PART I

SURVEY OF THE PROBLEMS IN RADIAL STELLAR INSTABILITY AND ITS RELATION TO STELLAR EVOLUTION

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THE THEORETICAL SITUATION

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1. Preliminary Remarks

The domain of this symposium is so wide – essentially anything at all having to do with stars – that it is impossible to do more than cursory justice to even a small portion of the matters to be discussed. My contribution will therefore be limited primarily to a discussion of the status of theoretical work bearing on the behavior of stars that evolve through the classical instability strip that extends from the region of Cepheids to the domain of RR Lyrae stars. Discussion of other extremely important variable stars such as cataclysmic variables, Mira and irregular variables, flare stars, β-Canis Majoris stars, and δ-Scuti and small amplitude variables will here be mentioned only in passing; presumably, most of these stars will be discussed at length by other speakers at this symposium. Further little attention will be paid to the thermal instability that is initiated in the helium-burning region of a double-shell-source star and to current thinking about the progenitors of type I and type II supernovae; presumably, these topics will be discussed in Warsaw in the symposium on advanced stages of evolution.

2. An Overview

I will first attempt to summarize: (1) what has been known for a long time, certainly by the time of the last IAU Symposium on Stellar Evolution and Pulsation, (2) what has happened since this last symposium, and (3) what some of the striking deficiencies in the theory are, that can and ought to be remedied.

2.1. What's old?

(1) Most of the regular variables of large amplitude are in the core helium-burning stage. Classical Cepheids are stars of intermediate mass (say 3–16 $M_\odot$); RR Lyrae stars are of low mass (say $\sim$0.5–0.7 $M_\odot$). Both pulsation theory and evolution theory, when compared with the appropriate observations, give these results. The beauty of the agreement is that the comparisons are, to a large extent, independent of one another. Pulsation theory deals primarily with the outer envelope of the star (temperatures below 10^6 K) whereas evolution theory deals in large part with the stellar core where the essential input physics is quite different from that in the envelope.

(2) The principal driving region for most regular variables is the He II ionization zone, although the hydrogen ionization zone can play a critical role, particularly in determining the mode in which a star will pulsate.

(3) The observational instability strip extends to luminosities considerably below

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those of RR Lyrae variables, thus extracting from an embarrassing situation theoreticians who are not able to easily stabilize such stars. Among the stars near the main sequence are the δ-Scuti variables, many of which may be burning hydrogen either at the center or at the edge of an inert helium core.

(4) The relationship between P, T_e, L, and M that is obtained by either linear or non-linear analysis is perhaps one of the most secure relationships in astrophysics.

(5) Blue edges for pulsation in all modes are sensitive to the envelope helium abundance as well as to mass. Lifetimes in different evolutionary phases and locations in the HR diagram are both sensitive to the envelope helium abundance. On comparison with the observations, these sensitivities both suggest a high helium abundance for population II variables.

(6) The fact that, at low T_e, pulsation in the fundamental mode seems to be favored over pulsation in the first harmonic mode and that, at high T_e, the reverse is true, leads to the suspicion that there may be a roughly composition-independent, mass-independent relationship between a 'transition' period and luminosity. A guess at this relationship permits one to estimate distances to RR Lyrae stars near the galactic center and in globular clusters. Stellar evolution calculations lead to an independent relationship between luminosity and composition that may also be used to estimate distances to RR Lyrae stars. Both methods (evolution and pulsation) agree on the sense and magnitude of the difference in luminosity between variables in clusters of intermediate metal content and variables in clusters of very low metal content.

(7) The red edge of the instability strip found in nature is probably connected with convection, which quenches the driving mechanism. Theory, however, cannot yet predict the location of a red edge.

(8) β-Cephei stars are probably near the overall contraction phase following the exhaustion of hydrogen in the core. There is the possibility that non-radial pulsation modes are involved and that a resonance coupling between such modes and rotation and/or with radial modes may occur. However, an excitation mechanism has not yet been demonstrated.

(9) Novae are probably binary systems, one member of which is a white dwarf and the other member of which is a red giant that transfers mass to its companion. When the temperature at the base of the newly deposited hydrogen layer exceeds a critical value, an explosion takes place which lifts off the new layer.

(10) Supernovae models of a non-exotic, non *ad-hoc* nature are almost nonexistent. Only one model has respectable antecedents – the carbon detonation model. Unfortunately, this model suffers from three serious flaws: no remnant; too much iron produced; possibly not enough energy liberated.

(11) Pulsars have been identified and, for the Crab, the only viable theoretical explanation is that of a rotating neutron star.

(12) Long period irregular variables are ubiquitous in the redder regions of the HR diagram. Scandalous to relate, neither the excitation mechanism nor the evolutionary status of the interior is understood.

(13) In young clusters, flare stars are common and unpredictable. Flares are

thought to be a surface phenomenon associated perhaps with the annihilation of magnetic field.

(14) A thermal instability occurs in the helium shell of stars that have exhausted helium at their centers. The amplitude of this instability increases with time. A convective shell that appears within the helium-burning shell and extends outward toward the hydrogen-helium discontinuity grows to a maximum during the peak of a thermal swing and may lead to mixing of hydrogen into the helium-burning region and/or mixing of helium-burning products into the hydrogen-rich region. In either case, *s*-process elements may be formed and eventually brought to the surface. The first property of the thermal instability – the increase in amplitude with each pulse – may be a clue to the formation of some planetary nebulae. The second property – the formation of an 'intershell' convective region – may be responsible for the peculiar abundances in carbon stars and the like.

(15) For masses in the neighborhood of 60 $M_{\odot}$, main sequence stars become unstable to radial pulsation driven by nuclear reactions in the core. Attempts at following the non-linear behavior of such objects suggest that, within the main sequence lifetime, pulsation amplitude does not grow without limit and that, therefore, pulsational instability is not the reason for the paucity of high mass stars. The reason for this paucity must reside in the conditions prevailing during star formation.

(16) U-Gem, W-Ursa Majoris stars, dwarf novae, and classical novae all fall into a regular scheme. In this scheme the separation between members of a binary pair figures as a crucial parameter.

2.2. What's new?

This, of course, is the subject of our conference, and we eagerly await tidings of new advances. Some advances are already in the literature and may therefore be reviewed.

(1) Extensive use has been made of P, L, T_e, M relationships and blue edge relationships to estimate the bulk properties of Cepheids and RR Lyrae stars. The uncertainties in these relationships have been extensively examined and appreciated.

(2) Evolutionary studies have clarified that many population II variables with period greater than a day are stars with a carbon-oxygen core that are burning nuclear fuel in two shells. BL-Herc stars are evolving through the instability strip on a nuclear burning time scale (in the suprahorizontal branch or asymptotic giant branch stage); W-Virginis stars are evolving through the instability strip on a thermal time scale (looping back from the asymptotic giant branch).

(3) The tentative relationship between a transition period and luminosity (discussed in 2.1.6) has fallen under a cloud. It appears that the relationship is really only a rough estimate of the location of a transition region where pulsation begun in either of two modes – fundamental or first harmonic – will persist in the initial mode for many periods. It possibly cannot be trusted to give luminosity differences as small as those which occur between RR-Lyrae stars in clusters of different composition.

(4) A powerful new technique for calculating full amplitude motion directly has been invented. Instead of integrating forward in time by brute force, a periodic solution

is sought by relaxing in time as well as in space. The final full amplitude motion can then be tested for stability. In principle one can thereby determine which of two unstable modes may survive over a long period of time. The new technique may remove the cloud now hanging over the relationship between a transition period and luminosity or place this relationship more permanently in limbo. Preliminary results suggest that the cloud will be reduced but not completely eliminated.

(5) The location of β-Cephei stars has been more carefully delineated. This location appears to be a band parallel to the main sequence and along the red edge of the main sequence band. The evidence is therefore rather compelling that β-Cephei stars are either in or near the overall contraction phase following the exhaustion of hydrogen at the center. Still, a completely acceptable excitation mechanism has not yet been identified.

(6) Dynamic studies of nova-like outbursts have been conducted on the assumption that the appropriate initial model consists of a white dwarf accreting hydrogen-rich matter from a red-giant companion. It develops that much of the power for the late stages of the outburst comes from the β-decay of ^{13}N and ^{15}O and that theoretical bolometric light curves are similar to those observed for nova outbursts in the visual. However, most of the energy associated with nova outbursts may not be in the visual.

(7) The carbon-detonation model of supernovae has been transformed into the carbon-fizzle model. Instead of exploding, the core now is thought to lose energy via Urca-process neutrinos at the edge of a convective core whose extent is determined by the threshold energy for β^- captures on ^{23}Na ($\rightarrow$ ^{23}Ne). Conveniently, ^{23}Na is one of the products of reactions that occur during carbon burning. As a consequence of the Urca energy drain, the incipient detonation may be quenched. As carbon burning proceeds quietly, neutron-rich elements become ever-more abundant (the electron molecular weight μ_e increases) until the Chandrasekhar limit ($\propto M_\odot/\mu_e^2$) exceeds the mass in the electron-degenerate carbon-oxygen core. The core may then collapse into a pulsar. It is possible that the beam from the pulsar may fill up the cavity between pulsar and envelope with electromagnetic energy which then expels the envelope. Thus, we are left with a condensed remnant that can act as a reservoir for large quantities of energy and with an expelled envelope that contains *no* new elements to speak of. For the heavy elements, it appears that we may be forced to rely on massive stars that don't develop instabilities until an iron or nickel core is formed.

(8) It is becoming increasingly probable that type I supernovae may initially be binary systems with rather long periods $\sim$1–10 yr. It is conjectured that one component has an initial main sequence mass on the order of 2–$3\frac{1}{2}$ $M_\odot$, the other an initial main sequence mass less than or equal to about 0.8 $M_\odot$. The time scale for the system is determined by the lighter star. The more massive star sheds mass due to radiation-pressure induced mass flow and becomes a carbon-oxygen white dwarf. Most of the mass is lost from the system. After $\sim 10^{10}$ yr, the secondary transfers mass to the white dwarf which heats up, reaches the Chandrasekhar limit and (maybe) explodes. Nice features of the conjectured model are a high degree of symmetry, the near uniqueness

of the immediate supernova progenitor, and an explosion energy that is not masked by a massive envelope (as may be the case for type II supernovae).

(9) Pulsars in binary systems of very short period provide a new puzzle. If the X-ray pulsars are indeed neutron stars, how could the companion possibly survive the explosion that presumably precedes the formation of a neutron star remnant? Maybe explosions don't always accompany the formation of neutron stars.

(10) Long period irregular variables may owe their variability to random convective motions that have their origin in the thermally unstable region defined by the helium and hydrogen-burning shells.

(11) Further exploration of *s*-process element formation under conditions anticipated in thermally unstable regions speaks more and more strongly for interplay between the intershell region and the surface. The key factors in achieving the appropriate relative abundances seem to be (a) mixing between hydrogen and products of helium burning and (b) repetition of the mixing process for hundreds of cycles. Further calculation of thermal instabilities has produced possibly consistent, but also possibly contradictory results – low mass stars may not mix; higher mass stars can't seem to avoid it. The observations (FG Sagittae) suggest that mixing may take place in the deep interior of low mass stars, but appearance of the exotic products at the surface may not occur until after the ejection of a planetary nebula.

(12) The rapid oscillations that occur following the nova outburst appear to be explicable only in terms of non-radial modes in a white dwarf.

(13) Some progress in understanding the structure of contact binaries has been made, but estimates of evolutionary behavior by different authors give contradictory results.

2.3. What should be done?

(1) In my mind, one of the most vital elements in the comparison between observation and theory has received insufficient attention – the conversion between observational quantities and theoretical properties. For example, how does the conversion between $B-V$ and $\log T_e$ depend on composition? Until this conversion has been fully esplored, we cannot properly understand the differences between Cepheids in our Galaxy, the SMC, the LMC, and Andromeda. Nor can we fully appreciate the dispersion in properties of Cepheids in our own Galaxy. Again, what are $\langle B-V\rangle$ and $\langle B\rangle - \langle V\rangle$ for a model pulsating star characterized by a specific $\log T_e$ at zero amplitude? Without a thoroughgoing study of these transformations we cannot make rigorous statements about masses. Exploration of these transformations is a task for theoreticians.

(2) We need a more complete study of the composition dependence of evolutionary tracks for stars of intermediate mass during core helium burning. Only then can we understand differences between Cepheids in the SMC, LMC, Andromeda, and our Galaxy.

(3) What is the excitation mechanism for β-Cephei stars? Is it sporadic convective motions that arise in massive stars during the phase of overall contraction when a

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large burst of energy is emitted by the contracting core? It may be significant that this burst of energy does produce a thick convective shell outside of the hydrogen-exhausted core.

(4) We need a convincing demonstration of the existence or non-existence of a transition period. Is there a unique period at any given L and M? If so, does it depend on composition and mass? Is there only a transition region of finite width? That is, is there a hysteresis effect so that a star will continue to pulsate in the mode in which it began as it passes through the transition *region* where both modes are possible?

(5) We need an explanation of the observed drifts in period which occur on time scales short compared to the drifts brought about by evolutionary changes. Are the rapid drifts due to perturbations that couple rotation and pulsation? Do magnetic fields play a role?

(6) Do we really understand how to estimate the effects of convective overshoot and of semi-convection at the outer boundaries of fully convective cores? The fact that the helium abundance in the envelopes of horizontal branch stars that is given by pulsation theory plus observations is significantly larger than that given by evolution theory (with overshoot and semi-convection treated in a particular way) plus observation suggests that something is wrong.

(7) The evolution of contact binaries needs further attention. Currently, theoretical results are contradictory.

(8) The possibility of supernova explosions connected with a carbon-oxygen core of mass near 1.4 $M_{\odot}$, whether inside of a star of intermediate mass or adjacent to a light donor that sends it over the brink, needs further extensive exploration.

(9) What is the cause of the planetary nebula phenomenon? An extremely large amplitude thermal instability initiated in nuclear burning regions? Radiation-driven mass loss when stellar luminosity exceeds a high, critical value? An exceedingly large amplitude envelope pulsation driven by a helium or hydrogen ionization zone? Or do all three mechanisms play a role?

3. Classical Cepheids

3.1. Gross features

Since the firm identification of main sequence stars as objects in the phase of core hydrogen burning, perhaps the most significant advance in our understanding of stars has been the firm identification of classical Cepheids and cluster variables as objects in the core helium-burning phase. This identification has provided a most convincing demonstration of the qualitative correctness of the theoretical framework of stellar structure and pulsation.

In Figure 1, the shaded areas represent where, according to evolutionary calculations (see, e.g., Iben, 1967a, b), stars of population I composition spend most of their time as consumers of nuclear fuel. Over 80% of a theoretical model's total lifetime is spent in the pure hydrogen-burning phase. Another 10 to 20% is spent burning both helium and hydrogen. The location of models in more advanced phases is on the giant

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branch. All details of evolutionary tracks have been suppressed in order to focus on the major qualitative intersection of evolution and pulsation theory: After its residence on the main sequence, a population I star of intermediate mass spends most of its time in a band of finite width that is widely separated from the main sequence band and is roughly orthogonal to a narrow band defined by pulsation theory as a region within which the stellar envelope is unstable to pulsation in the fundamental radial mode.

An immediate consequence of the theoretical results is that Cepheids should be limited to a finite range in luminosity and period. That this is in fact the case is beautifully demonstrated in Figure 2 where the distribution in apparent magnitude is shown for Cepheids in M31 (data from Baade and Swope, 1965).

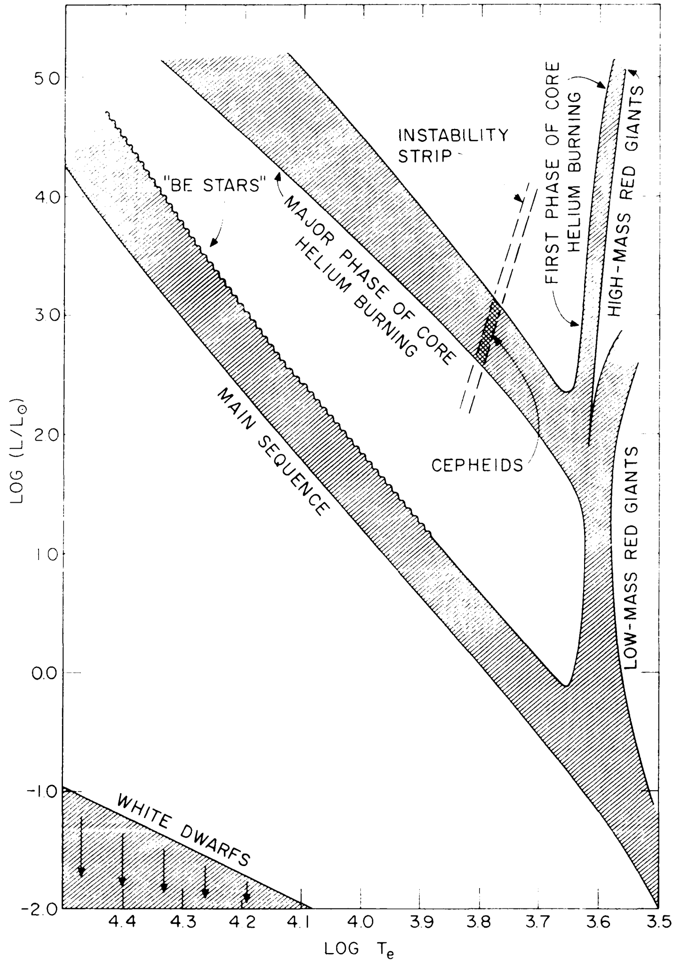

Fig. 1. The distribution of metal-rich stars in the Hertzsprung-Russell diagram expected on the basis of model calculations (from *Science* **155** (1967), 785).

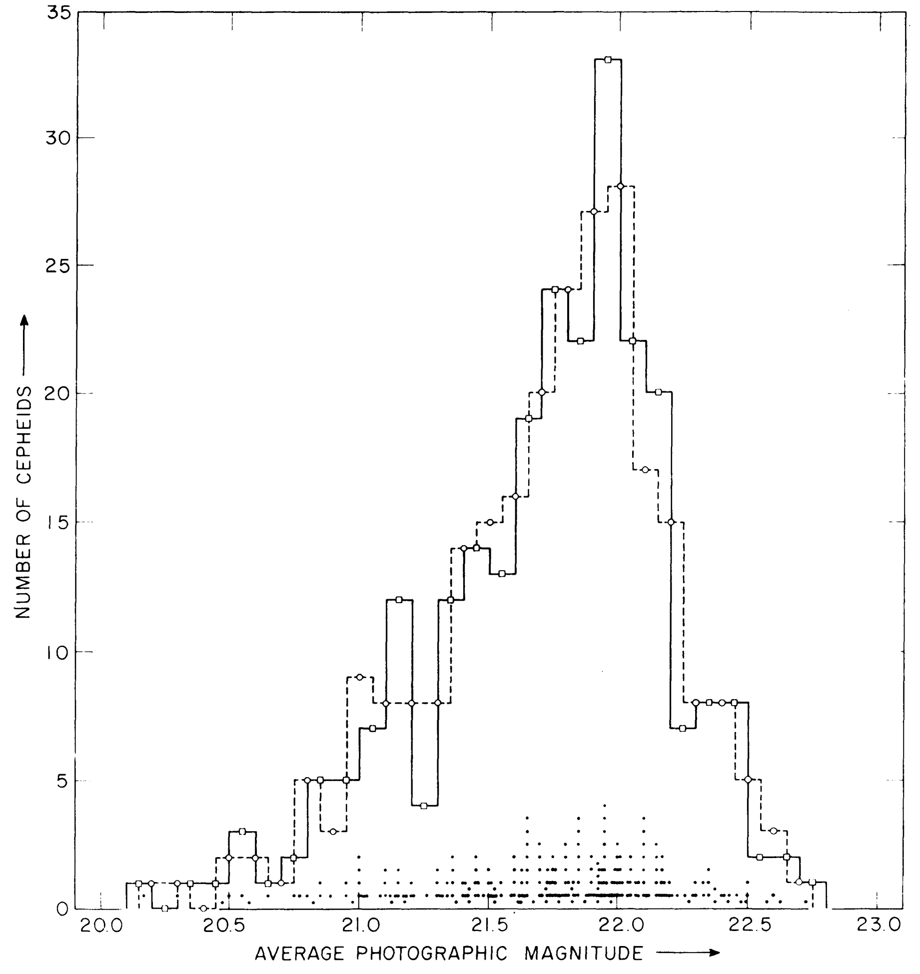

Fig. 2. Number vs magnitude relationship for Cepheids in Field III of M31 (after Baade and Swope, 1965). Unconnected points indicate the occurrence of a Cepheid at the designated magnitude (arithmetic mean of maximum and minimum magnitudes). For them, the vertical scale has no significance. The solid histogram is obtained by counting Cepheids in 0.1-mag intervals centered at 20.05, 20.15, etc. The dashed histogram is obtained by counting Cepheids in 0.1-mag. intervals centered at 20.1, 20.2, etc.

A second confirmation of the basic qualitative features of the theory is provided by the study of young clusters that contain stars massive enough to have reached the core helium-burning band (within the lifetime of the cluster). From the location of the cluster main sequence, one can estimate the absolute luminosity of the helium-burning stars and from the observed color one can estimate surface temperature. Theory suggests that, in younger clusters (that contain more massive stars), helium-burning stars will in general be brighter and bluer than such stars in older clusters. This trend is very nicely corroborated by clusters such as NGC2164 (Hodge and Flower, 1973), NGC 1866 (Arp 1967, Arp and Tackeray, 1967), and NGC1831 (Hodge 1963).

3.2. Uncertainties in the Theory and in the Interpretation of Observations

Although gross features of evolution and pulsation theory are in beautiful conformity with the gross features of the observations, attempts at detailed comparisons are plagued by large uncertainties. One may classify the uncertainties in three groups: (i) uncertainties in stellar structure; (ii) uncertainties in pulsation theory; and (iii) uncertainties attending an interpretation of observed properties.

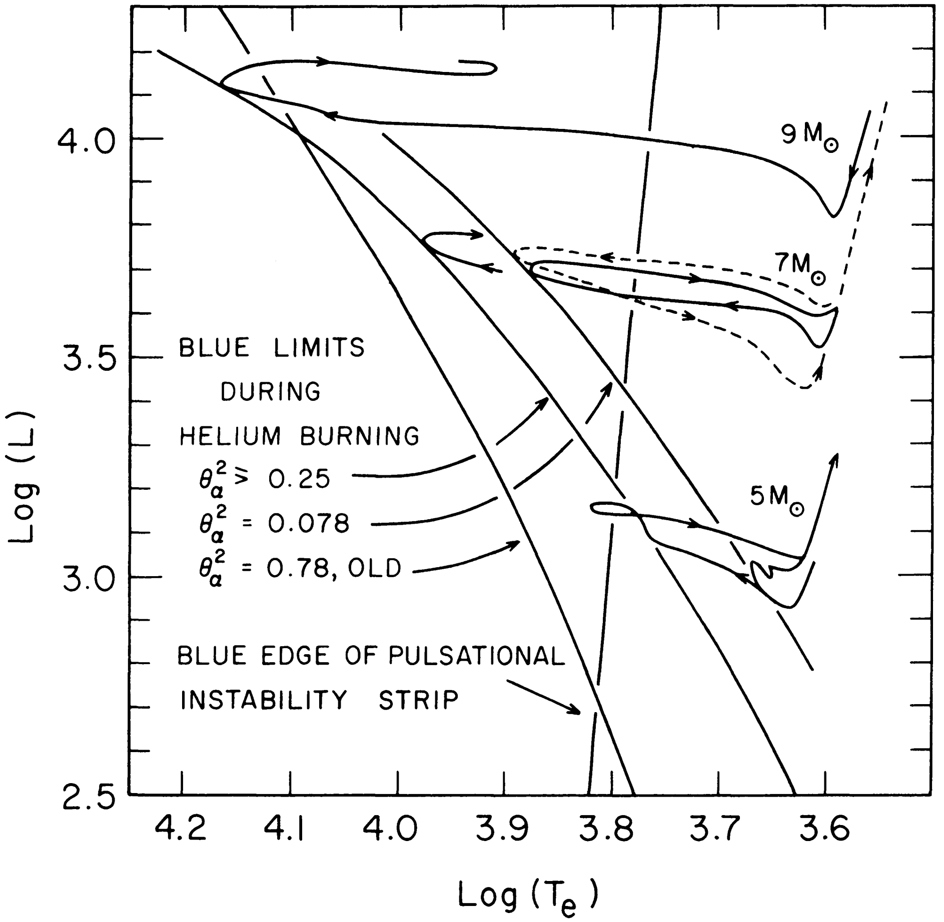

Fig. 3. Evolutionary tracks during core helium burning, blue limits, and the blue edge of a theoretical instability strip for radial pulsation in the fundamental mode. Initial composition parameters are $X=0.7$, $Y=0.28$, $Z=0.02$. Three choices of reduced width θ_α^2 are represented. For the $5M_\odot$ models, $\theta_\alpha^2=0.078$ and 0.25; for the $7M_\odot$ models, $\theta_\alpha^2=0.078$ and 0.78; for the $9M_\odot$ model, $\theta_\alpha^2=0.25$. The blue limits connect, for different masses, the bluest points reached during the major core helium-burning phase. A blue limit defined by an earlier set of models (see Iben, 1967a, b) is also shown.

(i) The uncertainties in stellar structure and evolution may be centered about the information in Figure 3, where tracks during the helium-burning phase for stars of 5, 7, and 9 $M_\odot$ are shown (Iben, 1972). Tracks have been constructed for specific choices of the input physics. An indicator of uncertainties is given by the 'blue limits', each limit being defined by connecting (for models of different mass) the highest surface temperature reached by each model during the core helium-burning phase.

The blue limits labeled '$\theta_\alpha^2 \gtrsim 0.25$' and '$\theta_\alpha^2 = 0.78$, old' are defined by models that differ *only* with regard to opacities. On examining the intersection of these two blue limits with the blue edge of the instability strip (for a similar composition), it is apparent that the two choices for opacity suggest mean Cepheid luminosities that differ by over 1.25 mag.! Thus, the choice of opacity excercises considerable influence over predicted Cepheid properties.

The only difference between models that define blue limits labeled '$\theta_\alpha^2 = 0.078$' and '$\theta_\alpha^2 \gtrsim 0.25$' is in the effective cross section for the reaction ^{12}C (α, γ) ^{16}O that occurs in the convective core of the models. Model stars constructed with a larger cross section evolve for a longer time and to a larger surface temperature than do model stars constructed with a smaller cross section. Laboratory nuclear physics experiments cannot yet exclude either choice of cross section. Thus, uncertainties in the nuclear physics introduce a large uncertainty in the predicted properties of Cepheids.

The treatment of mixing in the convective core of model stars introduces still further uncertainties. As helium turns into carbon and oxygen in this core, the opacity at the outer edge of the core grows larger than the opacity just outside the core (Castellani *et al.*, 1971a, b). The incipient discontinuity in opacity leads to finite convective velocities at the formal core edge and hence to overshooting. The consequence is that the effective region in which matter is mixed is larger than is the case when overshoot is neglected. The concomitant increase in nuclear fuel during the core helium-burning phase means that more time is spent in this phase. Therefore, the maximum surface temperature reached during this phase is also increased. The uncertainty introduced in the specification of a blue limit is as great as that introduced by the uncertainty in nuclear burning rates (Robertson, 1972).

Uncertainties in envelope opacity and in the extent of overshooting at the base of the convective envelope influence the location of the red limit of the helium-burning band (shown in Figure 1 but not in Figure 3). This red limit is related to the appearance of convection in the envelope.

A final uncertainty in theoretical models is the brightness of crossings of the instability strip following the core helium-burning phase. For the input physics used to construct models whose tracks are shown in Figure 3, these additional crossings are at roughly the same brightness as the first two major crossings. This is not true for other models in the literature (e.g. Kippenhahn *et al.*, 1966; Hofmeister, 1967). Uncertainties in opacity, in nuclear cross sections, and in the extent of mixing during core helium-burning will affect the details of the multiple crossings of the instability strip.

(ii) Among the uncertainties that affect pulsation results, I will here mention only those which affect the determination of blue edges for pulsation in the fundamental and first harmonic radial modes. These blue edges are best calculated in the linear, non-adiabatic approximation. Discussion can focus on the information in Figures 4 and 5 (from Iben and Tuggle, 1972a, b).

No satisfactory way of treating convection in a dynamic situation has been devised (not surprising, since neither has a treatment of convection under steady state conditions been devised). However, assuming that at any point the flux of energy in convective motions remains constant during pulsation and of the same magnitude as in the static initial model, some progress can be made in estimating the influence of convection (Baker and Kippenhahn, 1965). Adopting a mixing length treatment with mixing length given by a local pressure scale height, one finds that the surface temperature at a blue edge can increase by as much as $\Delta \log T_e \sim 0.02$–$0.03$ relative to the situation when convection is completely neglected. However, with this treatment of convection, the convective flux in some regions exceeds a theoretical maximum (Christy, 1966). Insisting that convective flux cannot exceed the theoretical maximum reduces the shift to $\Delta \log T_e \sim 0.005$ (Iben and Tuggle, 1972a).

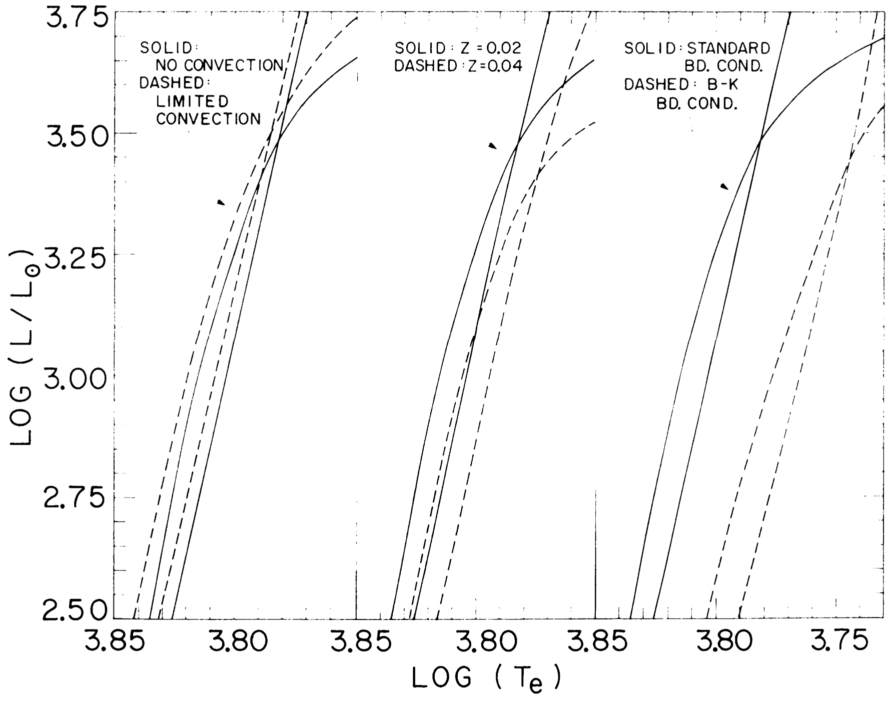

Fig. 4. The dependence of blue-edge location on convection, opacity parameter Z, and surface boundary conditions. In each panel, the solid lines are blue edges for pulsation in the fundamental and first harmonic modes for a $5M_{\odot}$ model of composition $X=0.7$, $Z=0.02$. The dashed lines are blue edges for models that differ from the standard set in one characteristic: (a) convection is included, (b) $Z=0.04$, (c) Baker-Kippenhahn surface boundary conditions are used. Lines with the least curvature are fundamental blue edges (from Iben and Tuggle, 1972a).

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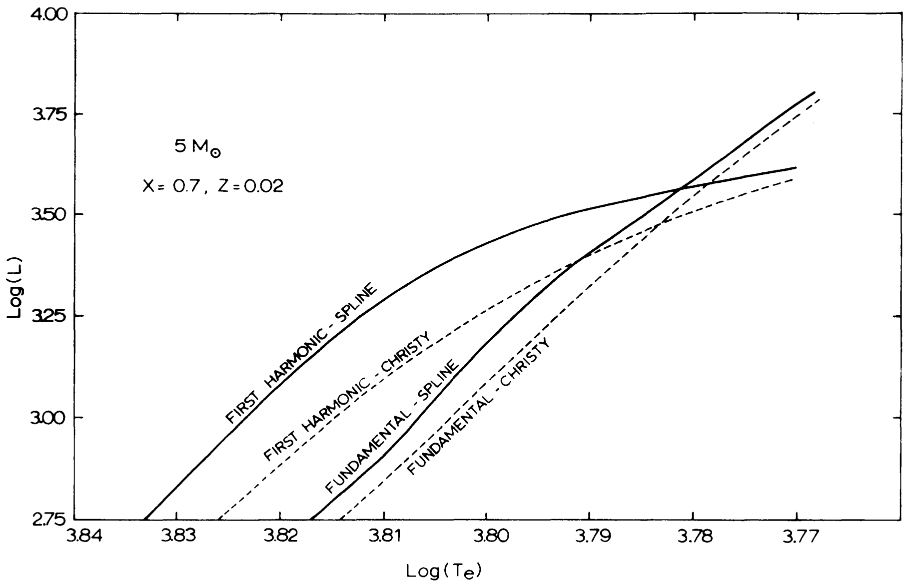

Fig. 5. Blue edges for pulsation in the fundamental and first harmonic modes. Stellar mass is $5M_{\odot}$ and composition parameters are $(X, Z)=(0.7, 0.02)$. Dashed lines are obtained by using Christy's (1966) analytic approximation to Cox and Stewart (1963) opacities. Solid lines are obtained by using cubic spline interpolation in fine-grain Cox and Stewart (1972) opacities (from Iben and Tuggle, 1972b).

The location of a theoretical blue edge is affected by the choice of surface boundary conditions, but the uncertainty thereby introduced is now probably negligible. This was not always true. When boundary conditions are incorrectly applied at the photosphere rather than at very small optical depth, blue edges can shift as much as $\Delta \log T_e \sim$ ~ 0.03–0.04 to the red (see Figure 4 and Iben and Tuggle (1972a)).

The choice of opacity has a major influence on the location of a calculated blue edge. This is not surprising since it is the nature of the opacity in the helium and hydrogen ionization zones that is in large part responsible for driving pulsation. What may be surprising is that even two very good approximations to the same set of opacity tables may give quite different results, as is demonstrated in Figure 5 (from Iben and Tuggle, 1972b).

(iii) The uncertainties attending an interpretation of observational characteristics of a star in terms of characteristics that may be directly compared with theory have already been alluded to in Section 2. One of the gravest uncertainties lies, of course, in the estimation of absolute magnitude. The so-called 'photometric' method works only for a dozen or so Cepheids in the Galaxy that happen to occur in young clusters with appreciable main sequence populations. One assumes that the distance to the relevant cluster can be estimated by fitting this main sequence to a 'standard' main sequence. One basic weakness of the method is the assumption that composition differences may be ignored both in constructing the standard and in fitting to the standard. There is the further difficulty of determining the distance to the Hyades cluster. This latter distance, of course, determines the normalization of the standard. One might expect an *a priori* uncertainty of 0.25–0.5 mag. in the bolometric magnitude of any of the dozen Cepheids estimated by the photometric method.

The current method for estimating magnitudes for Cepheids not in clusters rests very heavily on the slope of a mean relationship between period and color determined for galactic Cepheids and on the slope of a mean relationship between period and relative magnitude for Cepheids in systems external to our Galaxy (e.g., Sandage and Tammann, 1969). This latter relationship is normalized by means of the dozen Cepheids whose magnitudes have been estimated photometrically. Crude pulsation theory and estimates of a mean relationship between color and surface temperature are used to establish the form of a relationship between magnitude, color, and period. The coefficients in this final relationship are obtained by using the period-color and period-magnitude relationships. The most severe weakness of this PLC (period-luminosity-color) method of estimating magnitude is the difficulty in estimating a mean period-color relationship. The scatter of the observed distribution in the period-color plane is so large that only by luck will one choose a slope for the mean relation that is consistent with pulsation theory. Given $B-V$ and P (see Iben and Tuggle, 1972b), errors as large as ± 0.5 mag. in estimates of luminosity may be expected.

Another characteristic one wishes to obtain from the observations is surface temperature. Unfortunately theoreticians have not yet presented us with composition-dependent conversions between the observational means, $\langle B-V\rangle$ and $\langle B\rangle - \langle V\rangle$, and the surface temperature T_e that would characterize the star if its pulsational amplitude were vanishingly small. However, the necessary conversion formula, even if available, would be very complex and require accurate specification of additional quantities that are very difficult to obtain for distant Cepheids: the degree of microturbulence and the abundance of the major contributors to opacity (see Bell and Parsons, 1972).

3.3. Comparison between Observation and Theory

Considerable attention has been paid over the last three years to the fact that each of four different ways of estimating Cepheid masses gives a different result (Cogan, 1970; Rodgers, 1970; Fricke *et al.*, 1972; Iben and Tuggle, 1972a, b). I will discuss only two ways of estimating mass and argue that the apparent discordance is due simply to an underestimate of Cepheid luminosities (Iben and Tuggle, 1972a).

The first method is based on evolutionary calculations. For any choice of composition, evolutionary calculations establish a mean relationship between mass and luminosity during the core helium-burning phase. When $X=0.7$, $Z=0.02$, this relationship is approximately $\log M \sim 0.726 + 0.251 \times (\log L - 3.25)$. For any mass, the dispersion in luminosity about the mean is only ± 0.25 mag. For any luminosity, the dispersion in mass is only $\Delta \log M \sim \pm 0.025$. There are three studies in the literature that permit one to tentatively estimate the composition dependence of the normalization of the evolutionary $M-L$ relationship (Hallgren and Cox, 1970, Robertson, 1971; Noels and Gabriel, 1974). They suggest that for stars of mass $\sim 5\ M_{\odot}$, $\log M \sim 0.726 + 0.251 \times (\log L - 3.25) + (X - 0.7) + 3(Z - 0.02)$. Not enough information is available to determine whether or not the slope of the relationship (coefficient of $\log L$) depends strongly on the composition. However, it is clear that, given an estimate of luminosity and composition, one may in principle estimate a Cepheid's 'evolutionary' mass M_{evo}.

The second method is based on linear pulsation calculations which establish a relatively composition-independent relationship between mass M, fundamental period P_{F}, luminosity L, and surface temperature T_{e}. One approximation to this relationship (Iben and Tuggle, 1972a) is

$$\log M \sim 1.7 - 1.5 \log P_{\text{F}} + 1.26(\log L - 3.25) - 5.25 (\log T_{\text{e}} - 3.77).$$

Note that the dependence on P, L, and T_{e} given by this relationship,

$$M_{\text{puls}} \tilde{\propto} L^{5/4} T_{\text{e}}^{-5.25} P^{-3/2},$$

is significantly different from the relationship,

$$M_{\text{puls}} \tilde{\propto} L^{3/2} T_{\text{e}}^{-6} P^{-2},$$

so frequently used in earlier days to help devise a PLC relationship.

Pulsation masses for 13 galactic Cepheids for which luminosities can be estimated photometrically are shown in Figure 6. At any given luminosity, the mass suggested by evolutionary calculations for the composition $X = 0.7$, $Z = 0.02$ is larger by about 40% than the pulsation mass.

The difference between the two estimates of mass can be minimized in several ways: (a) by increasing the helium abundance used in evolutionary calculations by $\Delta Y \sim 0.15$; (b) by decreasing the surface temperature assigned to all Cepheids by about $\Delta \log T_{\text{e}} = 0.025$, or (c) by increasing the luminosity assigned to all Cepheids by about $\Delta \log L \sim 0.1$ (0.25 mag.). The first two alternatives can be discarded by examining the situation in the HR diagram (Figure 7). Each of these alternatives leaves too large a gap between fundamental mode blue edges (for the appropriate masses) and the location of the 13 Cepheids. If this argument is not convincing, one may argue that a helium abundance $Y \sim 0.45$ is far larger than Y estimated for other galactic objects and that an error of 0.025 in the conversion from $B-V$ to $\log T_{\text{e}}$ is not to be expected. The final alternative requires that the normalization of the standard main sequence is incorrect by about 0.25 mag. It is interesting that every method of estimating the distance to the Hyades, other than the convergent point method (which is now used to set the standard), makes the Hyades distance modulus greater by about 0.2 mag. than the one given by the convergent point method (van Altena, 1973). Thus, it appears very likely that the apparent discrepancy betweeen evolution and pulsation masses is simply due to an underestimate of Cepheid luminosities.

If, then, mass loss does not occur to an appreciable extent prior to or during the Cepheid stage, one may combine the results of evolutionary calculations with the results of pulsation calculations to derive a theoretical PLC relationship that should

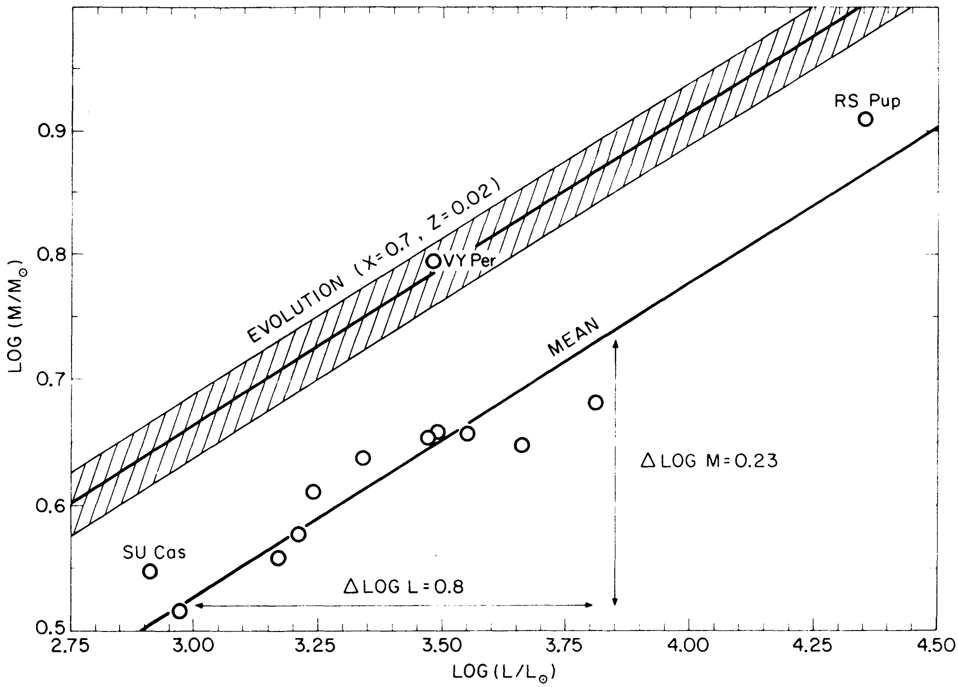

Fig. 6. Mass-luminosity relation for Cepheids given by evolutionary models ($X=0.7$, $Z=0.02$) without mass loss compared with one obtained by applying results of pulsation theory to a current interpretation of the observed properties of Cepheids in galactic clusters and associations. The shaded region is the evolutionary M-L relationship and the line labeled 'mean' is a fair representation of the pulsation M-L relation. Cepheid luminosities are tied to an assumed distance modulus of 3.05 mag. for the Hyades. (Iben and Tuggle, 1972a.)

be an even more reliable indicator of Cepheid luminosities (and hence distances) than is the PLC relationship one attempts to derive 'from observations alone.' For the composition $X=0.7$, $Z=0.02$, the result is (Iben and Tuggle, 1972b):

$$M_{\mathrm{Bol}} \sim -0.96-3.76 \log P_{\mathrm{F}}-13.0\,(\log T_{\mathrm{e}}-3.77).$$

Using the transformations $M_{\mathrm{Bol}} \sim M_V+0.145-0.332(B-V)$ and $\log T_{\mathrm{e}} \sim 3.886-0.175\,(B-V)$ which are thought appropriate for galactic Cepheids, the theoretical PLC relationship transforms into

$$M_V \sim -2.61-3.76 \log P_{\mathrm{F}}+2.60\,(B-V).$$

Neither this PLC relationship nor any other can be applied to all Cepheids without thinking. Above all, the effects of composition differences must be properly included. This means that sufficient evolutionary calculations must be done to determine, as a function of composition, the slope as well as the normalization of the mass-luminosity relationship. It means further that sufficient model atmosphere calculations must be done to determine the manner in which the conversion between T_{e} and $B-V$ depends on the composition (Bell and Parsons, 1972).

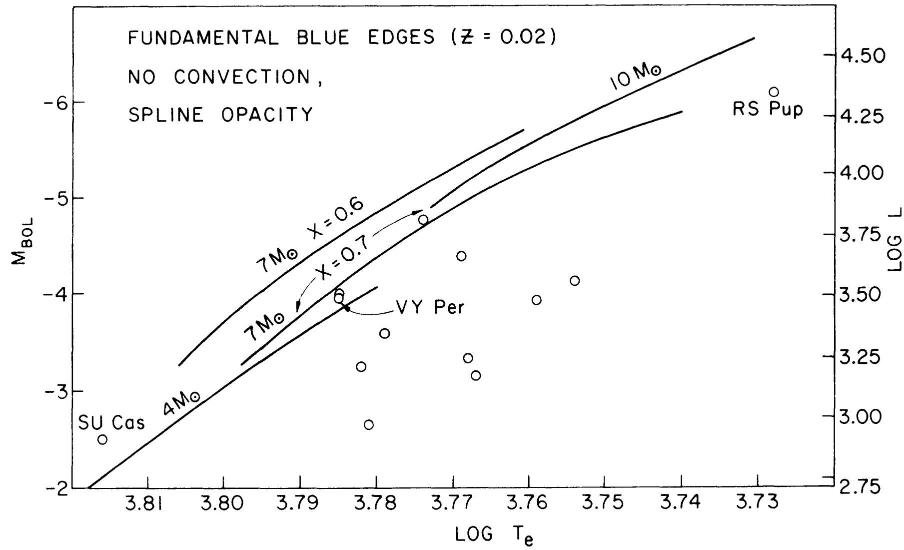

Fig. 7. Fundamental blue edges in the HR diagram for masses $M/M_{\odot}=4$, 7, 10 and composition ($X=0.7$, 0.6; $Z=0.02$; no convection) compared with the estimated surface temperatures and magnitudes of Cepheids in galactic open clusters and associations. Cubic spline interpolation in Cox and Stewart (1972) tables has been employed to obtain the opacity. The blue edge of $X=0.6$ is an estimate based on a linear extrapolation of blue edges for $X=0.7$ and 0.8. The introduction of convection shifts each blue edge to the blue by approximately $\Delta \log T_e \sim 0.005$. Data from Tuggle and Iben (1974).

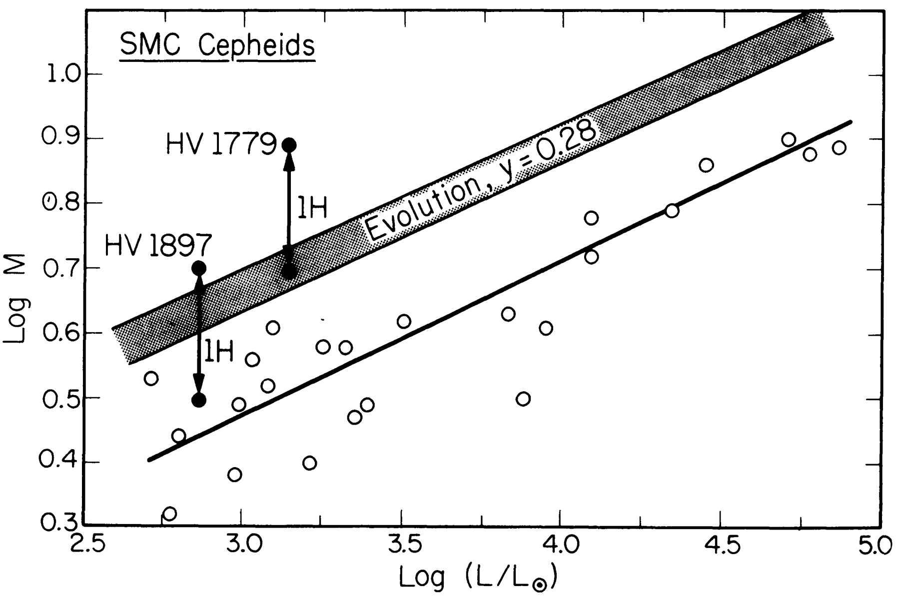

Fig. 8. Mass-luminosity relationship for Cepheids given by evolutionary models compared with a mass-luminosity relationship defined by Cepheids in the Small Magellanic Cloud if (1) the distance modulus for the SMC is 19.25 mag. and (2) if the $(B-V)$ to $\log T_e$ conversion is identical to that used for galactic Cepheids. Data from Gascoigne (1969); Theoretical estimates from Tuggle and Iben (1974).

To show how important these requirements are, let us look at estimates of pulsation and evolution masses for Cepheids in the Small Magellanic Cloud (Gascoigne, 1969). If luminosities are determined by adopting ('arbitrarily') a distance modulus of 19.25 mag. and if surface temperatures are determined by using the color-to-temperature conversion thought appropriate for galactic Cepheids (Kraft, 1961), the pulsation masses shown in Figure 8 result (Tuggle and Iben, 1974). On comparing with evolu-

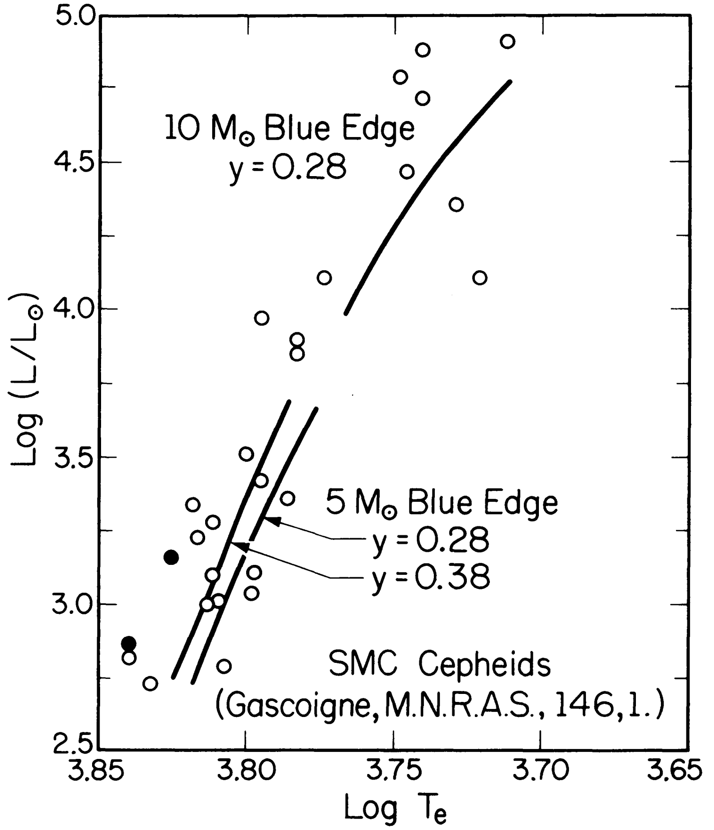

Fig. 9. Fundamental blue edges in the HR diagram for masses $M/M_{\odot}=5$ and 10 and composition ($X=0.7$, 0.6, $Z=0.02$; no convection) compared with estimates of surface temperature and luminosity for Cepheids in the Small Magellanic Cloud. Luminosities (Gascoigne, 1969) are based on a distance modulus of 19.25 mag. and temperatures are based on a conversion thought appropriate for Cepheids in our own Galaxy. Theoretical data are from Tuggle and Iben (1974).

tionary masses, the mass discrepancy again appears. However, a more serious discrepancy appears in the HR diagram (see Figure 9). The Cepheids are all much bluer than the blue edges for the relevant masses. The way out of both difficulties is to argue that the average 'metallicity' of SMC Cepheids is much smaller than that of galactic Cepheids and that the conversion from $B-V$ to $\log T_e$ that is appropriate for galactic Cepheids gives too large a surface temperature for the SMC Cepheids (Bell and

Parsons, 1972). If one arbitrarily moves all Cepheids in Figure 9 to the red by $\Delta \log T_e \sim 0.02$, then the discrepancy in the HR diagram is removed. And, wonder of wonders, the discrepancy between pulsation and evolution masses is also removed. Thus, by a lucky accident, the distance modulus remains unchanged from the original choice, for whatever reasons that choice was made.

4. Population II Variables

4.1. Gross differences between population I and population II variables

The distinction between the type of variable found in globular clusters and the type of variable found predominantly in the galactic disk is most strikingly displayed by

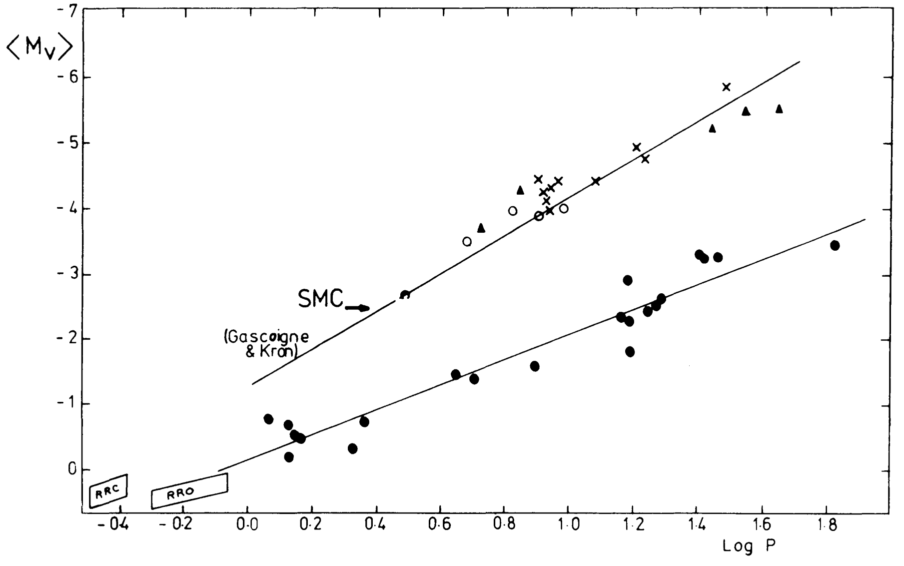

Fig. 10. Approximate magnitude-period relationships defined by variables belonging to two distinct populations. The upper curve is for population I and the lower curve is for population II. This figure is reproduced from Dickens and Carey (1967).

the difference in the magnitude-period relationships appropriate to the two types. As shown in Figure 10 (from Dickens and Carey, 1967), the population I variables are significantly brighter at any period than are the population II variables. At a period of 10 days, the difference of about 2 mag. is most easily accounted for if masses differ by approximately a factor of ten, the population I variables of this period having a mass near 5–7 $M_\odot$.

The second most striking difference between the two types is in the slope of the magnitude-period relationship, the slope for population I variables ($-\mathrm{d}M_V/\mathrm{d}\log \mathrm{P} \sim \sim 3$) being much larger than the slope for population II variables ($-\mathrm{d}M_V/\mathrm{d}\log P \sim \sim 1.9$). This difference is most easily accounted for if the mass of a population I variable

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is a monotonically increasing function of increasing luminosity and if the masses of all population II variables are roughly the same. Thus, simply by analyzing the appropriate magnitude-period relationships, one may discover that (a) the most evolved stars in globular clusters are of low mass and hence are representative of an old population, and that (b) the most evolved stars in the disk are of all different masses and hence are of all different ages.

4.2. Theoretical distribution in the HR diagram

We are all familiar with the observed color-magnitude distributions for globular cluster stars. Theory produces distributions that bear a reasonably close qualitative resemblance to the observed distributions. In Figure 11 is shown a typical theoretical distribution. Data from Simoda and Iben (1970), Iben and Rood (1970), Strom *et al.* (1970), Schwarzschild and Härm (1970), and Iben and Huchra (1971) has been used to construct this figure. All detail of individual tracks is suppressed in order to indicate where stars spend most of their nuclear-burning lives. A more realistic description of the distribution on the horizontal branch is given by Rood (1973).

Most of a star's lifetime (10^{10} yr for an 0.8 $M_{\odot}$ star) is of course spent on the main sequence where hydrogen is converted into helium at the stellar center. Appreciable time ($\sim 10^9$ yr) is also spent on the subgiant branch where hydrogen is converted into helium in a thick shell surrounding an inert helium core within which electrons are becoming degenerate. Comparable times ($\sim 10^8$ yr) are spent on the giant branch (hydrogen burning in a thin shell) and on the horizontal branch (helium burning at the center and hydrogen burning in a shell). As a star reaches the tip of the first giant branch, helium burning begins in an electron degenerate core of mass ~ 0.45 $M_{\odot}$. During the ensuing 'helium flash', a star jumps over to a position on the horizontal branch. The more mass it loses from the surface during the giant branch phase, the bluer the average position it adopts on the horizontal branch.

Following the exhaustion of helium at the center, a star rapidly moves to a position on the supra-horizontal branch or on the asymptotic branch, the mean position again being determined by the star's mass. There, hydrogen and helium burn in separate shells surrounding an inert carbon-oxygen core.

All stars eventually reach the asymptotic giant branch, which may extend to luminosities considerably brighter than is indicated in Figure 11. At some point high enough along the asymptotic giant branch, a thermal instability sets in. If it is sufficiently light, the star then loops back from the asymptotic giant branch in a series of relaxation oscillations (Schwarzschild and Härm, 1970). During this last phase, considerable mass may be lost from the star and it is possible that, as a consequence, the asymptotic branch may bend over toward cooler temperatures at luminosities greater than that at the tip of the giant branch proper (see Feast, 1974). It is also probable (on the basis of the theoretical calculations alone) that Mira-type pulsations of increasing amplitude may occur here and that a final large amplitude pulse may blow off all but a minute portion of the matter lying above the hydrogen burning shell (see Wood, 1974). This phenomenon may be the origin of many planetary nebulae – objects consisting of a shell of outwardly expanding matter surrounding a central star that, after burning brightly as a blue star for 10^4 yr or so, cools off to become a white dwarf.

Stars occurring in regions redward of the pulsational blue edges indicated in Figure 11 are expected to be unstable to radial pulsation in the indicated modes. Thus, one expects three groups of variables to be particularly conspicuous: (a) stars on the horizontal branch that pulsate in either the first harmonic mode or in the fundamental mode (c-type and ab-type RR Lyrae stars); (b) stars on the suprahorizontal branch

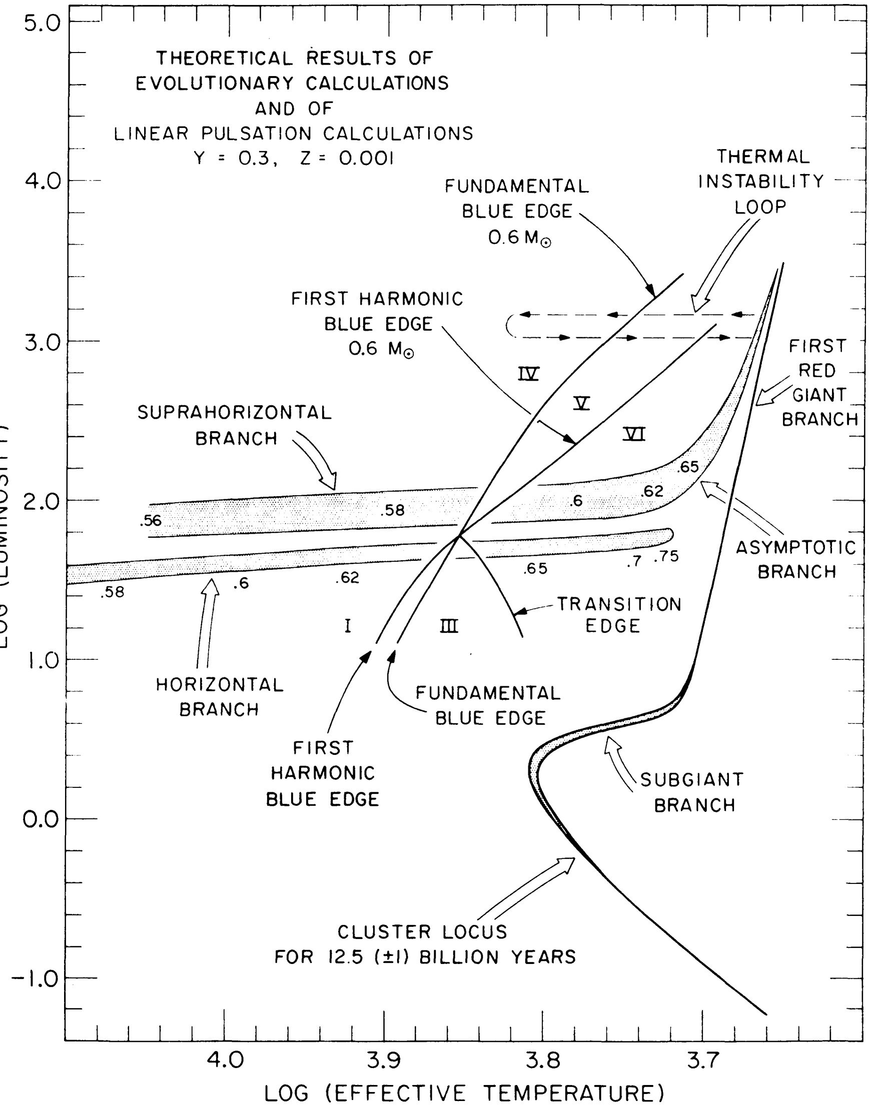

Fig. 11. Theoretical cluster locus compared with blue edges and a conjectured transition edge for $0.6 M_\odot$. Each number along the horizontal and suprahorizontal branches indicates the mass of a model whose mean location is correlated with the position of the number (from Iben and Huchra, 1971).

that pulsate primarily in the fundamental mode (BL-Herc stars or short period population II Cepheids); and (c) stars in the process of undergoing relaxation oscillations that cause them to swing back and forth from the asymptotic branch, pulsating in the fundamental mode while in the instability strip (W-Virginis stars or long period population II Cepheids).

Some additional detail should be mentioned. For sufficiently light stars, evolution during the double nuclear-shell-source stage occurs above the supra-horizontal branch, far to the blue of the asymptotic branch. Relaxation oscillations in such stars lead to wide amplitude excursions in the HR diagram that occupy a region far to the blue of the blue edges shown in Figure 11.

4.3. The driving regions for pulsation

In all variables that lie in the classical instability strip, the properties of hydrogen and helium ionization zones are responsible for instability against pulsation. In RR Lyrae stars with large pulsation amplitudes, these zones lie very near the surface in both mass and spacial displacement, as is illustrated in Figures 12 and 13. Figure 12 (from Iben, 1971b) describes the distribution of state variables and mass as a function of distance

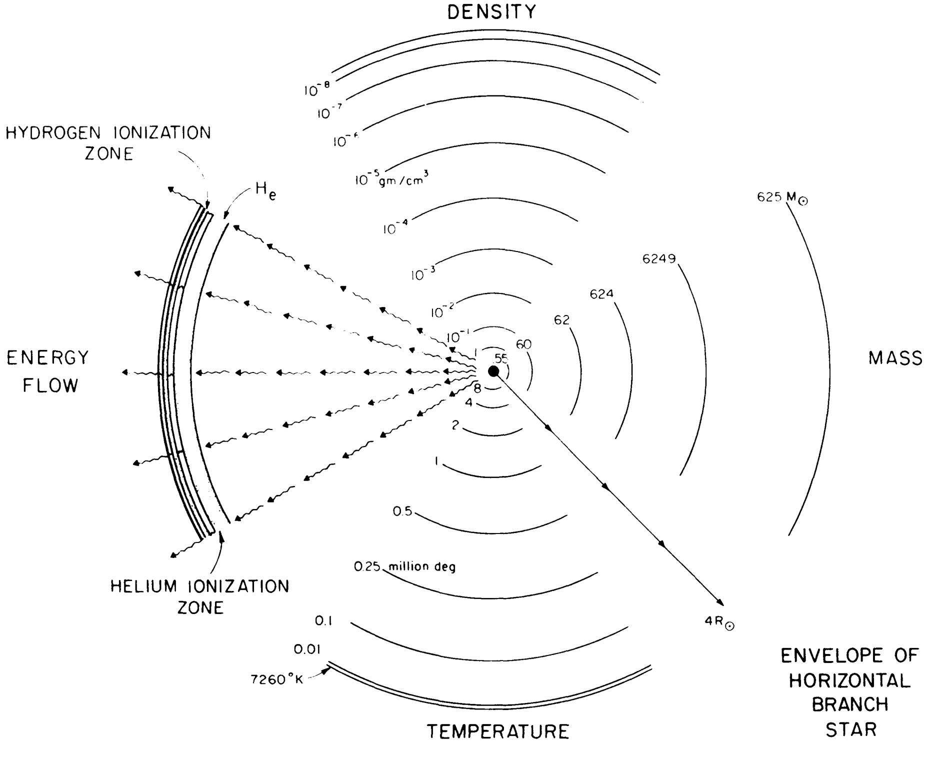

Fig. 12. Distribution of structure variables in the envelope of an initial horizontal branch model (from Iben 1971b).

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from the center within the envelope of a model horizontal branch star. The total mass above the second helium ionization zone is only $10^{-7}\,M_{\odot}$ and the total mass above the hydrogen ionization zone is only $10^{-9}\,M_{\odot}$. Figure 13 (from Iben, 1971a) shows that the only positive contributions to driving pulsation come from matter in the two major ionization zones.

Although in the illustration the second helium ionization zone is the major contributor to pulsation, it is clear that the presence of the hydrogen ionization zone plays

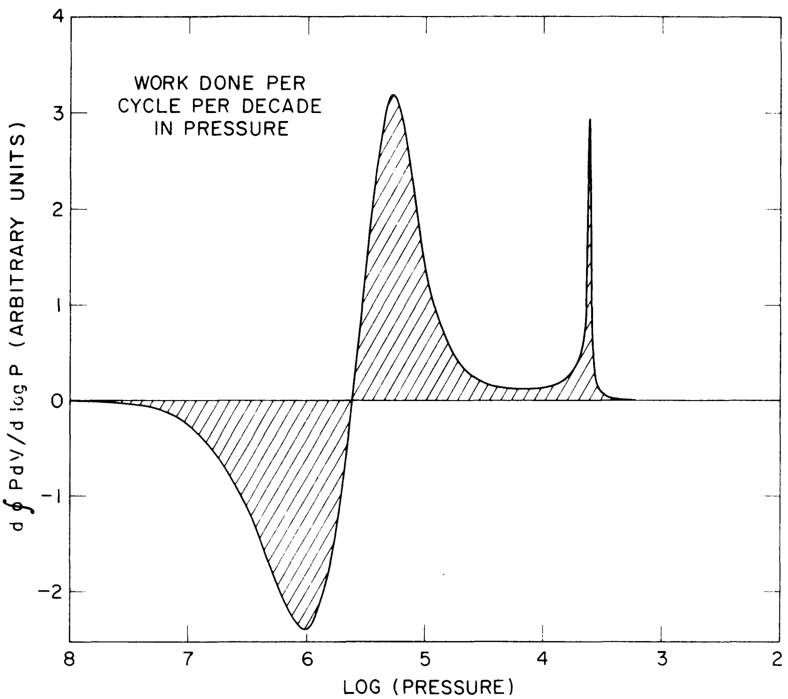

Fig. 13. Work done per cycle per decade in pressure for the first harmonic mode of the model described in Figure 12 (from Iben, 1971a).

an essential role. Certainly, the precise location of blue edges for pulsation in various modes and the phase relation between light and radius amplitudes are strongly influenced by the properties of the hydrogen ionization zone (Castor, 1971).

4.4. Blue edges for pulsation in the fundamental and first harmonic modes

For masses and compositions thought appropriate for horizontal branch stars in globular clusters, blue edges for pulsation in the fundamental and first harmonic modes intersect each other at luminosities in the neighborhood of those defined by

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horizontal branch stars. At luminosities below the intersection point, the blue edge for pulsation in the first harmonic mode is at a higher surface temperature than is the blue edge for pulsation in the fundamental mode. At luminosities above the intersection point, the reverse is true. This is illustrated in Figure 14 (from Tuggle and Iben, 1972).

Also demonstrated in Figure 14 is the sensitivity of blue edges and their inter-

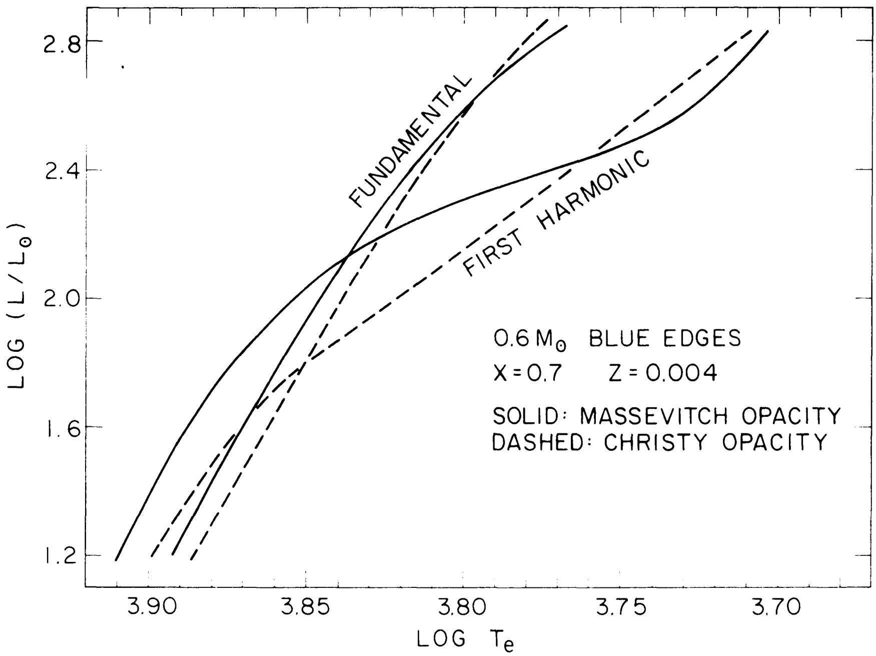

Fig. 14. Blue edges for pulsation in both the fundamental mode and in the first harmonic mode for a model star of mass $M = 0.6\ M_\odot$ and composition parameters $X = 0.7$, $Z = 0.004$. Solid curves are blue edges constructed with spline opacities (from Tuggle and Iben, 1972).

section points to the precise form of the opacity law used in making the calculations. Small differences in opacity approximations (see Figure 15, from Tuggle and Iben (1972)) are responsible for rather dramatic changes in the location of blue edges.

In all clusters that have been carefully examined and in which the RR Lyrae stars form a fairly broad distribution in color, the bluest RR Lyrae stars are pulsating in the first harmonic mode (Bailey c-type variables) whereas the reddest are pulsating in the fundamental mode (Bailey ab-type variables). This means, of course, that the intersection between blue edges must occur in the HR diagram above the horizontal branch. Since, for a given choice of opacity, the intersection point decreases in luminosity with decreasing model mass and also with decreasing envelope helium abundance, the observed relative location in color of c- and ab-type variables may be used in conjunction with observation-related estimates of horizontal branch luminosity to place lower limits either on mass (if the helium abundance is known) or on helium abundance (if the mass is known).

Estimates of helium abundance may be made in several ways. In principle, one of the

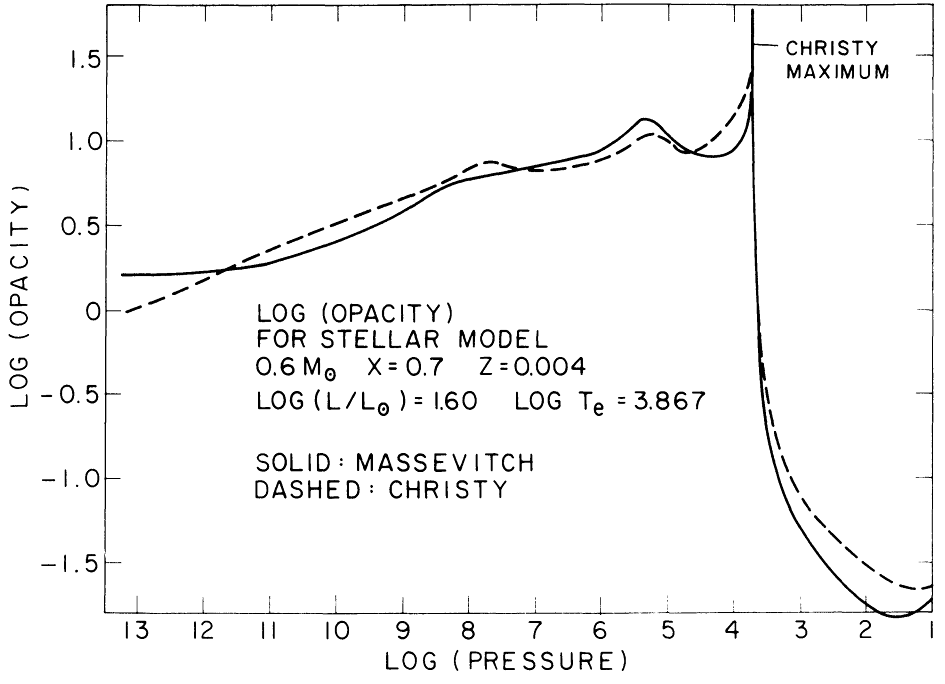

Fig. 15. Opacity derived by use of spline interpolation in fine grain opacity tables (Cox and Stewart, 1972) compared with opacity given by Christy's analytic approximation to Cox and Steward opacity tables. Opacities are plotted as functions of $\log P$, where P is the pressure (dyne cm^{-2}) in a model stellar envelope characterized by $M = 0.6 M_{\odot}$, $\log(L/L_{\odot}) = 1.6$, $X = 0.7$, $Z = 0.004$, $Y = 0.296$. This envelope has been constructed with the 'spline opacity' (from Tuggle and Iben, 1972).

cleanest ways makes use of the relationship between period P and surface temperature T_e along first harmonic blue edges. As is illustrated in Figure 16 (from Tuggle and Iben, 1972). this relationship is relatively insensitive to mass and is primarily a function of the helium abundance Y. Given that, in a particular cluster, the periods of the shortest period c-type variables are in the range $\log P \sim -0.6 \rightarrow -0.5$ and that an estimate of the surface temperature of the bluest variables is in the range $\log T_e \sim 3.86 \rightarrow 3.87$, one may estimate a helium abundance in the range $Y \sim 0.20 \rightarrow 0.25$ (Tuggle and Iben, 1972; Cox *et al.*, 1973).

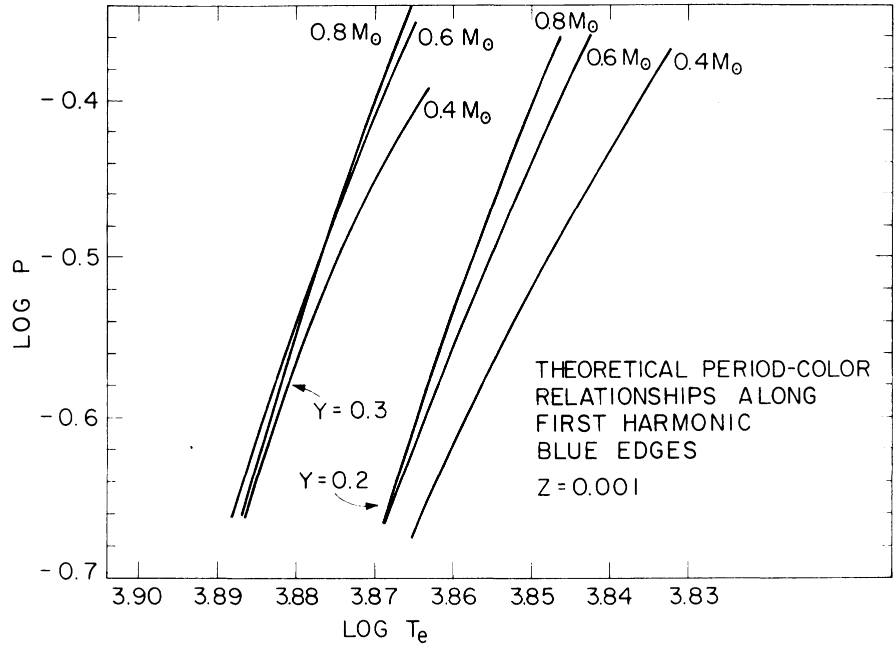

Fig. 16. The relationship between period and surface temperature for stars along first-harmonic blue edges in the major region of relevance to RR Lyrae stars in globular clusters. Curves for $M/M_{\odot}=0.4$, 0.6, 0.8, $Z=0.001$, and $Y=0.3$ and 0.2 are shown (from Tuggle and Iben, 1972).

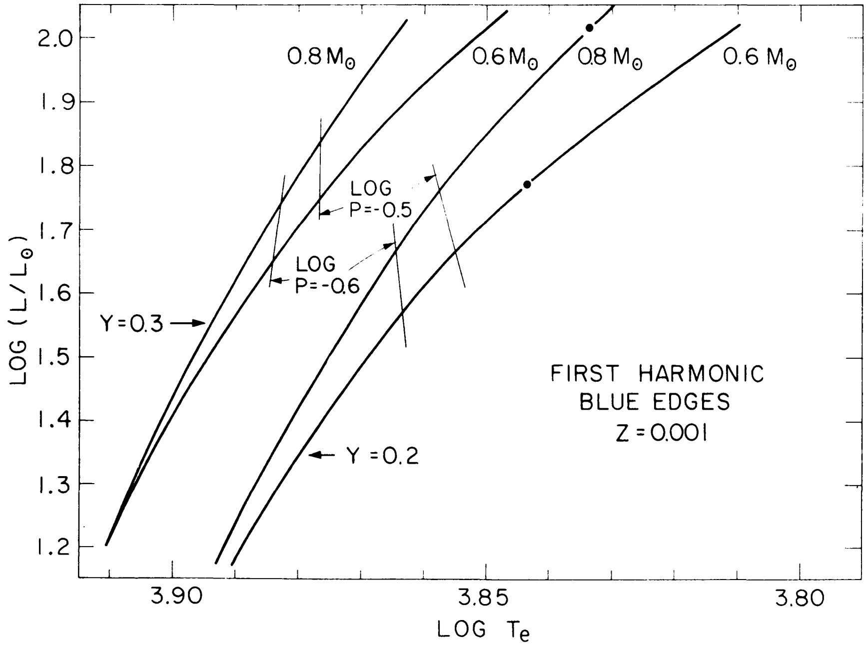

Fig. 17. First-harmonic blue edges for $M/M_{\odot}=0.6$, 0.8; $Y=0.2$, 0.3 (from Tuggle and Iben, 1972).

Once helium abundance has been estimated, one may then use additional properties along blue edges to determine a relationship between mass and luminosity, as is made clear in Figure 17. Even a rough estimate of mass permits a relatively tight estimate of luminosity. Or, given a rough estimate of luminosity, a rough estimate of mass can be derived.

Other estimates of mass and luminosity can be obtained by combining additional theoretical relationships with observational data. Evolution theory permits one to estimate both L and M versus T_e for horizontal branch stars as a function of Y and Z. Estimates of Z can of course be provided in principle by spectroscopy and/or photometry. Thus, with estimates of T_e for variable stars, one may determine both L and M as functions of Y. The composition-independent relationship between P, L, T_e, and M given by pulsation theory may be used to give a further relationship between L and M for variables. Combining the information from all approaches one may again obtain estimates of all bulk properties (see, for example, Baker, 1965, van Albada and Baker, 1971; Iben, 1971b).

4.5. The transition region

One of the most interesting problems in pulsation theory that is yet to be resolved is the full-amplitude behavior of stars in the region where models are unstable to radial pulsation both in the fundamental mode and in the first harmonic mode (Christy, 1966). The current situation may be described in terms of the curves in Figure 18 (from Iben, 1971b). Brute-force calculations show that the region where both modes can be excited may be partitioned into three groups according as (a) final, full-amplitude motion is in the first harmonic mode, regardless of initial conditions (region 1); (b) final motion is in the fundamental mode, regardless of initial conditions (region 2); and (c) final motion, after a finite number of cycles, depends on the initial mode of excitation (region 3).

This latter region might be called the 'transition region' between pulsation in the first harmonic and pulsation in the fundamental mode. Its size and shape are difficult to determine by standard brute-force calculations and one might expect that conclusions could depend on the number of cycles for which the motion is followed.

The brute-force calculations suggest that, within region 3, pulsation begun in the fundamental mode will continue in the fundamental mode with no indication of switching to the first harmonic. Similarly, pulsation begun in the first harmonic mode persists with no sign of switching to the fundamental mode. Thus, a kind of 'hysteresis' occurs for models in region 3. But, is this 'type-3' behavior independent of the number of cycles for which motion is followed? Or will switching to a truly favored mode occur after a sufficiently long time?

Recent calculations by von Sengbush (1973) and by Stellingwerf (1973) may help provide an answer. Using a technique developed by Baker and von Sengbush (1969), they are able to construct the final, full-amplitude model pulsating in a pure mode and test this model for stability. Preliminary results by von Sengbush (1973) suggest that there is a finite region 3 in the HR diagram where true hysteresis occurs. However, the

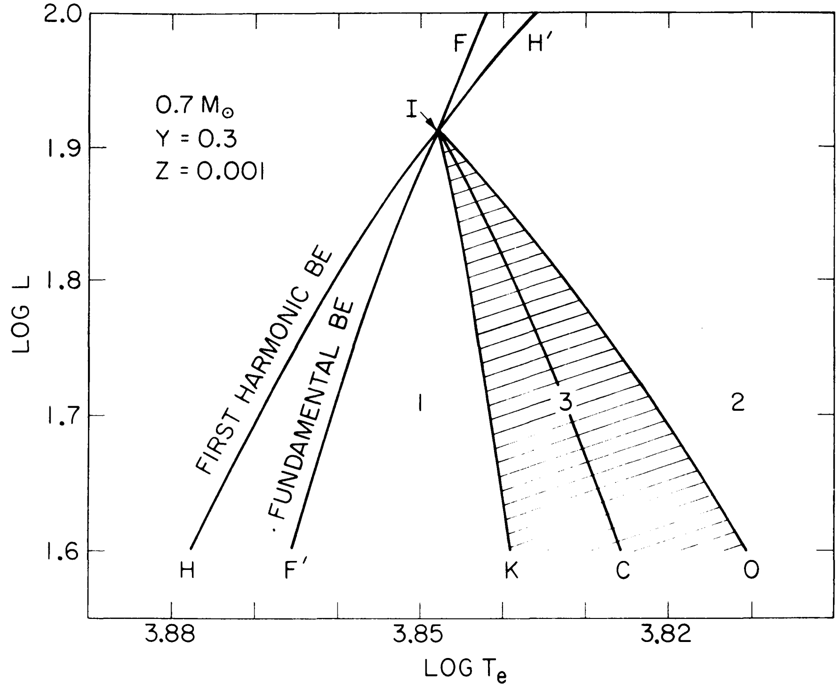

Fig. 18. Schematic description of the relative locations of different types of instability. Above point *I* and to the red of the fundamental blue edge, pulsation at full amplitude is in the fundamental mode. Below point *I*, the situation is more complex. Between the first harmonic blue edge and the curve *IK*, full amplitude motion is in the first harmonic mode, regardless of initial conditions. The region between the fundamental blue edge and the curve *IK* has been christened by Christy the region of type 1 instability. Beyond the curve *IO*, full amplitude motion is in the fundamental mode, regardless of initial conditions (Christy type 2 instability region). In the region between *IK* and *IO*, pulsation may persist in the mode initially excited (Christy type 3 instability region). The angle *KIO* may well depend on the number of cycles for which motion can be practically followed (from Iben, 1971b).

site of this region is much smaller than was suggested by the earlier brute-force calculations (Christy, 1966).

If horizontal branch stars evolve in both directions in the HR diagram the hysteresis effect should show up in the form of overlap in color between c-type (first harmonic) and ab-type (fundamental) variables. Or, if no overlap occurs, one might deduce that horizontal branch stars all evolve predominantly is one direction only.

The possible existence of a hysteresis effect permits one to account for the fact that clusters with many RR Lyrae stars fall clearly into one of two Oosterhoff (1939) types. Suppose that the mean luminosity of all horizontal branch stars is roughly the same, independent of composition, but that the direction of evolution along the horizontal branch is critically dependent on composition. In clusters in which evolution along the horizontal branch is predominantly to the right (high T_e to low T_e), stars which begin pulsating in the first harmonic in region 1 will continue to pulsate in the first harmonic as they evolve through region 3. Only when they reach region 2, will they pulsate in the fundamental. Thus there will be a high percentage of c-type variables and the 'transition period' will be large compared to the case where stars evolve predominantly from low T_e to high T_e. In this later case, stars which begin pulsating in the fundamental mode in region 2 will continue to pulsate in the fundamental mode as they evolve through region 3, switching into the first harmonic only on evolving into region 1.

It may well be, as van Albada and Baker (1971, 1973) suggest, that hysteresis plus direction of evolution are the dominant contributors to the Oosterhoff effect. However, there are other factors that almost certainly make a contribution. The additional factors include differences between clusters of the two types: (a) in the mean mass of RR Lyrae stars (Stobie, 1971; Castellani *et al.*, 1973), (b) in the mean luminosity of the horizontal branch (e.g., Iben, 1971b), and (c) in the distribution in number vs color on the horizontal branch (Iben, 1971b).

That the distribution in color along the horizontal branch plays a role in determining the ratio of c-type to ab-type variables is made indisputably clear in Figures 19 and 20

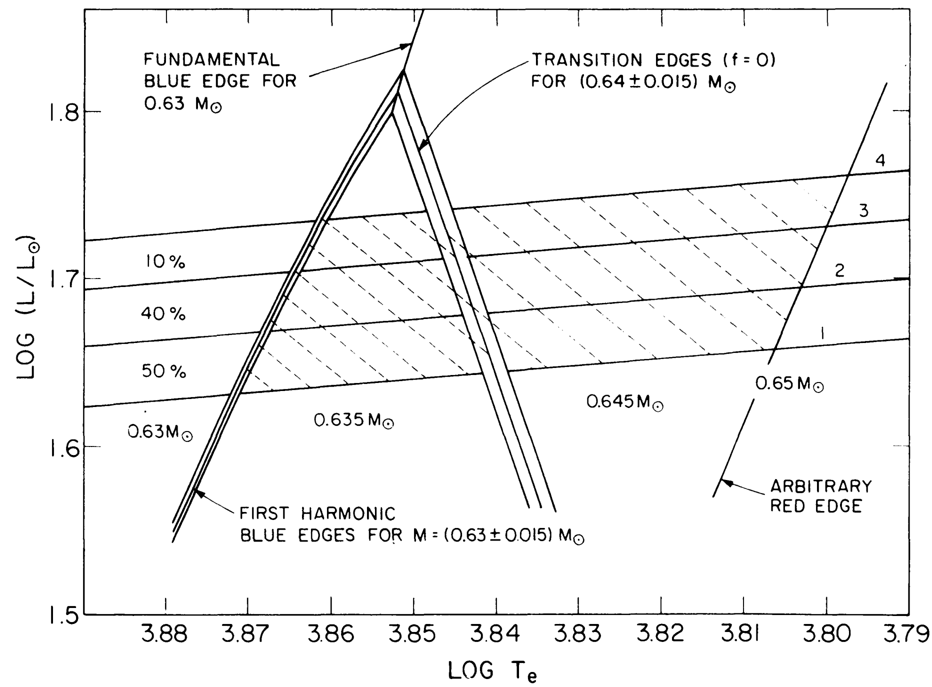

Fig. 19. Theoretical blue edges, constant period lines, transition edges, and the theoretical horizontal branch for $Y=0.3$, $Z=0.001$. Mean mass is indicated as a function of $\log T_e$. In the truncated pyramid bounded by the first harmonic blue edges and the adopted transition edges, stars are assumed to pulsate at full amplitude in the first harmonic mode. Between the transition line and the semi-empirical red edge, stars are assumed to pulsate at full amplitude in the fundamental mode. The mean position of a horizontal branch star is near line number 2, approximately one third of the way up from the base of the horizontal branch and two thirds of the way down from the top. Lines (dashed) of constant period have the slope $d \log L/d \log T_e \sim 4.04$ (from Iben and Huchra, 1971).

(from Iben and Huchra, 1971). The theoretical distribution in number vs period, shown in Figure 20, follows from the location of the blue and red edges, the location of transition edges (transition *regions* of zero width), and the location of the horizontal branch, shown in Figure 19, *if the distribution in number versus log T_e along the horizontal branch is constant.* On comparing (Figure 20) the theoretical distribution with the distribution defined by variables in the cluster M3, it is clear that the *relative* locations of edges and of the horizontal branch have been specified correctly. However, the ratio of ab-type variables to the number of c-type variables in the theoretical distribution is smaller (by a factor of 2.5) than the observed ratio. The *only* way to achieve agreement is to replace the assumption of uniform density (in number vs log T_e) along the horizontal branch by an assumed distribution that has a much higher

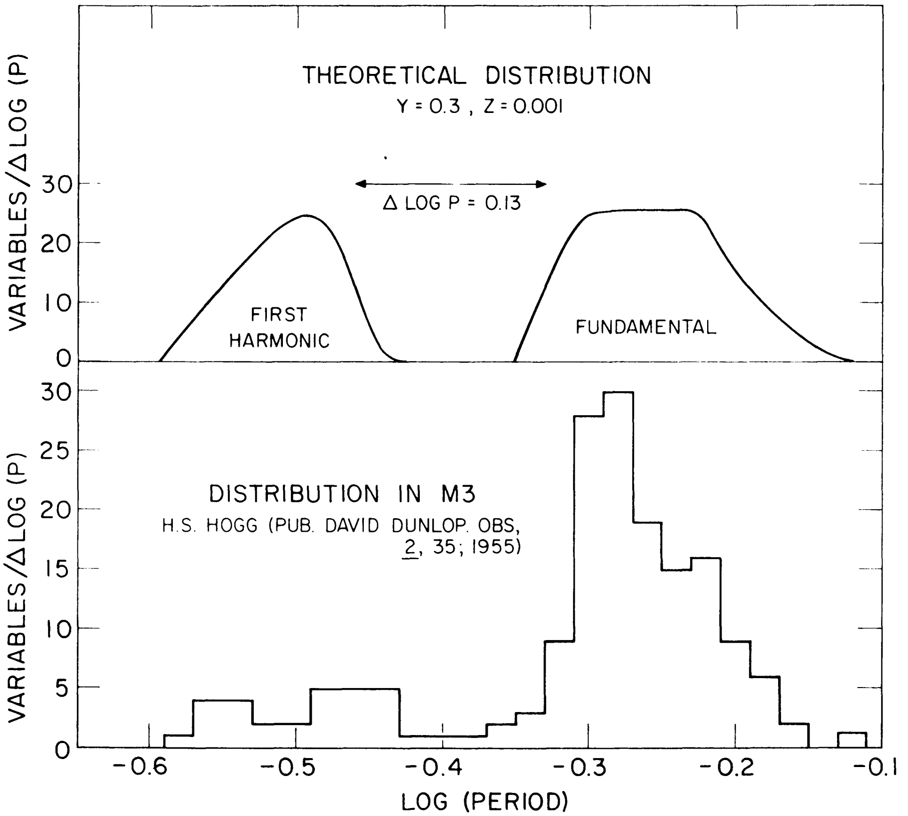

Fig. 20. Theoretical and observed (M3) distributions in number vs period. The theoretical distribution has been derived on the assumption that the density of horizontal branch stars is constant with respect to log T_e. It has been further assumed that at a given surface temperature, horizontal branch stars are distributed in the following way (refer to Figure 19): 50% between curves 1 and 2, 40% between curves 2 and 3, 10% between curves 3 and 4. In the theoretical distribution there are roughly twice as many stars in the fundamental peak as there are in the first harmonic peak. The actual ratio in M3 is approximately 5 to 1 (from Iben and Huchra, 1971).

density in the region of ab-type variables than in the region of c-type variables (see Iben and Rood, 1970, for further discussion).

One must also worry about the assumption that the dominant direction of evolution within the instability strip can alter abruptly and discontinuously when composition parameters are varied through some 'critical set.' This assumption amounts to a rather cavalier dismissal of many studies of evolution along the horizontal branch that show track behavior varying smoothly and continuously with change in composition (e.g., Iben and Rood, 1970; Sweigart and Gross, 1973).

Is it not as likely that the location of the 'transition region' changes rapidly when composition parameters vary past some 'critical set'? Certainly, the properties of the transition region are less well understood than any other element that enters into the analysis.

To this reviewer, it seems quite possible that the mean position of the transition region may move to the red (at any given magnitude and for any given mass) with decreasing abundance of the heavy elements and/or with decreasing abundance of helium in such a way as to give a much more natural accounting of the Oosterhoff effect than has yet appeared in the literature.

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DISCUSSION

Rodgers: Are you saying to us that the Sandage-Tammann period-luminosity-colour relation is wrong? That is well founded on Cepheids in open clusters.

Iben: No, only that the period-colour relation is subject to uncertainty.

Rodgers: It is based on stars whose luminosities and intrinsic colours were obtained by main sequence fitting.

Iben: Yes.

Rodgers: But you say that if anyone uses the Sandage P-C relation, he is wrong and that it is the cause of the mass discrepancy in Cepheids.

I do not know of convincing evidence that η Aql is a first harmonic oscillator. There are strong reasons (e.g. beat Cepheids and Cloud Cepheids) to believe that transition periods occur around two to three days.

Iben: I have nothing against a period-luminosity-colour relationship. It is absolutely essential if one wants to estimate the bulk properties of the Cepheids in the Galaxy: their distances, luminosities and so on. You have to get it somewhere. However, what I am arguing is that one of the basic elements used in obtaining a P-L-C relationship, focussing primarily on the observations, is the P-C relation. You can get all sorts of different slopes here which will give you fantastically different results with regard to the luminosity of the given Cepheid, whereas each one of the different lines fits equally well with the observations. That is all I am saying. There is by now a lot of theory that is not all that bad that could be made use of in obtaining a P-L-C relationship. The line, given by the evolution plus pulsation theory can be passed through the relevant points to show that there is consistency with that particular observational data.

Tayler: Would it be fair to say that you were arguing, when you were comparing the evolutionary masses and the pulsation masses, that because it just looked as if you had shifted a line bodily, that you were rather unhappy with the idea of a sort of mass loss that was causing the difference. You felt you would not have got a shifted line with more or less the same slope?

Iben: It is just that if you look in the three different planes, P-L, M-L and H-R diagram, you find that you can get consistency without mass loss.

Rodgers: I think that a statement which says there is no reason to have any greater degree of mass loss rate in more massive stars than in low mass stars, is a bit sweeping because there is direct observational evidence for mass loss and where it does occur: It occurs in M supergiants and in long period Cepheids whereas it does not occur in short period Cepheids nor K giants nor ordinary giants.

Iben: I am in full agreement. It could be that this is the case. On the other hand, more massive stars spend less time as bright supergiants than do less massive stars as less bright ones. So when you fold in integration time with the lower mass loss rate, you might get the slope of that mass loss curve having exactly the opposite sign.

Tayler: The other possible argument is that if there is mass loss when they are late type stars, then you have got a very much deeper convection zone (in mass fraction) in the high mass stars than in the low mass stars. If the surface convection zone is responsible for the mass loss and if the cores of the stars do not differ very much in mass though the total masses differ a lot, then if you are going to strip something down somewhere towards the core, you would have a bigger mass loss for the high mass stars than the low mass stars.

Stobie: Are you saying that a change in slope of the $[(B-V)-\log P]$ relation actually gets rid completely of the mass discrepancy or does it change the slope?

Iben: No, it changes the slope.

Stobie: So you still need a change in the distance modulus of the Hyades.

Iben: Yes, or mass loss may be the answer.

Breger: You have a limited distribution of points. You have 13 points in this diagram and they are not randomly distributed in period, luminosity and colour, so by projecting the points into a two-dimensional diagram like this, it may not be valid to draw any slope, because part of the slope is due to the over-run from the luminosity. You have to consider all three. My question is: have you made a new solution of the data to see whether a shift like you would like is still okay with the data?

Iben: Yes, I just plonk down a line and say it looks good.

Breger: But three-dimensional?

Iben: Yes!