66. DENSITY WAVES IN GALAXIES OF FINITE THICKNESS

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Abstract. The effect of the finite thickness of a galaxy on the propagation of density waves of the type described by Lin and his collaborators has been calculated. The calculated effect does not differ appreciably from what has been estimated previously on the basis of heuristic arguments.

In investigations of the equilibria and stability of galaxies in the context of the problem of spiral structure, it has been customary to neglect the finite thicknesses of galaxies. However, in the application of these theoretical developments to the interpretation of observed spiral structure, as in the work of Lin *et al.* (1969), corrections for finite thickness are significant. Attempts to estimate these corrections by Shu (1968) and others are usually based on heuristic arguments formulated in order to avoid a detailed consideration of the dynamics of galaxies perpendicular to their principal planes.

The work reported here is part of a program to investigate the equilibria and departures from equilibrium of rapidly rotating galaxies of small but finite thickness. These investigations are based on simultaneous solutions of Liouville's equation (the encounterless Boltzmann equation) and Poisson's equation. Although the solution of these equations involves several features which do not appear when the effects of finite thickness are neglected, only the essential feature of the present treatment of the dynamics of a galaxy in the direction perpendicular to its principal plane will be described here.

It can be shown that in a galaxy which is flattened sufficiently, the frequency of the perpendicular oscillation of a star is large compared with the other frequencies of its motion. Under this condition, the perpendicular oscillation is characterized by the approximate constancy of an adiabatic invariant. In a system of cylindrical polar coordinates (ϖ, θ, z) oriented so that the plane of the galaxy is the (ϖ, θ)-plane, the energy of the perpendicular oscillation is

$$E_z = \tfrac{1}{2}Z^2 + \mathfrak{B}(\varpi, \theta, z, t) - \mathfrak{B}(\varpi, \theta, 0, t), \tag{1}$$

where Z is the component of the velocity in the perpendicular direction, $\mathfrak{B}$ is the gravitational potential, and t is the time. The adiabatic invariant is the action integral

$$J = \sqrt{2} \oint \left[E_z - \mathfrak{B}(\varpi, \theta, z, t) + \mathfrak{B}(\varpi, \theta, 0, t)\right]^{1/2} \mathrm{d}z, \tag{2}$$

where the integration extends over one period of the perpendicular oscillation. The methods of solving Liouville's equation in the present work are based on the approximate constancy of J.

Part of this work is an investigation of the effects of the finite thickness of a galaxy on the propagation of density waves of the type considered by Lin *et al.* (1969; see

Becker and Contopoulos (eds.), The Spiral Structure of Our Galaxy, 341–342. *All Rights Reserved.*

this paper for references to the other publications of these authors). In considering this problem, Shu (1968) writes the dispersion relation in a form equivalent to

$$\kappa^2 - m^2(\Omega_p - \Omega)^2 = 2\pi G\sigma\,|k|\,\mathfrak{F}\mathfrak{T}, \tag{3}$$

where κ is the epicyclic frequency, the integer m is the multiplicity of arms in the spiral pattern of the wave, Ω_p and Ω are the angular velocities of the pattern and the galaxy, respectively, G is the constant of gravitation, σ is the surface density of the galaxy, k is the radial wave number of the wave, $\mathfrak{F}$ is a reduction factor representing the effect of the peculiar motions of stars in directions parallel to the plane of galaxy, and $\mathfrak{T}$ is a reduction factor intended to represent the effects of the finite thickness. The reduction factor $\mathfrak{T}$ depends on the model adopted for the unperturbed structure of the galaxy. Shu has made estimates of the values of $\mathfrak{T}$ for a self-consistent model which leads, in the lowest order of approximation, to a density distribution

$$\varrho(\varpi, z) = \varrho_0(\varpi)\,\mathrm{sech}^2(z/\Delta), \tag{4}$$

where $\varrho_0(\varpi)$ and $\Delta(\varpi)$ are arbitrary functions. The present calculations of $\mathfrak{T}$ are based on substantially the same model. For typical conditions in a galaxy, for example those in the solar neighborhood, the values of $\mathfrak{T}$ calculated in the present work are 5–10% larger than the values estimated by Shu. It does not appear that these differences are large enough to alter significantly the discussion by Lin *et al.* (1969) of their interpretation of the spiral structure of the Galaxy.

A detailed account of this work will soon be submitted to the *Astrophysical Journal.*

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References

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