J. Fluid Mech. (2024), *vol.* 1000, F7, doi:10.1017/jfm.2024.834

JFM FOCUS ON FLUIDS

Dynamic mode decomposition for analysis of time-series data

Ivan Marusic†

Department of Mechanical Engineering, University of Melbourne, Parkville, VIC 3010, Australia

(Received 9 June 2024; accepted 30 August 2024)

Since its publication in 2010, the paper by Schmid (*J. Fluid Mech.*, vol. 656, 2010, pp. 5–28) has wielded considerable influence, an impact we examine here. That seminal work introduced dynamic mode decomposition, a method for performing flow-field spectral analysis of snapshot sequences of data. As a data-driven approach aimed at uncovering spatial and temporal patterns or modes within datasets, its applicability has extended far beyond fluid mechanics, reaching into a wide array of fields.

Key words: low-dimensional models

1. Introduction

The paper by Schmid (2010) stands out as the only paper in the *Journal of Fluid Mechanics* to be the subject of two Focus of Fluids articles. The initial article, authored by Henningson (2010), focused on the potential implications of the work. Now, after 14 years, we reflect on the enduring impact of the paper and why it was selected as the most significant paper in the *Journal of Fluid Mechanics* published in the century of volumes 601–700.

Schmid (2010) introduced dynamic mode decomposition (DMD), which was initially presented at the 2008 APS-DFD meeting (Schmid & Sesterhenn 2008). The DMD approach serves as a factorization and dimensionality reduction technique for sequential data streams. Essentially, it provides a means to analyse and extract dynamic behaviour from time-series data, particularly within the context of dynamical systems and fluid dynamics. It represents an equation-free, data-driven approach aimed at identifying spatial and temporal patterns or modes within the data.

The rapid adoption and success of DMD in fluid mechanics since Schmid (2010) has likely coincided with the concurrent rise of data science and the rapid development of time-resolved, planar and volumetric velocity field measurement techniques (particle image velocimetry and its variants) and numerical simulations (direct numerical

† Email address for correspondence: imarusic@unimelb.edu.au

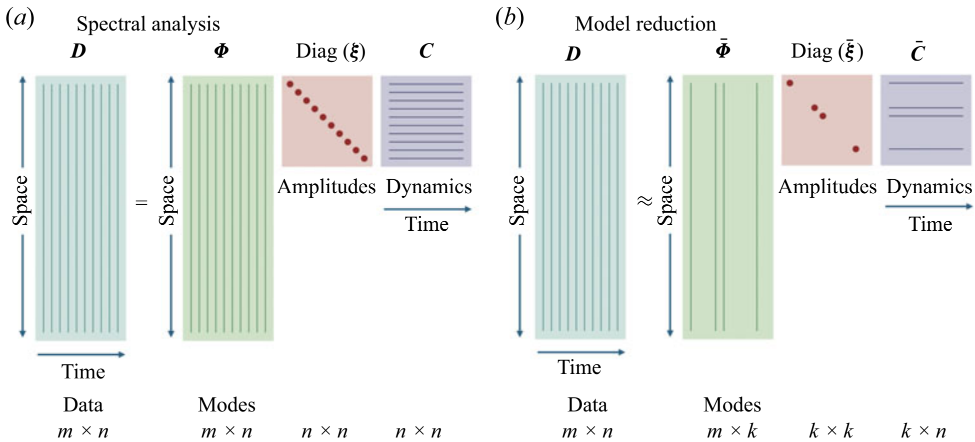

Figure 1. Dynamic mode decomposition: (*a*) factorization and (*b*) dimensionality reduction. Adapted with permission from Schmid (2022).

simulations and large-eddy simulations). These techniques provide the necessary input to DMD, as described by Schmid (2010), in the form of constant-sampling-rate time-series data.

2. Overview

The fundamental aspects of the DMD approach are depicted in figure 1. The time-series data are arranged into a data (snapshot) matrix $\boldsymbol{D}$ where each column represents a snapshot of the system at a specific time. The aim is to produce a decomposition of this data matrix into three matrices capturing spatial structures ($\boldsymbol{\Phi}$), the amplitudes (diag($\boldsymbol{\xi}$)) and temporal dynamics ($\boldsymbol{C}$).

The reduced-order representation detailed in figure 1(*b*) may be obtained by manually selecting modes that correspond to large amplitudes and/or frequencies of interest, or via algorithmic methods such as the sparsity-promoting DMD variant developed in Jovanović, Schmid & Nichols (2014). This reduced subset aims to capture the dominant dynamics of the system. The selected (dynamic) modes depict spatial patterns or structures evolving over time, with each mode representing a specific oscillatory behaviour or dynamic feature of the system. Furthermore, the eigenvalues associated with each dynamic mode are produced by the DMD algorithm, providing insights into the growth or decay rates of the respective modes.

The DMD approach is closely associated with Koopman analysis, a technique used in dynamical systems theory. Specifically, DMD produces the best (least-squares) fit linear dynamical system to the nonlinear dynamical system generating the data; its dynamic modes are approximations of Koopman modes (Rowley *et al.* 2009). Hence, DMD provides a straightforward and powerful algorithm for computing Koopman eigenfunctions and their associated eigenvalues (Brunton *et al.* 2022), which can be further enhanced through the use of nonlinear transformations of the original data (Williams, Kevrekidis & Rowley 2015). Moreover, as pointed out by Henningson (2010), the dynamic modes of DMD simplify to global linear eigenmodes for linearized problems or to Fourier modes for (nonlinear) periodic problems.

The DMD methodology is well suited for analysing flows where the processed data capture relevant temporal and/or spatial scales of the dynamic process. Schmid (2022)

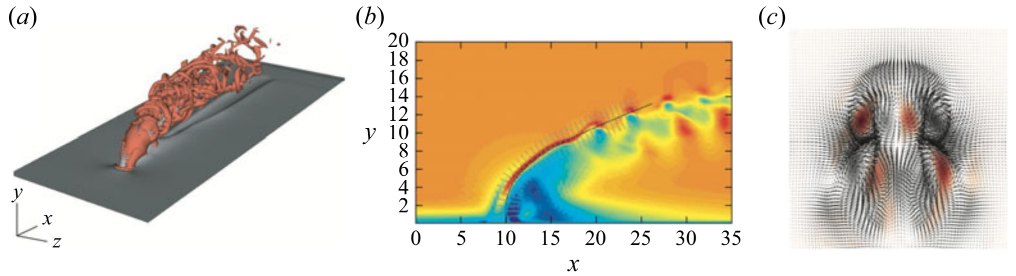

Figure 2. Jet in cross-flow. (*a*) Isocontour of Q for representative snapshot. (*b*) Layout of the spatial slices normal to the base-flow streamline. (*c*) First dominant dynamic mode from streamline-based DMD analysis of the spatial evolution of fluid structures along the counterrotating vortex sheet. Velocity vectors shown in black and vorticity contours in red–grey colour in a plane normal to the base-flow streamline emanating from the jet exit. Adapted with permission from Schmid (2022).

presents an illustrative example demonstrating its effectiveness in complex flows, particularly in describing the evolution of structures along curved trajectories in space. For instance, as depicted in figure 2, a turbulent jet in cross-flow exhibits a breakdown of vortical structures developing along a curved counterrotating vortex pair. Streamline-based DMD analysis reveals this phenomenon by projecting the three-dimensional velocity fields into in-plane and plane-normal components along the curved streamline and incorporating these data into the DMD data matrix.

While DMD has predominantly been utilized in fluid mechanical studies for flow-field analysis, its application extends to various other domains, including thermal, reactive, acoustic or fluid-structural time-series data. In addition to serving as a diagnostic tool through low-rank spatiotemporal features, DMD facilitates state estimation and future-state prediction, paving the way for its application in purely data-driven control strategies (Kutz *et al.* 2016). These attributes, coupled with the approach's versatility in handling a variety of evolution processes and data formats, underscore the broad appeal and utility of DMD.

3. Broader applications and future

Schmid (2022) provides a comprehensive review of recent advancements in DMD, emphasizing its strengths while also recognizing its limitations. Since DMD is limited to linear analysis, it may struggle to capture nonlinear transients and strong nonlinearity effects. It can also face robustness challenges when dealing with noisy or insufficient data, which are usually unknowns, as well as sparse measurements or data with sub-Nyquist sampling. Consequently, considerable attention has been directed towards extending, generalizing and improving DMD, making it an active area of ongoing research (Kutz *et al.* 2016; Schmid 2022).

The utility of the DMD approach, relying solely on data without the need for an underlying model or governing equations, has led to its rapid adoption across numerous fields beyond fluid mechanics. Examples range from its utilization in the analysis and forecasting of highway traffic (Avila & Mezić 2020) to its application in facial recognition schemes. In this latter context, DMD's capability to represent the temporal information of an entire video as a single image with dimensions similar to those of the video's constituent images serves as a critical preprocessing step, enabling efficient detection of spoof samples (Tirunagari *et al.* 2015). Furthermore, DMD finds application in medicine, such as in the real-time forecasting of tumour ablation treatment simulations (Bourantas *et al.* 2014), and in robotics, where Berger *et al.* (2015) integrated DMD with machine learning to detect, estimate and compensate for external perturbations using only input from standard sensors. Numerous other application areas, as described by Schmid (2022), extend to neuroscience, epidemiology, medical imaging, climate science, oceanography and financial engineering.

The above list of application areas is far from complete and will only continue to grow. Indeed, the citation numbers of Schmid (2010) continue to increase exponentially year on year. This growth is unlikely to halt anytime soon, especially given the incorporation of DMD into many modern machine learning approaches (Brunton, Noack & Koumoutsakos 2020), whose uptake in research has been staggering. Coupled with the ever-growing generation of data, in both quality and quantity, this trend ensures that the impact of data-centric techniques such as DMD will only continue to expand.

Acknowledgements. I wish to thank Dr S. Illingworth and Dr S. Dawson for their informative insights and helpful discussions, and the ONR (grant N62909-23-1-2068) for financial support.

Declaration of interests. The author reports no conflict of interest.

Author ORCID.
Ivan Marusic https://orcid.org/0000-0003-2700-8435.

REFERENCES

AVILA, A.M. & MEZIĆ, I. 2020 Data-driven analysis and forecasting of highway traffic dynamics. *Nat. Commun.* **11** (1), 2090.

BERGER, E., SASTUBA, M., VOGT, D., JUNG, B. & BEN AMOR, H. 2015 Estimation of perturbations in robotic behavior using dynamic mode decomposition. *Adv. Robot.* **29** (5), 331–343.

BOURANTAS, G.C., GHOMMEM, M., KAGADIS, G.C., KATSANOS, K., LOUKOPOULOS, V.C., BURGANOS, V.N. & NIKIFORIDIS, G.C. 2014 Real-time tumor ablation simulation based on the dynamic mode decomposition method. *Med. Phys.* **41** (5), 053301.

BRUNTON, S.L., BUDIŠIĆ, M., KAISER, E. & KUTZ, J.N. 2022 Modern Koopman theory for dynamical systems. *SIAM Rev.* **64** (2), 229–340.

BRUNTON, S.L., NOACK, B.R. & KOUMOUTSAKOS, P. 2020 Machine learning for fluid mechanics. *Annu. Rev. Fluid Mech.* **52**, 477–508.

HENNINGSON, D.S. 2010 Description of complex flow behaviour using global dynamic modes. *J. Fluid Mech.* **656**, 1–4.

JOVANOVIĆ, M.R., SCHMID, P.J. & NICHOLS, J.W. 2014 Sparsity-promoting dynamic mode decomposition. *Phys. Fluids* **26** (2), 024103.

KUTZ, J.N., BRUNTON, S.L., BRUNTON, B.W. & PROCTOR, J.L. 2016 *Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems*. SIAM.

ROWLEY, C.W., MEZIĆ, I., BAGHERI, S., SCHLATTER, P. & HENNINGSON, D.S. 2009 Spectral analysis of nonlinear flows. *J. Fluid Mech.* **641**, 115–127.

SCHMID, P.J. 2010 Dynamic mode decomposition of numerical and experimental data. *J. Fluid Mech.* **656**, 5–28.

SCHMID, P.J. 2022 Dynamic mode decomposition and its variants. *Annu. Rev. Fluid Mech.* **54**, 225–254.

SCHMID, P.J. & SESTERHENN, J. 2008 Dynamic mode decomposition of numerical and experimental data. In *Sixty-First Annual Meeting of the APS Division of Fluid Dynamics San Antonio, Texas, USA*. American Physical Society.

TIRUNAGARI, S., POH, N., WINDRIDGE, D., IORLIAM, A., SUKI, N. & HO, A.T.S. 2015 Detection of face spoofing using visual dynamics. *IEEE Trans. Info. Forensics Security* **10** (4), 762–777.

WILLIAMS, M.O., KEVREKIDIS, I.G. & ROWLEY, C.W. 2015 A data-driven approximation of the Koopman operator: extending dynamic mode decomposition. *J. Nonlinear Sci.* **25** (6), 1307–1346.