# 15

# The hadronic decays of the Z and W bosons

In Chapter 13 we described the results on the leptonic decays of the Z boson, obtained from experiments using $e^+e^-$ colliders. These results are in striking agreement with the predictions of the Weinberg–Salam electroweak model. In this chapter, we shall consider some of the wealth of data that has been accumulated at CERN and SLAC on the hadronic decays of the Z, and we shall find equally striking agreement between experiment and theory.

## 15.1 Hadronic decays of the Z

In the Standard Model, a hadronic decay of the Z is most likely to be triggered by an initial decay to a quark–antiquark pair. The subsequent hadrons produced are mostly confined to two *jets*, back-to-back in the Z rest frame and made up of stable, or long lived, particles (see Fig. 15.1). The precise details of the processes involved in the creation of a jet are not fully understood.

The momentum of a jet may be defined as the total momentum of the particles associated with it, and may be presumed to be equal to the momentum of the initiating quark or antiquark. The Z has sufficient rest energy to decay to any quark–antiquark pair other than a $t\bar{t}$ pair, but it has so far not been possible to identify jets as arising specifically from u, d or s quarks, or their antiquarks. However, many b quark jets can be identified with some confidence from the recognition of B mesons ($b\bar{u}$, $b\bar{d}$), which have a high probability of being produced in b quark jets, and a low probability of being produced in other jets. Similarly, $\bar{B}$ mesons are used to identify $\bar{b}$ jets. The observation of charmed hadrons in jets has likewise been used to identify jets arising from c quarks and $\bar{c}$ antiquarks.

Associating the observed jets with the initiating quarks, comparisons can be made with the Standard Model predictions of Z decay rates to quark–antiquark pairs. We shall first consider the decay of a Z that is in a definite spin state. The interaction Lagrangian (14.4) has the same form for the d, s and b quarks, and in



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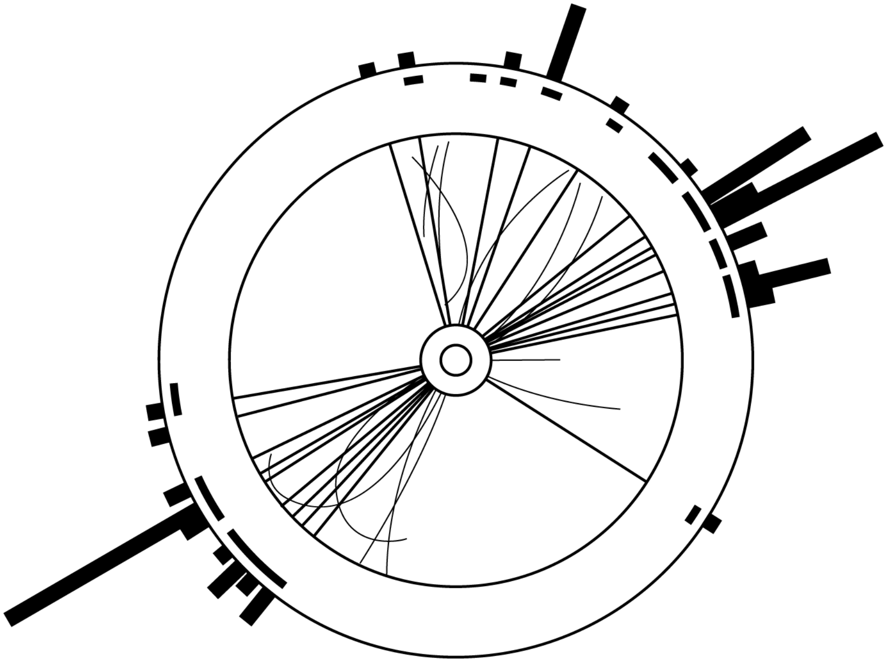

Figure 15.1 A Z hadronic decay recorded by the OPAL detector at CERN. The charged particle tracks can be seen in the inner region. The dark bands around the outer circle indicate the angular distribution of energy deposited in the outer calorimeter. The figure gives a projection of the event onto a plane perpendicular to the beam axis (see Dydak (1990)).

the lowest order of perturbation theory gives a differential decay rate into a $\mathrm{d}_k\bar{\mathrm{d}}_k$ pair ($\mathrm{d}_1 = \mathrm{d}$, $\mathrm{d}_2 = \mathrm{s}$, $\mathrm{d}_3 = \mathrm{b}$)

$$\frac{\mathrm{d}\Gamma(\mathrm{d}_k\bar{\mathrm{d}}_k)}{\mathrm{d}\cos\theta} = \frac{3G_{\mathrm{F}}{M_Z}^3}{32\sqrt{2}\pi}\left[\left(1-\frac{2}{3}\sin^2\theta_w\right)^2(1-\cos\theta)^2 + \left(\frac{2}{3}\sin^2\theta_{\mathrm{w}}\right)^2(1+\cos\theta)^2\right], \qquad (15.1)$$

where $\theta$ is the angle between the direction of the $\mathrm{d}_k$ quark momentum and the direction of the Z spin. Similarly, the decay rate to a $\mathrm{u}\bar{\mathrm{u}}$ or $\mathrm{c}\bar{\mathrm{c}}$ pair is

$$\frac{\mathrm{d}\Gamma(\mathrm{u}_k\bar{\mathrm{u}}_k)}{\mathrm{d}\cos\theta} = \frac{3G_{\mathrm{F}}{M_Z}^3}{32\sqrt{2}\pi}\left[\left(1-\frac{4}{3}\sin^2\theta_w\right)^2(1-\cos\theta)^2 + \left(\frac{4}{3}\sin^2\theta_w\right)^2(1+\cos\theta)^2\right]. \qquad (15.2)$$

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The colour factor of 3 is included in these rates. Terms in $m_q/M_Z$ are neglected. Integrating over $\theta$ gives the total decay rates

$$\Gamma(\mathrm{d}_k\bar{\mathrm{d}}_k) = \frac{G_F {M_Z}^3}{4\sqrt{2}\pi}\left[1 - \frac{4}{3}\sin^2\theta_\mathrm{w} + \frac{8}{9}\sin^4\theta_\mathrm{w}\right] = 0.3677\,\mathrm{GeV}, \quad (15.3)$$

$$\Gamma(\mathrm{u}_k\bar{\mathrm{u}}_k) = \frac{G_F {M_Z}^3}{4\sqrt{2}\pi}\left[1 - \frac{8}{3}\sin^2\theta_\mathrm{w} + \frac{32}{9}\sin^4\theta_\mathrm{w}\right] = 0.2853\,\mathrm{GeV}. \quad (15.4)$$

These numbers are obtained taking $\sin^2\theta_\mathrm{w} = 0.2315$ (see Section 11.4). Adding the decay rates to all pairs gives a total decay rate

$$\Gamma_{\mathrm{q\bar{q}}} = 1.6737\,\mathrm{GeV}.$$

This lowest order calculation is in quite good agreement with the experimental total hadronic decay rate, which is

$$\Gamma_{\mathrm{experiment}} = 1.741 \pm 0.006\,\mathrm{GeV}.$$

At the high energy of the Z boson, the effects of the strong interaction can be estimated with some confidence (Chapter 17). When additional gluon radiation is taken into account, the theoretical $\Gamma_{\mathrm{q\bar{q}}}$ is modified by a factor $f = 1.038$, and gives

$$\Gamma_{\mathrm{theoretical}} = f\,\Gamma_{\mathrm{q\bar{q}}} = 1.737\,\mathrm{GeV},$$

in very close agreement with experiment.

The identification of $\mathrm{b\bar{b}}$ jets and (less precisely) $\mathrm{c\bar{c}}$ jets enables these partial decay modes also to be compared with the Standard Model. The estimates from experiment are

$$\Gamma(\mathrm{b\bar{b}}) = 0.385 \pm 0.006\,\mathrm{GeV},$$
$$\Gamma(\mathrm{c\bar{c}}) = 0.275 \pm 0.025\,\mathrm{GeV}.$$

The Standard Model values, (15.3) and (15.4) corrected by the factor $f$, are

$$\Gamma(\mathrm{b\bar{b}})\,(\text{theoretical}) = 0.3817\,\mathrm{GeV},$$
$$\Gamma(\mathrm{c\bar{c}})\,(\text{theoretical}) = 0.2961\,\mathrm{GeV}.$$

The agreement between theory and experiment is satisfactory.

## 15.2 Asymmetry in quark production

We noted in Section 13.6 that the SLC electron beam can be polarised to produce Z bosons with a much higher degree of polarisation than those produced at CERN

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by unpolarised beams. From (15.1) there is a forward–backward asymmetry, with respect to the Z spin direction, in the angular distribution of b quarks in a $b\bar{b}$ pair produced by Z decay, given by

$$\frac{\Delta\Gamma}{\Gamma} = \frac{\Gamma(0 < \theta < \pi/2) - \Gamma(\pi/2 < \theta < \pi)}{\Gamma(0 < \theta < \pi/2) + \Gamma(\pi/2 < \theta < \pi)}$$
$$= -\frac{3}{4}\left(\frac{1 - (4/3)\sin^2\theta_w}{1 - (4/3)\sin^2\theta_w + (8/9)\sin^4\theta_w}\right).$$

Taking $\sin^2\theta_w = 0.2315$ gives $\Delta\Gamma/\Gamma = -0.7016$. At the peak of the Z mass distribution electromagnetic interference effects are very small, and one can expect a forward–backward asymmetry in the b quark jets relative to the electron beam direction. Measurements of b quark jets at SLC give a value of $\Delta\Gamma/\Gamma = -0.630 \pm 0.075$ (Prescott, 1996).

At LEP the Zs produced in $e^+e^-$ collisions are polarised along the direction of the electron beam with polarisation $P$, to give a forward–backward asymmetry of b quark jets with respect to the electron beam direction of

$$A^b_{FB} = P\frac{\Delta\Gamma}{\Gamma}.$$

From Section 13.6, taking $\sin^2\theta_w = 0.2315$ gives $P = -A_{LR} = -0.148$, so that

$$A^b_{FB}(\text{theory}) = 0.104.$$

The experimental value (Renton, 1996) is

$$A^b_{FB}(\text{experimental}) = 0.0997 \pm 0.0031.$$

The corresponding numbers for the $c$ quark jets are

$$A^c_{FB}(\text{theory}) = 0.0719,$$
$$A^c_{FB}(\text{experimental}) = 0.0729 \pm 0.0058.$$

Again the Standard Model and experiment are in accord.

A significant aspect of these asymmetry measurements is that an assignment of the right-handed rather than the left-handed quark fields to the $SU(2)$ doublet would lead to an asymmetry of opposite sign. (The total widths would be unaffected.) The results vindicate the left-handed assignment.

## 15.3 Hadronic decays of the $W^\pm$

The $e^+e^-$ colliders give a clean source of Z bosons, but there is as yet no clean source of $W^\pm$ bosons. Consequently the experimental data on $W^\pm$ decays is less

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precise than that for Z decay. The hadronic decays of a W$^\pm$ are, in its rest frame, like those of the Z: principally into two back-to-back jets, which are interpreted as the signatures of the initiating quark–antiquark pairs.

Consider for example the decay of the W$^+$ to a quark $\mathrm{u}_i$ ($\mathrm{u}_1 = \mathrm{u}$, $\mathrm{u}_2 = \mathrm{c}$) and an antiquark $\bar{\mathrm{d}}_j$ ($\bar{\mathrm{d}}_1 = \bar{\mathrm{d}}$, $\bar{\mathrm{d}}_2 = \bar{\mathrm{s}}$, $\bar{\mathrm{d}}_3 = \bar{\mathrm{b}}$). The coupling of the W$^+$ to the quark fields is given by $\mathcal{L}_{\mathrm{qw}}$ (equation (14.15)), and depends on the elements $V_{ij}$ of the Kobayashi–Maskawa matrix. In the lowest order of perturbation theory, and neglecting quark masses, the differential decay rate to a pair $\mathrm{u}_i\bar{\mathrm{d}}_j$ is

$$\frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}\cos\theta} = \frac{3G_{\mathrm{F}}{M_{\mathrm{w}}}^3}{16\sqrt{2}\pi}|V_{ij}|^2(1-\cos\theta)^2, \tag{15.5}$$

where $\theta$ is the angle between the direction of the $\mathrm{u}_i$ momentum and the direction of the W$^+$ spin. Integrating over $\theta$ gives the total decay rate

$$\Gamma(\mathrm{W}^+ \to \mathrm{u}_i\bar{\mathrm{d}}_j) = \frac{G_F {M_w}^3}{2\sqrt{2}\pi}|V_{ij}|^2 = (0.677 \pm 0.006)|V_{ij}|^2\ \mathrm{GeV}. \tag{15.6}$$

There is no data that resolves both initiating quark jets, so that we have no information from W decay on individual components of the KM matrix. However, we can sum over $j$, and since the KM matrix is unitary

$$\sum_{j=1}^{3}|V_{ij}|^2 = \sum_{j=1}^{3} V_{ij}V_{ij}^* = \sum_{j=1}^{3} V_{ij}{V_{ji}}^\dagger = 1 \quad \text{for } i = 1, 2, 3.$$

Then summing over the possible $u_i$, the u and c quarks, and including the factor $f$, we have

$$\Gamma(\text{all possible } \mathrm{q}\bar{\mathrm{q}}' \text{ pairs}) = \frac{G_{\mathrm{F}}{M_{\mathrm{w}}}^3 f}{\sqrt{2}\pi} = 1.41 \pm 0.008\,\mathrm{GeV}.$$

This value is in close agreement with the observed hadronic decay rate of the W$^+$:

$$\Gamma(\text{hadronic}) = 1.44 \pm 0.04\,\mathrm{GeV}.$$

Also, c quark jets can be identified with some confidence. From the above we would expect

$$\frac{\Gamma(\text{all possible } \mathrm{c}\bar{\mathrm{q}}' \text{ pairs})}{\Gamma(\text{all possible } \mathrm{q}\bar{\mathrm{q}}' \text{ pairs})} = 0.5$$

close to the measured value $0.51 \pm 0.08$.

In conclusion, it would seem that we have no reason to doubt the efficacy of the Standard Model in describing the interactions of the Z and W$^\pm$ bosons with both leptons and quarks. The details of the KM matrix $V_{ij}$ remain undetermined by these experiments, but it does pass two tests of unitarity. We have to rely on lower energy hadron physics to investigate the KM matrix more thoroughly, as will be discussed in Chapter 18.

## Problems

**15.1** Obtain the decay rates (15.3), (15.4) and (15.6). Note that quark masses have been neglected in these expressions (cf. Problem 13.3).