# PART 3

# HELIOSEISMOLOGY

# A REVIEW OF OBSERVATIONAL HELIOSEISMOLOGY

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## Introduction

There have been several excellent reviews of observational helioseismology in recent years. These include the reviews by Harvey (1988), Libbrecht (1988), and van der Raay (1988) presented at a recent conference in Tenerife. The present effort will concentrate on the progress made on solar rotation recently.

## Basic Helioseismology

The Sun is a resonant cavity that supports many (~ $10^7$) modes of oscillation. The modes that are most easily observed are the acoustic or p-modes. The eigenfunctions for these modes are:

$$E = f_{nl}(r) Y_{lm}(\theta,\phi) e^{i 2\pi \nu_{nlm} t}$$

$f_{nl}(r)$ is the radial part of the separable eigenfunction where $r$ is the radial coordinate measured from the center of the star. $n$ is the number of radial nodes in the eigenfunction. $Y_{lm}(\theta,\phi)$ is the spherical harmonic function, where $\theta$ is the colatitude and $\phi$ is the longitude. The spherical harmonic degree $l$ is the number of nodes of the spherical harmonic measured along a great circle that makes an angle $\cos^{-1}(\frac{m}{l(l+1)})$ with the equator. The azimuthal order $m$ is the number of nodes around the equator. The frequency of the eigenmode, $\nu_{nlm}$ , depends on the mode. Much of our information derived about the solar interior from helioseismology comes from the measurement of these frequencies.

Some examples of the spherical harmonics for $l = 40$ and different $m$ values are shown in Fig. 1. The limiting of these functions in latitude is not obvious at the lowest degrees. There is a term in the spherical harmonic function that multiplies the overall function that is $\sin^m(\theta)$ . This term causes the latitudinal falloff at high $m$ values. So for the sectoral modes ( $m = l$ ), we are observing equatorially concentrated quantities, while for zonal modes ( $m = 0$ ), we have an approximately equal weighting in latitude. This is true both in latitude and in radius: we observe integral quantities with the integral extending over different ranges of the independent variable. To learn about a solar parameter versus depth or latitude requires an equivalent differentiation of the data, a somewhat noisy process. If the eigenfunction of a mode does not extend into a certain region, we cannot learn anything directly about the region from this mode. This is why it is difficult to learn much about the deep interior from p-modes as not many of the



*G. Berthomieu and M. Cribier (eds.), Inside the Sun*, 253–264.

modes extend into that region.

The p-modes satisfy a dispersion relation that is best seen as an observational power spectrum ( Fig. 2 ). The $m$ variation of power has been suppressed. The dispersion relation, which shows the frequency $\nu_{nl}$ versus $l$ would consist of closely spaced dots along the "ridges" of power in Fig. 2. Each ridge corresponds to a constant value of $n$ or radial harmonic.

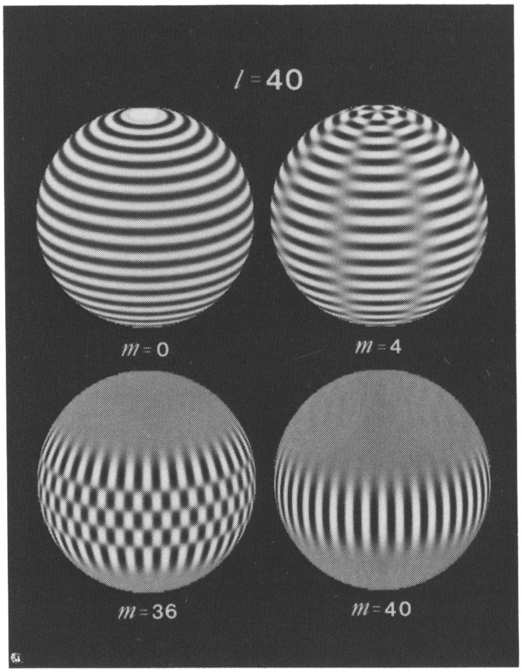


Fig. 1. Some examples of the spherical harmonic functions for degree $l = 40$. The white and black areas would correspond to receding and approaching areas in velocity observations. Note the increasing equatorial concentration as the azimuthal order $m$ approaches its maximum value at $l = m$.

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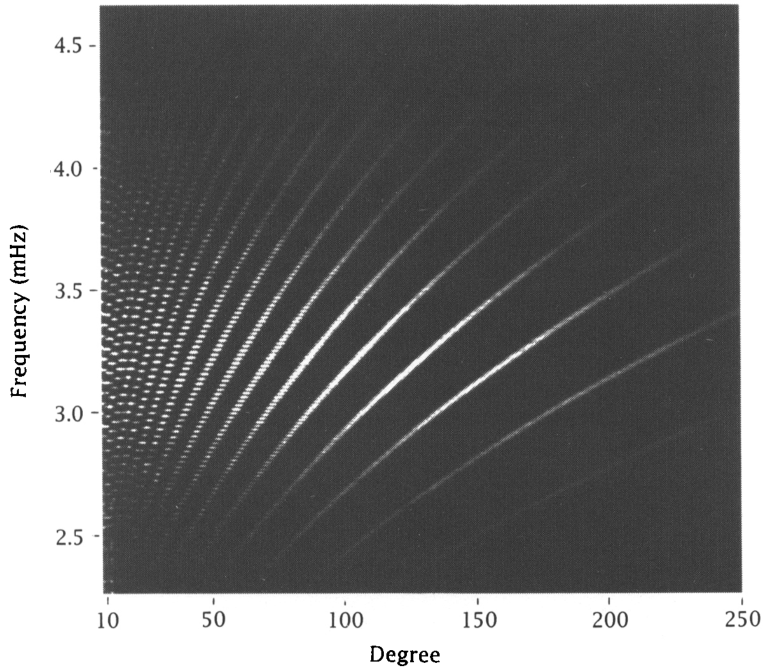


Fig. 2. This is a portion of the space-time spectrum of intensity oscillations of 50 hours of images from Duvall *et al.* (1987).

## Global modes

Our use of the frequencies to infer properties about the solar interior depends critically on the assumption that the waves are giving a true global average of solar properties. This may not be true for all the waves that we observe. Which waves are global modes? If waves can travel a circumference coherently to interfere with themselves, then they are global. So, the lifetime must be greater than the travel time for a circumference. The energy from a wave will travel at a velocity given by the group velocity: $\frac{d\omega}{dk} = 2\pi R_\odot \frac{d\nu}{dl}$ . So the time to travel a circumference is $T=2\pi\frac{R_\odot}{d\omega/dk}=\frac{1}{d\nu/dl}$. If the frequency width of a feature is $<\frac{1}{T}$, then it will be global. Or, equivalently width $<\frac{d\nu}{dl}$. $\frac{d\nu}{dl}$ is the frequency spacing between modes of the same $n$ and adjacent degree $l$. In our observational spectra, we always have the modes from several adjacent $l$ values in a single frequency spectrum because of our inability to see the back side of the Sun. The spherical harmonic functions are a complete set over the whole sphere. An example of a well-separated set of modes is shown in Fig. 3a. So the condition for waves to be global reduces observationally to the condition of being able to separate the adjacent $l$ "sidebands". There are areas of the $k$–$\omega$ diagram in which the modes are global as *e.g.* Fig. 3a. In addition,

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there are currently observed areas where this is not the case. An example is shown in Fig. 3b, where we see the normal phenomenon of the mode width increasing with frequency. At lower frequency we are separating the adjacent $l$-values while at higher frequencies we are not.

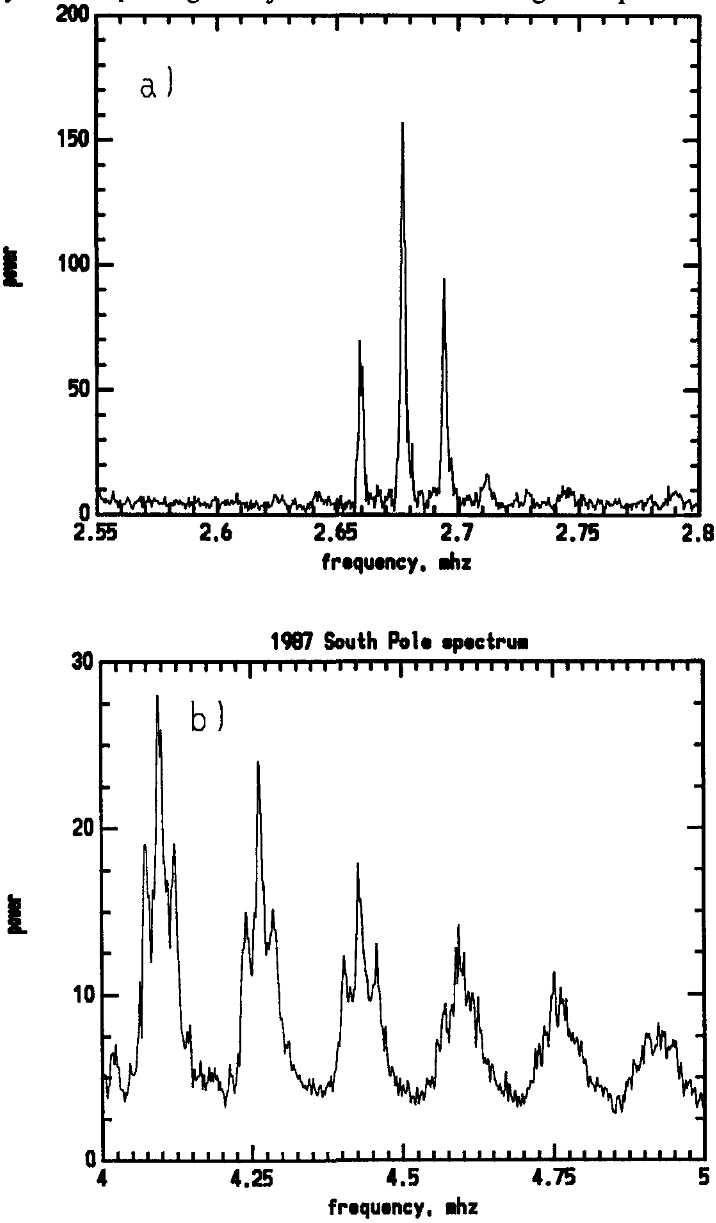


Fig. 3. Some examples of high resolution spectra at degree 50 from the data described by Jefferies *et al.* (1988). In a) we see a low frequency sprectral area where the spatial sidebands are well resolved. In b) we see the transition from resolving the spatial sidebands to not

resolving them as frequency increases. Both spectra have been averaged over the azimuthal order with rotation removed. In addition, the high frequency spectrum has been smoothed by convolution with a Gaussian of full width at half maximum of $5\mu Hz$.

Some care needs to be exercised in using results derived from such an area of the spectrum. The waves are not really a global average of solar properties but more of a local average over the observing aperture. Also for frequency estimation if the system response to adjacent degree modes is asymmetric ( which it often is ), then frequency estimates will be biased by the unequal weighting of the unresolved waves. This is a serious source of systematic error at high degrees, where the frequency separation of adjacent degree modes is small and unresolved in short observation sequences.

To how high in degree are any waves global? This is a question for which we do not currently have a good answer, with a lower limit of $l = 150$ for the Antarctic data of 1987. This question is currently under active investigation by several observational groups. It is an important question to answer for designing analysis techniques. If waves are not global, we may not need to go to the expense of computing a spherical harmonic decomposition at these degrees.

**Frequency estimation**

A problem that has not received enough attention from solar oscillation observers is the estimation of mode frequencies from the spectra that we observe. The problem has some subtle difficulties that are not always appreciated. The most important of these is probably that the statistics of the spectrum are not Gaussian. The standard deviation of the power at a certain frequency is equal to the power at that frequency. This means that points with high signal should not have a very high weight as they are sometimes given. This problem of the statistics is sometimes ignored and a standard unweighted nonlinear leastsquares fit is made to a region of the spectrum to a line profile ( *e.g.* Libbrecht (1986), Lazrek *et al.* (1988)). There should not be any systematic errors associated with this procedure as long as lines are symmetric ( which is generally assumed anyway). The incorrect weighting will lead to random frequency errors that are larger than those given by an optimum technique. Also, some fitting algorithms will tend to settle on one sharp spike as being the profile in question as reported by Sorensen (1988).

An advance was made in our understanding of the properties of the spectrum with the doctoral thesis of Woodard (1984). He showed that for the case of a harmonic oscillator excited by random noise that the power spectrum will be distributed as chi-square with two degrees of freedom, or that the distribution function at a given frequency of the power as measured in a large number of independent trials would be given by

$$\gamma(p_0) = \frac{1}{p_0} e^{-p/p_0}.$$

$p_0$ is the mean or expectation value of the power that one would obtain from doing a large number of experiments and averaging the power spectra. $p$ is the power in a given realization. He then showed that the observed spectra were consistent with this distribution. One consequence of this is the innate uncertainty of mode frequencies. Even if there is no instrumental noise and no solar background at the frequency of the mode in question, there is still an uncertainty in measuring the mode frequency because of the stochastic nature of the excitation process. An approximate expression for this uncertainty is

$$\sigma = \sqrt{\frac{w}{4\pi T}} ,$$

where $w$ is the fwhm of the mode and T is the length of the observing sequence. This leads to nonnegligible errors. As a concrete example, consider $w = 1\mu Hz, T = 90 days$. The result is $\sigma = 0.1\mu Hz$. For long observing sequences, this noise source, which we might call realization noise, dominates in many situations according to the simulations of Duvall and Harvey (1986). $p_0$ is a function of frequency in our spectra with Lorentzian profiles representing the modes on top of an underlying smooth background.

The Lorentz profile is the one expected for a harmonic oscillator excited by random noise. To date there has not been a good observational demonstration that this is the correct profile to use. A logical consequence of the random oscillator model is that the power and phase are random from point to point in the observed spectrum (Jenkins and Watts, 1968) at frequencies separated by at least $1/T$ where $T$ is the length of the time series. This will not be exactly true for gapped data sets. A way to simulate an observed power spectrum is then to assume a mean or expected spectrum and then at each "observed" frequency to pick a random number consistent with the above distribution. An example of a simulated realization and its associated expectation value are shown in Fig. 4 from Woodard (1984).

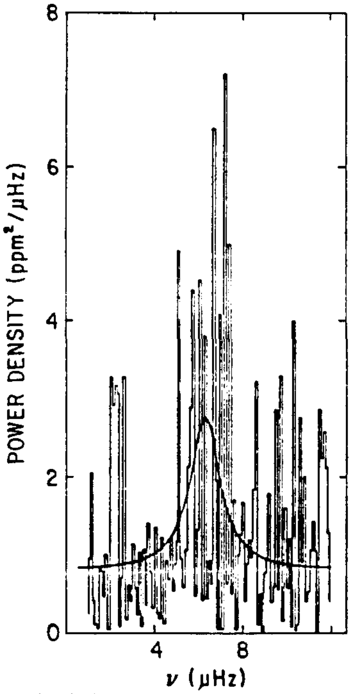


Fig. 4. A simulated mode ( the histogram ) along with its limit spectrum (the smooth curve). The power is independent from point to point with only a statistical relation to the limit spectrum. If one did a large number of realizations and averaged the power spectra, the limit spectrum would result.

The problem we would normally like to solve is to estimate the parameters of the underlying mean spectrum given a realization. The maximum likelihood technique described by Duvall and Harvey (1986) is a good way to do this. If we consider the mean spectrum as a function of frequency $p_0(\nu)$ to be a sum of Lorenztians plus background of unknown parameters, we can then construct the joint probability density that we would observe the spectrum that we did as the product of individual probabilities of the form shown above:

$$L = \prod_i \gamma(p_0(\nu_i)) = e^{-\sum_i [\frac{p(\nu_i)}{p_0(\nu_i)} + \ln p_0(\nu_i)]}$$

The method then consists of maximizing this probability distribution or likelihood function as a function of the parameters describing the spectrum. Maximizing the likelihood function is equivalent to minimizing the negative of the argument of the exponential:

$$-\ln L = \sum_i [\frac{p(\nu_i)}{p_0(\nu_i)} + \ln p_0(\nu_i)]$$

The above function is minimized using standard techniques.

## Solar Rotation

If the sun were spherically symmetric and nonrotating, its mode frequencies would be independent of the azimuthal order $m$. Fortunately for helioseismologists this is not the case as it permits some of the most interesting inferences about the solar interior. The solar rotation is the largest departure from sphericity in its effect on the mode frequencies. The largest part of the frequency shift is due to the advection of the mode. That is, the mode is fixed to the rotating sun and the observer sees the pattern moving. This causes a Doppler shift of the mode's frequency which varies linearly with the azimuthal order $m$. The mode is advected at a rate which depends on the rotation rate in the region in which it is concentrated. By examining modes with different radial and latitudinal regions of concentration, we can learn about the rotation versus depth and latitude in the solar interior.

Observationally we express the variation of frequency in a multiplet ( fixed $n, l$, varying $m$ ) as a Legendre polynomial series:

$$\nu_{nlm} - \nu_{nl} = L \sum_i a_i P_i(-m/L),$$

where $P_i$ is the Legendre polynomial and $L = \sqrt{l(l+1)}$. Some observers use $l$ instead of $L$ in this relation with the result being small differences in the $a_i$'s. The odd terms in this sum yield the direct effect of solar rotation while the even terms contain information about latitudinal variation of the mean structure and about internal magnetic fields.

The coefficient of the first term, $a_1$, is by far the largest coefficient, being about a factor of 20 larger than the next largest, $a_3$. It signifies roughly a latitudinal average of the rotation. If the rotation were only a function of depth, it would be the only nonzero coefficient. The next odd coefficient, $a_3$, is a measure of latitudinal differential rotation. $a_5$ is similarly a measure of latitudinal differential rotation but is somewhat smaller than $a_3$ and is not well determined as yet.

The $a_i$ coefficients have been measured by a number of observers in the intermediate degree range of $l = 10$–$60$ ( *e.g.* Libbrecht (1988) and Brown and Morrow (1987)). The results have led to a consistent picture of the internal rotation versus depth and latitude for the outer half by radius of the sun. In this picture, the rotation is constant with depth and latitude

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throughout the convection zone and the decrease of rotation rate between equator and pole of 20% that we see at the surface persists throughout the convection zone. Below the convection zone is a transition zone of depth at most $0.1R_\odot$ ( it has not been resolved yet; Christensen-Dalsgaard and Schou (1988)). Below this depth the sun rotates as a solid body: no differential rotation in latitude or depth.

To see how well ( or not ) this model compares to the results, it is useful to consider the "forward" problem. That is, given a model of the interior rotation, what values of the $a_i$ will we observe? Morrow (1988) has considered several interesting cases which I will show here. In all of these figures the calculations are compared with the data of Brown and Morrow (1987). The variation of the $a_i$ is shown versus the degree $l$ of the mode which is a proxy for depth, lower $l$ corresponding to larger depths.

A model of the rotation in the solar interior that has received much attention is the fluid dynamic calculation of Gilman (1977), Gilman and Miller (1986), and Glatzmaier (1987). This model suggests that the rotation of the convection zone should be constant on cylinders. In Fig. 5, we show (following Morrow (1988)) a comparison of the $a_1$,$a_3$and$a_5$ coefficients for a model with constant rotation on cylinders (the bottom curve). The top curve in this figure is a model with rotation constant with depth in the convection zone but having the surface latitudinal differential rotation. The model with rotation constant with depth but with normal surface latitudinal differential rotation obviously fits the data pretty well while the rotation constant on cylinders model is inadequate. It is on the basis of this figure that rotation constant on cylinders is considered to be excluded by the helioseismic data.

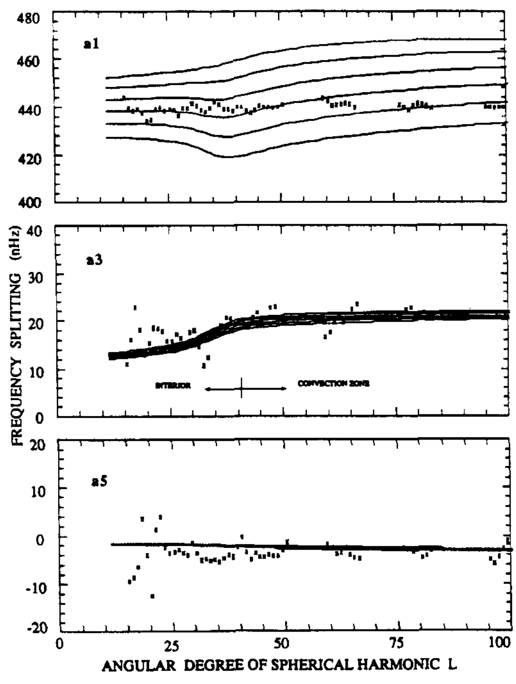


Fig. 5. The odd rotation coefficients $a_i$ for models with rotation constant on cylinders ( bottom curve for $a_1$), rotation constant with radius throughout the convection zone ( top curve for $a_1$ ), and intermediate models from Morrow (1988).

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The approximate constancy with the degree of the mode $l$ of the $a_1$ coefficient suggests that the rotation averaged over latitude is approximately constant. This point is brought home clearly in Fig. 6 from Morrow (1988) which shows several models compared that all have differential rotation only in latitude in the convection zone and rotation constant in latitude and depth below this level but at a rate that varies from model to model. The center rate is near the value derived for the correct latitudinal averaging to get a constant $a_1$.

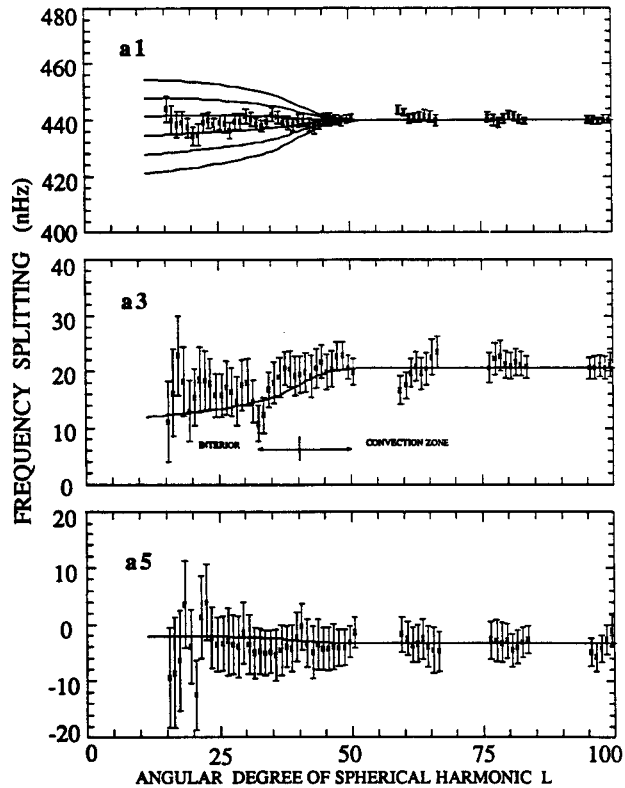


Fig. 6. The odd rotation coefficients $a_i$ for models in which the rotation in the convection zone is independent of radius and has the same latitudinal structure as the surface but the interior constant rotation rate is varied for the different models. The rate is varied by 11% of the mean value between the bottom and top curves for $a_1$.

The depth at which the rotation switches from surface latitudinal differential rotation to constant rotation in latitude is investigated in Fig. 7 again from Morrow (1988). $a_3$ is seen to be the sensitive parameter in this case. As the depth of the rotation transition is varied over a total range of 0.25 $R_{\odot}$, $a_3$ varies by an amount that is distinguishable by the data. The top curve corresponds to a deeper region of latitudinal differential rotation.

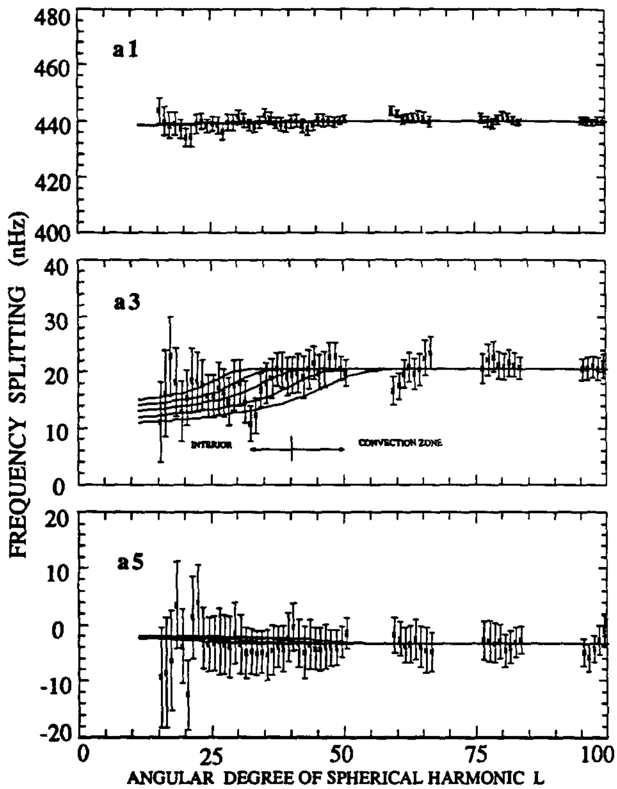


Fig. 7. Rotation coefficients for models in which the depth of the convection zone is varied. The bottom curve for $a_3$ is for the shallowest convection zone.

These curves show that we can make some pretty strong statements about the solar interior rotation, at least over the outer half of the sun by radius. The convection zone does not have rotation constant on cylinders but the rotation looks much as it is at the surface. The rotation immediately below the differentially rotating layer is constant with latitude at a rate that corresponds to about 30 degrees latitude. The actual depth at which the transition occurs between differential and rigid rotation is somewhat uncertain because of slight differences in the results of different observers. The current results should provide significant input to workers studying the solar dynamo.

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