BULL. AUSTRAL. MATH. SOC. MOS 16A26
VOL. 3 (1970), 73-74.

# The identity of certain representation algebra decompositions

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Let $G$ be a finite group and $F$ a complete local noetherian commutative ring with residue field $\bar{F}$ of characteristic $p \neq 0$. Let $A(G)$ denote the representation algebra of $G$ with respect to $F$. This is a linear algebra over the complex field whose basis elements are the isomorphism-classes of indecomposable finitely generated $FG$-representation modules, with addition and multiplication induced by direct sum and tensor product respectively. The two authors have separately found decompositions of $A(G)$ as direct sums of subalgebras. In this note we show that the decompositions in one case have a common refinement given in the other's paper.

We adopt the notation of [2]; suppose then that $H$ is a normal $p$-subgroup of $G$ and $H \leq R \leq G$. We shall show that the decompositions (25) and (26) of [2] have a common refinement given in Proposition 5.5 of [1]. The problem will be resolved by showing that the idempotents $E_S$ introduced in [2, p. 400] are the sums of idempotents $u_S$ giving the decomposition 5.5 of [1]. As the $E_S$ are linear combinations of the $I_R$ of [2, p. 398] it will be sufficient to show the same for the $I_R$. The idempotents $F_K$ mentioned in (27) of [2, p. 401] have been discussed in [1] and have been shown there to be sums of the $u_S$'s already. The intersection of the decompositions given by the $E_S$ and the $F_K$ in (27)

Received 3 April 1970.

of [2] is similarly refined by 5.5 of [1].

We recall that $P(G)$ is the projective ideal of $A(G)$ and $P^G(R) = \left(P(R)\right)^G$ , that is, the image of $P(R)$ in $P(G)$ under the induction map $\mathrm{ind}^{\cdot}_{R\to G}$ . $P^G(R)$ is an ideal of $P(G)$ . All the idempotents $I_R$ can be considered to lie in $P(G/H)$ and so we may without loss of generality assume that $H$ is the trivial subgroup of $G$ . $I_R$ is then the idempotent generator of $P^G(R)$ .

From Proposition 5.11 of [1], we have that $I_G \in \Sigma_R P^G(R)$ , where the sum runs over a complete set $Y$ of non-conjugate $p'$-cyclic subgroups $R$ of $G$ . For $R \in Y$ , write $P'^G(R) = \Sigma_{R'} P^G(R')$ , the sum being over those $R' \in Y$ which, to within conjugacy in $G$ , are *properly* contained in $R$ . Using the Mackay formula for the tensor product of induced representations, we see that the element $u_R = \left(1/|N(R) : R|\right) \left\{\left(1_R\right)^G\right\}$ of $A(G)$ is the identity of $P^G(R)$ modulo $P'^G(R)$ . Using induction on $|R|$ , $R \in Y$ , we see that each $I_R$ is a combination of the $u_S$'s , as required.

## References

[1] S.B. Conlon, "Decompositions induced from the Burnside algebra", *J. Algebra* 10 (1968), 102-122.

[2] W.D. Wallis, "Decomposition of representation algebras", *J. Austral. Math. Soc.* 10 (1969), 395-402.

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