constrained, SD tries to estimate them as well as the orbital elements. Thus, the range of applicability is different for the two approaches: when the spectra are not well constrained, SD can be used to provide the individual spectral features. When they are relatively known, trying to estimate them would unnecessarily increase the errors of the orbital elements, and it would be better to use all the available information. Another important difference between the two techniques is the fact that TODCOR works on individual spectra, whereas SD needs a good coverage of orbital phases to obtain satisfactory results in estimating the spectra. Once again, this relates to the fact that SD tackles a much more difficult problem.

I concluded my talk by referring to a future application of SB2s in general, which is mass detemination through relativistic effects. We presented this idea in 2007 (Zucker & Alexander 2007), and it seems to be quite a challenge, as it requires very precise RVs of SB2s. Whether TODCOR is the right way to solve this problem or not, I believe it is a challenge that is worth exploring, as the return of having more dynamically determined stellar masses is significant.

References

Hadrava, P. 1995, *A&AS*, 114, 393
Ilijic, S., Hensberge, H., & Pavlovski, K. 2002, *Fizika B*, 10, 357
Konacki, M., Muterspaugh, M. W., Kulkarni, S. R., & Helminiak, K. G. 2009, *ApJ*, 704, 513
Konacki, M., Torres, G., Jha, S., & Sasselov, D. D. 2003, *Nature*, 421, 507
Mandushev, G., *et al.*, 2005, *ApJ*, 621, 1061
Mayor, M., Queloz D. 1995, *Nature*, 378, 355
Reipurth, B., Lindgren, H., Mayor, M., Mermillod, J.-C., & Cramer, N. 2002, *AJ*, 124, 2813
Schaefer, G. H., Simon, M., Prato, L., & Barman, T. 2008, *AJ*, 135, 1659
Simkin, S. M. 1974, *A&A*, 31, 129
Simon, K. P. & Sturm, E. 1994, *A&A*, 281, 286
Simon, M. & Prato, L. 2004, *ApJ*. 613, L69
Tonry, J. & Davis, M. 1979, *AJ*, 84, 1511
Torres, G., Claret, A., & Young, P. 2009, *ApJ*, 700, 1349
Torres, G., Latham, D. W., & Stefanik, R. P. 2007, *ApJ*, 662, 602
Zucker, S. 2003, *MNRAS*, 342, 1291
Zucker, S. & Alexander, T. 2007, *ApJ*, 670, 1326
Zucker, S. & Mazeh, T. 1994, *ApJ*, 420, 806
Zucker, S., Mazeh, T., Santos, N. C., Udry, S., & Mayor, M. 2003, *A&A*, 404, 775
Zucker, S., Torres, G., & Mazeh, S. 1995, *ApJ*, 452. 863

Discussion

P. Harmanec: Would you agree that there is also another difference between the correlation technique and disentangling? The correlation technique performs better as the

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studied wavelength interval increases. The danger hidden in this is that if for some reason (e.g., systematic errors in wavelength calibration, unrecognized star spots affecting different lines differently etc.) the velocity amplitude differs from one line to another, you will not know that. For disentangling, you can investigate each stronger line separately (on the premise that you have enough spectra available) and see whether you are obtaining considerable results.

S. Zucker: I agree, in the sense that TODCOR and spectral disentangling are not applicable in the same cases. TODCOR is probably not the best approach for cases when one has to analyze individual lines separately.

A. Prša: Regarding the relativistic corrections, wouldn't it be more convenient to apply them to the RV curves that were already extracted, i.e., run TODCOR in a classical way and incorporate the corrections in an EB model?

S. Zucker: It was probably not clear in my talk. Indeed, there's no need to introduce relativistic effects at the stage of calculating the velocities. You first calculate the velocities in the regular way, and only introduce relativity at the orbital solution.