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The CREAM conjecture and certain abelian-by-nilpotent varieties

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The work in this thesis has arisen from an attempt to confirm the claim (now withdrawn) by Brady, Bryce and Cossey [1] that the subvarieties of $\underline{\underline{A}}_m(\underline{\underline{N}}_2 \wedge \underline{\underline{B}}_n)$ are CREAM in the sense of Higman [2] when m, n are coprime. This attempt has succeeded for certain values of n, namely when n is odd and not divisible by q^4 for any prime q. In the course of this work, we obtain results of independent interest (Chapters 2 and 4).

Chapter 1 is essentially an elaboration of certain remarks and observations made by Higman [2]. First we clarify and make precise Higman's definitions of CREAM functions and CREAM varieties. Some elementary properties of CREAM functions and CREAM varieties are also given. The rest of the chapter supplies the details of the methods sketched by Higman [2] to study the CREAM question for subvarieties of $\underline{\underline{A}}_m\underline{\underline{W}}$, where $\underline{\underline{W}}$ is a locally finite variety of exponent prime to m.

The finite q-groups of class 2 with cyclic centre have been explicitly described in [1]. Using this description, we solve in Chapter 2 the isomorphism problem for such groups when q is an odd prime, and assign to each of these groups a set of invariants. The methods used are elementary. This enables us to classify in Chapter 3 the closed classes of irreducible linear groups over $GF(p)$ belonging to $\underline{\underline{N}}_2 \wedge \underline{\underline{B}}_{q^n}$ where p, q are different primes.

In Chapter 4, we study the structure of non-degenerate alternating

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forms on a module U over Z_{q^α} (the ring of integers modulo q^α, $\alpha \geq 1$). This is applied to calculate the order of the group of linear automorphisms, lin aut G, of a finite q-group G of class 2 with cyclic centre, q being an odd prime. It turns out that lin aut G is closely related to a certain subgroup of the group of isometries of U. We also obtain some extensions of certain results of Winter [3].

We prove in Chapter 5 that the subvarieties of $\underline{A}_m(\underline{N}_2 \wedge \underline{B}_n)$ are CREAM when m, n are coprime, and n is odd and not divisible by q^4 for any prime q.

References

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[3] David L. Winter, "The automorphism group of an extraspecial p-group", *Rocky Mountain J. Math.* 2 (1972), 159-168.