25. THE SECONDARY MAXIMUM OF T CORONAE BOREALIS

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1. T Coronae Borealis is a well-known recurrent nova which has undergone two outbursts: in 1886 and in 1946. In both cases the star declined very fast after maximum and reached in less than two months the premaximum magnitude; shortly afterwards, there was a small increase to a secondary very flat maximum followed by a slower decrease.

The symbiotic spectrum observed at minimum is usually interpreted as due to a binary star involved in a small nebula; the excitation of the nebular spectrum is provided by the hot fainter star, which undergoes the nova outbursts, while the brighter star (a normal M giant) is constant in brightness. Sanford (1) has found the radial velocity of the M star to be variable with a period of 774 days and a semi-amplitude of 21 km./sec.

The spectrum of the 1946 outburst was at first not unlike that of common novae with broad emission and highly displaced absorption lines (by -4500 or -5000 km./sec.), but very soon the absorption lines faded away and an emission spectrum appeared with rather sharp lines, practically identical with the nebular spectrum before the outburst into which it gradually changed as the star light decreased. At minimum the M spectrum was again visible so that the star showed more or less the usual symbiotic spectrum; but at the secondary maximum, the spectrum had again changed: both the M and the nebular spectrum had disappeared and there was only left a strong continuous spectrum with some weak emission lines and a strong shell spectrum, whose lines were often multiple.

Gratton and Krüger (2) showed that the spectrum during the first maximum could be explained by the usual binary star model, if we admit that the sharp emission spectrum is due to the nebula, the enormous increase of the intensity being due to the increase of the intensity of the exciting source (due to the outburst) whose light reached the nebula a couple of days after the maximum. They called the attention to the fact that, in this case, the main envelope ejected during the outburst reached the nebula about the time of the secondary maximum, which might therefore be caused by the encounter of the envelope with the nebula.

The purpose of this report is to give a more detailed discussion of this encounter (3).

2. We may fix the date of the ejection of the envelope at about 9 February 1946, the day of the maximum light; the beginning of the secondary maximum is about 17 May, which gives, with a velocity of 4500 km./sec. a distance of $3{\cdot}8\times10^{15}$ cm. $=250$ A.U. This is of the order required by the appearance of the nebular spectrum. If Δv is the dispersion of the velocity among the ejected particles, the thickness of the envelope is

$$l=\frac{3{\cdot}8\times10^{15}}{4{\cdot}5\times10^{8}}\,\Delta v.$$

If Δv were due to thermal motion, it should be of the order of ± 16 km./sec.; this is of course a lower limit on account of turbulence. The corresponding volume of the envelope (supposed spherical for simplicity) would be $2{\cdot}4\times10^{45}$ cm.^{3}. The mass ejected in a nova outburst is of the order of 10^{28} or 10^{29} gr. (4); taking the lower value to compensate in part the lower limit of Δv, we get for the density ρ_e of the envelope

$$\rho_e=4{\cdot}2\times10^{-18}\ \text{gr./cm.}^3.$$

Here and in many other cases the two figures are of course not significant and are given only for the sake of computation.

The density ρ_n of the nebula was determined by Hachenberg and Wellman (5), who estimated it at about 10^6 atom/cm.^{3}; this seems to be rather high as compared with planetary nebulae, whose densities are from 10^3 to 10^4 atoms/cm.^{3}. We assume

$$\rho_n=10^{-19}\ \text{gr./cm.}^3.$$

As a result of the encounter between the expanding envelope and the nebula, two shock fronts will form, whose initial velocities may be easily determined by means of the usual formulae of shock waves (6). Let us call V_1 the initial velocity of the shock front in the nebula, V_2 the initial velocity of the shock front in the envelope, u the initial velocity of the surface of contact between envelope and nebula, all referred to the (undisturbed) envelope; then it is easy to show that

$$V_1 = \frac{1 - \frac{1}{3}\sqrt{(\rho_e/\rho_n)}}{1 + \sqrt{(\rho_e/\rho_n)}} V, \qquad u = -\frac{1}{1 + \sqrt{(\rho_e/\rho_n)}} V \qquad (1)$$
$$V_2 = \frac{4/3}{1 + \sqrt{(\rho_e/\rho_n)}} V$$

where V is the relative velocity of the envelope and the nebula (4500 km./sec.).

With $\rho_e/\rho_n = 42$, we get $V_1 = 8 \times 10^7$ km./sec.; that is, in two days the front shock will cross the whole envelope leaving it with a density about four times the initial value (at the beginning of the encounter). Due to various uncertainties, we shall make the computations that follow for two different densities of the compressed envelope $\rho_e = 2 \times 10^{-17}$ (case A) and $\rho_e = 10^{-16}$ (case B).

3. The problem of the encounter of a gaseous mass with a more rarefied gas was studied by Burgers(7). We can apply his results to our case and in order to avoid new numerical computations we will consider the moment at which the function ϕ of the first Burgers paper has the value 1·50; being in our case $\bar{\rho}$ (mass of the colliding cloud for unit area) equal to $5{\cdot}5 \times 10^{-5}$ gr./cm.^{2}, V_0 ($= -u$, initial velocity at the surface of contact) equal to $3{\cdot}9 \times 10^8$ cm./sec., $a = \frac{36}{7}\frac{\rho_n}{\bar{\rho}} = 0{\cdot}9 \times 10^{-14}$ cm.$^{-1}$, the corresponding value of $\tau = 0{\cdot}50$ is equivalent to about one week.

This means that after one week from the beginning of the encounter the density of the nebula at the surface of contact is

$$\rho = 4\frac{\rho_n}{\phi} = 2{\cdot}7 \times 10^{-19} \text{ gr./cm.}^3,$$

and the temperature is given by

$$RT = \frac{1}{3}\frac{V_0^2}{\phi^{\frac{2}{3}}} = 3{\cdot}9 \times 10^{16} \text{ cm.}^2/\text{sec.}^2,$$

which gives for ionized hydrogen $T = 2{\cdot}4 \times 10^8$ °K.

At the surface of contact the pressure inside the nebula and the envelope are the same; thus we obtain for the temperature of the envelope

$$T_e = 3{\cdot}0 \times 10^6 \text{ (case } A) \text{ or } T_e = 0{\cdot}6 \times 10^6 \text{ (case } B).$$

4. At temperatures as high as these the emission of light must be due mainly to bound-free and free-free transitions of H atoms. The corresponding probabilities have been computed by Cillié; the resulting emissive power per cm.^{3} per unit solid angle and per unit frequency are

$$J_n(\nu) = 2{\cdot}17 \times 10^{-32} \frac{n_1 n_e}{n^3 T^{\frac{3}{2}}} e^{-(h\nu - \chi_n)/kT} \quad \text{(c.g.s.)}$$

for bound-free transitions and

$$J_f(\nu) = 6{\cdot}88 \times 10^{-38} \frac{n_1 n_e}{T_e^{\frac{1}{2}}} e^{-h\nu/kT}, \quad \text{(c.g.s.)}$$

for free-free transitions, where n_1 and n_2 are the numbers of protons and electrons per cm.^{3}, χ_n is the ionization energy for the n level and ν the frequency(8). At $T = 10^6$ the exponentials are practically equal to unity and we get $J_n/J_f = 0{\cdot}01$ for $n = 3$ and $J_n/J_f = 0{\cdot}04$ for $n = 2$; thus in all cases the bound-free transitions may be neglected.

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We obtain, on the hypothesis that the envelope consists mainly of hydrogen,

$$J = 1{\cdot}4 \times 10^{-27} \text{ (case } A) \text{ or } 8{\cdot}0 \times 10^{-26} \text{ (case } B),$$

for all frequencies.

To obtain the total emissive power Σ_λ in wave-length units, we must still multiply by the factor $4\pi c/\lambda^2$; for $\lambda = 5{\cdot}3 \times 10^{-5}$ (5300 A.), we get thus

$$\Sigma_\lambda = 1{\cdot}9 \times 10^{-7} \text{ (case } A) \text{ or } 1{\cdot}1 \times 10^{-5} \text{ (case } B).$$

Finally, the visual intensity (for unit of wave-length) of the envelope is obtained by multiplying Σ_λ by the volume, that is, by the mass divided by the density. The result is approximately

$$I_{\text{env.}} = 10^{38} \text{ (case } A) \text{ or } I_{\text{env.}} = 10^{39} \text{ (case } B)$$

erg./sec. × unit of wave-length.

These values may be compared with the visual intensity of the Sun; taking a mean value of 2×10^{14} erg./sec. cm.^{2} for unit of wave-length as the emissive power of the Sun in the visual region, the average visual intensity of the Sun is of about 10^{37} erg./sec. × unit of wave-length. In other words the visual absolute magnitude of the envelope shortly after the beginning of the encounter is

$$M_v = +2{\cdot}2 \text{ (case } A) \text{ or } M_v = -0{\cdot}3 \text{ (case } B).$$

If the absolute magnitude of the system at minimum is that of the M component, or $+0{\cdot}5$ (the average absolute magnitude of an M giant), the corresponding increase in magnitude is

$$\Delta m = 0{\cdot}2^{\text{m}} \text{ (case } A) \text{ or } \Delta m = 1{\cdot}2^{\text{m}} \text{ (case } B).$$

We conclude that the encounter of expanding envelope with the nebula may fully account for the secondary increase of magnitude of T Coronae Borealis.

5. A problem which presents itself in this case is the following: where is the shell absorption spectrum observed during the secondary maximum formed? The gaseous layers between the radiating envelope and the observer are, in fact, those of the nebula (where the shock wave has not arrived as yet) and these at first might seem incapable of forming absorption lines on account of their small optical thickness. If, however, one considers that the emissive power (for unit surface) of the radiating envelope is also extremely small, it is seen that the formation of absorption lines is not impossible. Indeed, the situation is the same as if a normal atmosphere with a sharp division between photosphere and reversing layer is allowed to expand enormously without changing the total energy output of the photosphere. The optical thickness of the expanding reversing layer is of course exceedingly small, but if we divide the photosphere into elements which radiate the same energy in the continuum as an element of unit area did before the expansion, then the total number of atoms above one of them is just the same as before the expansion and we may expect that, other conditions being unchanged, the intensity of the spectral lines will be the same. In fact, the only change is that of the unit of length.

It seems therefore that the usual model of a binary involved in a nebula might explain all the essential spectral features observed during the outburst of T Coronae Borealis.

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