THE SHORTAGE OF LONG-PERIOD COMETS IN ELLIPTICAL ORBITS

Edgar Everhart
Physics Department, University of Denver

Based on the number of 'new' comets seen on near-parabolic orbits, one can predict the number of comets that should be found on definitely elliptical orbits on their subsequent returns. We show here that about three out of four of these returning comets are not observed.

In this study a Monte Carlo model, which follows hypothetical comets, yields distributions that can be compared with those for real comets. The model takes hundreds of these hypothetical random comets entering the solar system with original 1/a-values, here called u-values, equal to 50×10^{-6} AU^{-1} . This is a typical value for near-parabolic new comets. Jupiter and Saturn perturb the total energies of the comets. Those that gain sufficient total energy are lost to infinity on hyperbolic orbits, but others lose total energy (i.e. their u-values become more positive) and return again. Sometimes comets return many times during their random walk on the u-axis, but eventually u becomes negative and all are lost to infinity.

The model also includes perturbations by passing stars. Comets with very small positive u-values have enormous aphelia. For these tne gravitational impulses caused by passing stars can change their perihelion distances, sometimes so drastically that the comets do not then come near enough to the sun to be visible.

For the class of comets with perihelia between 0 and 1 AU we form a cumulative distribution N(u) vs u, where N(u) is the number of comets with u-values equal to or less than u. Figure 1 compares N(u) from the Monte Carlo experiment with the same distribution for real comets, normalizing both curves to 1000 new comets.

The data shown for real comets are found from the 1/a (original) column of Table III in Marsden, Sekanina, and Everhart (1978). Of the 82 comets in that table with perihelia less than unity we take 28 to be new, based on their original u-values. The observed N(u) curve is far lower than the predicted curve. This paper discusses the discrepancy.

R. L. Duncombe (ed.), Dynamics of the Solar System, 273–275.

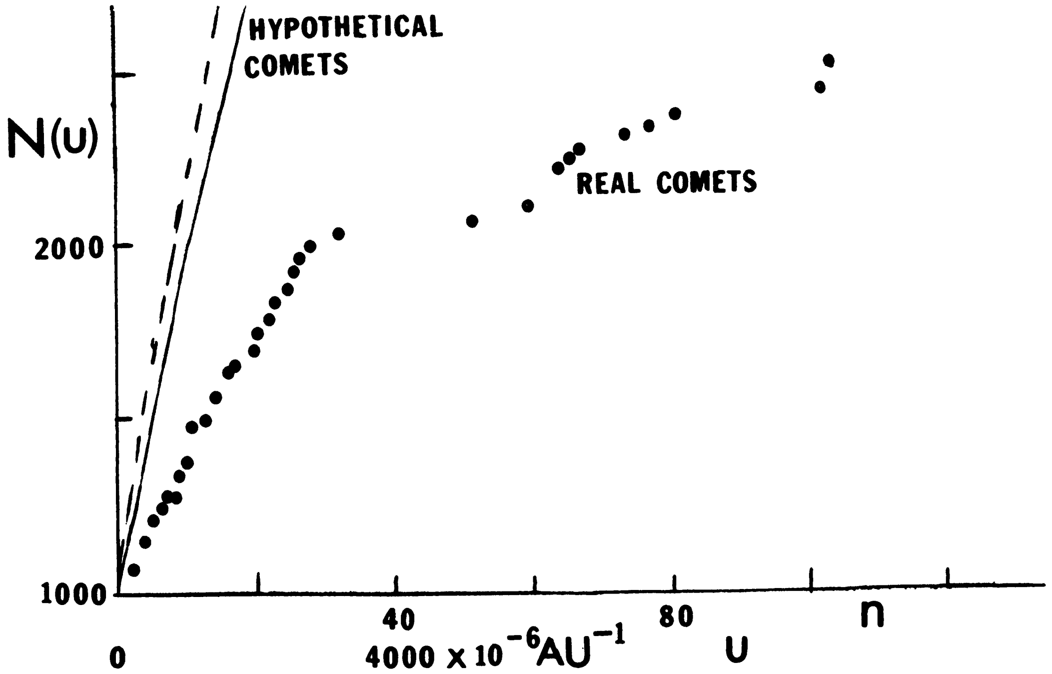

Fig. 1. Cumulative Distributions for Comets with $0<q<1$.

The dashed line in the figure shows the effect of not including stellar perturbations in the model. Allowing for them does lower the predicted curve, but not nearly enough to agree with the data.

Let us look at the Monte Carlo results in more detail. Let m be the return number of a hypothetical comet. Thus when m = 3 it is on its third return, not counting its initial appearance. Let n be the index specifying its u-value, as in the scale in Fig. 1. Define S_{mn} as the number of events in the (m,n) category, normalizing to 1000 entering comets. One can form the table of S_{mn} values shown below. We see that on their 3rd return there was 41 comets with a u-index of 2. This is only

n \ m	1	2	3	4	...
1	42	59	65	50	...
2	30	39	41	30	...
3	17	23	26	22	...
4	...	...	...	...	...

Table I. The S_{mn} Matrix

the upper left corner of a 100 by 200 matrix, since the experiment followed hypothetical comets up to 100 returns, and there were 200 u-indices. Summing S_{mn} of m for each row and then forming a cumulative sum up to index n yields the predicted N(n)-values of Figure 1.

Now let w_m be the probability that a comet is observed on its mth return. One might expect this to be considerably less than unity because of dissipation and observational selection. Then

$$w_1 S_{1n} + w_2 S_{2n} + w_3 S_{3n} + \dots = \Delta N_0 / \Delta n, \tag{1}$$

where the right side is the slope of the data curve for real comets in Fig. 1 per unit change in n.

There are 200 such equations, each with 100 terms. Such a large matrix is not tractable. Accordingly, we group the m and n indices so that there are only 22 equations with 22 w-values to solve for.Although the 22 by 22 system is readily solved,the results turn out to be useless. The w-values, being probabilities, should be in the range of 0 to 1, but the solution has them wildly oscillating. Values of -94 and +80 are seen. Reasonable results are achieved only after restricting the w-variation to 2 parameters and solving for these in a least-squares sense.

For the set of comets where $0 < q < 1$ we require w to change only linearly as m increases with the grouped data. We then find w to be about 0.20 for the first return and about 0.23 for the group of returns numbered 90 to 100. This means that we see only about 1/5 to 1/4 of the returning comets in this range of perihelia. For $1 < q < 2$ we find w to be 0.28 for the first return and 0.13 for returns 90 to 100. For the range $2 < q < 3$ these probabilities are 0.33 and 0.04.

An improved model would show that the actual probabilities are lower than the above numbers. Because of observational selection not all real comets are actually observed on their initial appearance when they are 'new'. If we saw only 50% of these, this would change the normalization in Fig. 1 so as to double the discrepancy. Then the probabilities given above would be cut in half.

It is well known that comets with $q > 3$ AU are rarely seen, except as new comets. We see here nearly the same thing for comets of smaller perihelia. The model is now being improved, allowing for distributions in absolute magnitude of comets and observational selection. With the better model we ought to be able to make a quantitative estimate of dissipation, which is (at this writing) the only effect we know of which could account for so many returning comets being missed.

The support of the National Science Foundation is appreciated.

Reference:

Marsden, B.G., Sekanina, Z., and Everhart, E. "Astron. J." 83, pp.64-71.