This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background material in case the problem is unsolved. Send all communications concerning this department to I. G. Connell, Department of Mathematics, McGill University, Montreal, P. Q.

ON A LEMMA OF M. ABRAMSON

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Kaplansky's Lemma [3] states: the number of k-combinations of 1, 2, ..., n with no two consecutive integers in any selection is $\binom{n-k+1}{k}$. Using this, Abramson [1; lemma 3] solves the problem: find the number of k-combinations so that no two integers i and i+2 appear in any selection. (This is generalized by Abramson in [2].) An interesting solution, also using Kaplansky's lemma, is obtained as follows.

If n = 2m, we choose s from among the m even integers, no two consecutive, and k-s from among the m odd integers, no two consecutive. Summation on s gives the solution

$$\sum_{s=0}^{k} \binom{m-s+1}{s} \binom{m-(k-s)+1}{k-s}.$$

Similarly, if n = 2m+1, we get

$$\sum_{s=0}^{k} \binom{m-s+1}{s} \binom{m-(k-s)+2}{k-s}.$$

Summation on k gives F^2_{m+2} and $F_{m+2}F_{m+3}$, respectively, where the F_m are the Fibonacci numbers defined by $F_0 = 0$, $F_1 = 1$, and $F_m = F_{m-1} + F_{m-2}$ for $m \geq 2$.

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