7.2 A MAGNETIC DIPOLE MODEL FOR THE CRAB EXPLOSION

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Dr. Ostriker suggested that the main features of the Crab Nebula, in particular the value of its magnetic field and the energy of the particles, could be derived very simply and directly from the known characteristics of the pulsar. The link was through the radiation from a spinning dipole. The following summary was prepared by the Editors.

The power radiated from a rotating dipole with a skew axis, as noted by Pacini in a discussion of neutron stars (and even earlier by Deutsch in connection with A stars) is proportional to $\omega^4 B^2 a^6$, where ω is angular velocity, B a field strength, and a the radius. This power is calculable directly from observations of ω and $\dot{\omega}$, given a value of the moment of inertia I from the neutron star theory. Since a and I do not vary during the evolution of the pulsar, we can assume that ω varies with time as

$$\frac{\omega^2}{\omega_0^2} = \left(1 + \frac{2t}{\tau_0}\right)^{-1}$$

where τ_0 is obtained from observations as

$$\tau_0 = \frac{\omega}{\dot{\omega}}.$$

Since the power radiated varies as ω^4, we can now follow the history of the power injected into the nebula through the dipole radiation.

The magnetic field B close to the pulsar is a dipole field. At larger distances it becomes a spiral, then it forms a spherical wave in which the field strength falls as R^{-1}. (This is possibly observable, providing a test of the theory.)

It is certain that energy is supplied continuously to the nebula. Woltjer has emphasised the requirement for injecting energy into the electrons which radiate X-rays and light. The electrons which radiate radio waves also require a continuous input, since their present energy of $10^{49\pm1}$ erg could not have been achieved after a large adiabatic expansion from a radius of say 10^{13} cm to 10^{18} cm. This would require an initial energy of 10^{54} erg.

Even the supernova event itself cannot be impulsive. When it becomes visible the thermal energy is of order 10^{49} erg and the radius 10^{15-16} cm. Again an adiabatic expansion cannot have taken place, since an energy of over 10^{54} erg would be required.

We therefore consider the whole process of expansion of the nebula as controlled by the energy output of the pulsar. The mass motions are consistent with this, if we equate kinetic energy to the loss of pulsar rotational energy.

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The model ignores only the initial phase of sudden contraction of a white dwarf core to a neutron star. The remaining outer parts of the star form the nebula, and the present characteristics are obtained from simple integrations involving the pressure in the cavity. The boundary conditions are unimportant, since, whatever processes of reflection or absorption occur, the cavity will be filled by a gas of relativistic particles with $\gamma = \frac{4}{3}$.

Numerical values are satisfactory if the initial mass of the star is about 10 to 12 $M_{\odot}$. The calculations give present values as follows: velocity 990^{+59}_{-30} km sec^{-1}, radius $2.6^{+0.3}_{-0.2} \times 10^{18}$ cm, magnetic field 7×10^{-5} G at the edge, energy in the cavity $1.6 \times$ $\times 10^{49}$ erg.

Although the detailed processes are not understood, the general theory of magnetic dipole radiation provides the right values for relativistic particle energies and magnetic field. It is very attractive both for the Crab Nebula and for the extragalactic objects which are sources of strong synchrōtron radiation.

A detailed solution will be found in the Astrophysical Journal dated March 1971.

Discussion

A. T. Moffet: Could you relate some of the features of your model to some of the observed features of the Crab Nebula?

J. P. Ostriker: The model is a shell, but if a bird's-nest is preferred there will be a scaling factor of order unity. The shell should be expanding at 990 km sec^{-1}, with radius 3.6×10^{18} cm. Within the shell the main field should fall off as r^{-1} to 7×10^{-5} G; its structure will have rotational symmetry, and it will define the polarization of the radiation. The particles giving optical radiation will fill the nebula, moving isotropically; the lowest energy particles giving radio will have a small zone of avoidance and will exist in the outer regions only.

F. D. Kahn: The magnetic field you describe in the nebula is associated with an electric field. Does this affect the nature of the synchrotron radiation produced by the relativistic electrons?

J. P. Ostriker: Yes, the radiation is not truly synchrotron radiation, but something we have called NIC (non-linear inverse Compton). It is synchrotron-like. J. E. Gunn and I have written a paper on the characteristics of NIC radiation which should appear soon.

F. D. Drake: Your model appears to predict a decay in the electromagnetic radiation from the nebula. Perhaps this is an observable quantity – can you tell us what it is.

J. P. Ostriker: Since the particles currently being accelerated in the Crab do not contribute to the observed radio radiation we would predict the usual result that changes are only due to the adiabatic losses.