COMMENT ON 'DETERMINATION OF THE RADIOCARBON AGE OF PARCHMENT OF THE VINLAND MAP'

K R Ludwig

Berkeley Geochronology Center, 2455 Ridge Road, Berkeley, California, 94709, USA. Email: kludwig@bgc.org.

I wish to point out some statistical errors in the uncertainty calculations for the radiocarbon age of the Vinland Map (Donahue et al. 2002). First, the authors state that the 14c measurements of samples with chemical treatments B, C, D, and E "... result in consistent radiocarbon ages", and that "... two (of these analyses) are 1.5 sigma, and the others are one sigma or less, from the weighted average." In fact, the overall scatter of the data show that it is improbable that these data are statistically coherent, largely because analysis J21C lies 2.1 sigma from the weighted mean.

The usual parameter for examining the coherence of data with known, Gaussian errors is the "reduced χ^2 ", or in the usual geochronological terminology, MSWD (= Mean Square of Weighted Deviates; McIntyre et al. 1966; Wendt and Carl 1991), and defined as

$$\frac{\sum (x_i - \bar{x})/\sigma^2_{x_i}}{\sum (1/\sigma^2_{x_i})}$$

where \bar{x} is the weighted average of the individual data points, x_i .

The MSWD for the five Vinland Map samples considered coherent by Donahue et al. (2002) is 2.58, indicating that the observed amount of scatter from the weighted mean is a factor of $\sqrt{2.58}$ greater than expected from the assigned errors. The product of the MSWD and the 4 degrees of freedom has a χ^2 distribution about the degrees of freedom, from which the probability of the assigned errors yielding as much as the observed amount of scatter can be calculated. This probability is only 0.035—significantly less that the usual cutoff of 0.05. Thus, according to usual statistical practice, the five analyses selected by Donahue et al. (2002) do not agree within their assigned errors.

Second, though the standard deviation of the weighted average is indeed ± 0.0033 as given in the paper, the value should not be taken as equivalent to a Gaussian, 68.2% confidence error, since it was calculated from only 4 degrees of freedom. Expansion of this value by the appropriate Student's-t factor is required to obtain the correct error at a given confidence limit, which would result in a much larger 95%-confidence age-error than stated by Donahue et al. (2002). The essential point, however, is that because the 5 analyses are not in statistical agreement, the relevance of their average value (weighted or otherwise) to the true age of the Vinland Map is unclear, and calculation of confidence limits of the resulting age from any of the standard algorithms is not straightforward.

The three samples that were treated with chemical processes C, D, and E, however, *are* statistically indistinguishable (MSWD = 0.96, probability = 0.38), with a weightedaverage 14 C modem-carbon fraction of 0.9391 \pm 0.0025 (68.2% conf.), where the error is calculated in the statistically appropriate way for a weighted mean of coherent measurements (Bevington 1969),

$$\sigma_{\bar{x}}^2 = \frac{1}{\sum (1/\sigma_{x_i}^2)}$$

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The resulting age range (from OxCal 3.5; Bronk Ramsey 1995) is 1404–1440 AD (95.4% conf.)—not much different than the 1411–1468 AD calculated by Donahue et al. (2002), but in this case statistically justified.

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