

BERRICK, A. J., *An approach to algebraic K-theory* (Research Notes in Mathematics 56, Pitman, 1982), 108 pp. £7.95.

SILVESTER, J. R., *Introduction to algebraic K-theory* (Chapman and Hall, 1981), xi+255 pp. £15 (hardback) or £6.95 (paperback).

These two books are of very different characters, as their titles suggest. Silvester aims to treat the subject at the most elementary level possible. His book has undergraduate algebra as its only prerequisite, and its theme is the extension of linear algebra from fields to more general rings. The material consists of the easiest parts of the subject. Algebraic  $K$ -theory is the study of a sequence of functors  $K_q$  ( $q$  an integer) from rings to abelian groups. In general their definitions need topological ideas, but  $K_0$ ,  $K_1$  and  $K_2$  have purely algebraic definitions. Silvester's book studies  $K_0$ ,  $K_1$  and  $K_2$ , giving their algebraic definitions, their basic properties, and some computations.

Berrick's book is aimed at algebraists, topologists and algebraic number-theorists. It is intended to "complement, not reproduce, established references", and so hopes to interest veterans as well as novices. Its interests are structural rather than computational. The bulk of the book is designed to give a natural definition of  $K_q$  for any integer  $q$ , and thus explain the properties of the algebraically defined  $K_0$ ,  $K_1$  and  $K_2$ . When  $q$  is positive  $K_q A$  ( $A$  a ring) is in fact the  $q$ th homotopy group of a topological space  $X$  depending on  $A$ . The construction of  $X$  involves a long excursion through the byways of elementary, but subtle, homotopy theory; in particular there is a detailed study of Quillen's plus-construction. There are a few original results.

How well do the books succeed? Silvester's book really is written at an elementary level; it gives its arguments carefully and in detail. As with all books, it helps to have more than the bare prerequisites; for this book, a little knowledge of categories, modules and group presentations would be useful. The material, particularly given the few prerequisites, is well-chosen: the functors  $K_q$  with  $q=0, 1$  or  $2$  are the ones that usually appear in applications. On the other hand there is very little guidance for the reader; definitions, theorems and proofs follow directly on one another, and the proofs tend to be computational rather than explanatory. The scope of the book is similar to that of J. Milnor's *Introduction to algebraic K-theory* (Annals of Mathematics Studies 72, Princeton, 1971). Milnor's book is written at a more advanced level, but gives more guidance, and I think people who can follow it will prefer it. Silvester's book would be suitable for a taught course.

Berrick's book has an interesting combination of aims. It gives a summary of the main results of algebraic  $K$ -theory, many without proof, and this should be useful to anybody. It also gives a detailed study of certain parts of the theory. This is worth skimming in any case, but a thorough reading needs a background in algebraic topology. The book is written in a lively and humorous style, and the structures of the arguments are made clear. The author goes to some trouble to find slick proofs. In places the style is allusive, as though the book were really aimed at a veteran rather than a novice. A good example is the opening sentence: "The beginning is predictable enough..." Unfortunately there are several errors, both typographical ("For  $q \leq 2$ " instead of "For  $q \geq 2$ " in statement (12.1)) and mathematical (the alleged ring  $CA$  defined on page 22 is not closed under multiplication). To sum up, much of the book is for a veteran with some knowledge of topology; the novice will enjoy it, but should be careful.

Both books are reproduced from typescript, and it is interesting to compare their appearances. Silvester's has type of uniform thickness and symbols in italics, while Berrick's has type of variable thickness and symbols in roman. Berrick's has larger type and more space. I found Berrick's easier to read, and I deduce that it is not worth putting symbols in italics. I hope this conclusion will be agreeable to typists.

RICHARD STEINER

KASCH, F. *Modules and rings* (translated by D. A. R. Wallace) (Academic Press, London 1982), xiii + 372 pp. £33.80.

This book has grown out of lectures and seminars given over the years by Professor Kasch. It only assumes a very basic knowledge of the elementary concepts of ring and module theory, and