IV
Hot hadronic matter

10 Relativistic gas

10.1 Relation of statistical and thermodynamic quantities

The first law of thermodynamics describes the change in energy dE of a system in terms of a change in volume dV and entropy dS:

$$dE(V,S) = -P\,dV + T\,dS, \tag{10.1}$$

$$T = \left(\frac{\partial E}{\partial S}\right)_V, \qquad P = -\left(\frac{\partial E}{\partial V}\right)_S. \tag{10.2}$$

The coefficients of the first law are the temperature T and the pressure P. Both can be introduced as the partial derivatives of the energy $E(V,S)$. E is a function of the extensive variables V and S, i.e., variables that increase with the size of the system. Below, we include into this consideration the baryon number, see Eq. (10.12), which is also an extensive variable.

The free energy,

$$F(V,T) \equiv E - TS, \tag{10.3}$$

is the quantity in which, as indicated, the dependence on the entropy is replaced by the dependence on temperature, an intensive variable that does not change with the size of the system. Namely,

$$dF(V,T) = dE - T\,dS - S\,dT = -P\,dV - S\,dT, \tag{10.4}$$

and, as a consequence of the transformation Eq. (10.3), we obtain in analogy to Eq. (10.2)

$$S = -\left(\frac{\partial F}{\partial T}\right)_V, \qquad P = -\left(\frac{\partial F}{\partial V}\right)_T. \tag{10.5}$$

For an extensive system with $F \propto V$, a very useful relation for the entropy density σ follows from Eq. (10.5):

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