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Green's relations in semigroups of functions

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The first two chapters in this work are devoted to finding characterizations of Green's relations in various semigroups of continuous functions.

The main concern of the thesis is the concept of a pseudo-idempotent in a semigroup of functions. Basically a pseudo-idempotent function f is a function which maps its image bijectively onto itself. Chapter 3 is devoted to proving one basic theorem on pseudo-idempotents in various settings. For the Green's relation H we look at the corresponding equivalence classes, called H-classes. For a number of semigroups of functions we show that an H-class contains an idempotent if and only if it contains a pseudo-idempotent if and only if it consists of pseudo-idempotents. For $S(X)$, the semigroup of continuous functions on a topological space X, the theorem is established for various choices of X. For X discrete the theorem is established for the following semigroups: $S_1(X)$, the one-to-one functions in $S(X)$; $Q(X)$, the semigroup of functions defined on arbitrary subsets of X; $Q_1(X)$, the one-to-one functions in $Q(X)$; $S(X, Y)$, the functions in $S(X)$ which take a fixed subset $Y \subset X$ into Y; $S(X, p, Y)$, the functions from X into Y where multiplication is given by $f \circ g = fpg$ for a fixed function p from Y into X.

In Chapter 4 we examine semigroup homomorphisms φ which have the

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property that for some Green's relation G we always have f and $\varphi(f)$ in the same G-class. We say that such homomorphisms are G-class-preserving. The typical result here is that the only G-class-preserving homomorphism φ is the identity isomorphism. For instance, if $\varphi : Q(X) \to Q(X)$ is H-class-preserving and X is Hausdorff, then φ is the identity.

In Chapter 5 we look at pseudo-idempotents and G-class-preserving homomorphisms in the context of semigroups of relations. Most of the chapter is devoted to $B_1(X)$, the semigroup of relations T on a set X such that $x \neq y$ implies $T(x) \cap T(y) = \emptyset$. We show that an H-class in $B_1(X)$ contains an idempotent or pseudo-idempotent if and only if it consists of pseudo-idempotents. And we show that the only H-class-preserving homomorphism $\varphi : B_1(X) \to B_1(X)$ is the identity isomorphism.

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