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Corrigenda

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'A Kahn-Priddy sequence and a conjecture of G. W. Whitehead'

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In this note we show that Propositions 5·2 and 5·3 of [1] are incorrect as stated. Thus statement (5) of Theorem 1·3 must be considered as a conjecture and certainly does not follow from our constructions. Our error does not affect any other results in the paper, and, indeed, § 5 was included mainly to provide some conceptual motivation for our constructions.

We use the notation of [1].

Recall that X_k is a wedge summand of $\Sigma^\infty D_2^k S^1$ and Y_k is an 'iterated cofiber' space mapping to $\Sigma^{k-1} D_2^k S^1$ such that

$$H_*(X_k) = \bigcap_{i=1}^{k-1} \operatorname{Im} T_i \quad \text{and} \quad H_*(\Sigma^{1-k} Y_k) = \bigcap_{i=1}^{k-1} \ker p_{i*}$$

in $H_*(D_2^k S^1)$. Our mistake was in the assertion, made in the last line of the proof of Proposition 5·2, that

$$\bigcap_{i=1}^{k-1} \operatorname{Im} T_i = \bigcap_{i=1}^{k-1} \ker p_{i*}. \qquad (*)$$

In fact, if $k \geqslant 4$, only the relation $\subseteq$ is true. We neglected to consider elements in the kernel of $H_*(D_2 D_2 Y) \to H_*(D_4 Y)$ of the form

$$(a \bar{*} b) \bar{*} (c \bar{*} d) + (a \bar{*} c) \bar{*} (b \bar{*} d)$$

where a, b, c, and d are elements of $H_*(Y)$.

The small size of $H_*(S^1)$ easily implies that (*) is true when $k = 2$. With more care one can check that (*) is even true when $k = 3$. We now give what is essentially the simplest example showing that (*) is false when $k = 4$.

Notation. Let Σ_5 denote the symmetric group on $\{1, \ldots, 5\}$ and let $x \in H_1(S^1)$ be the generator. If $\sigma \in \Sigma_5$ and

$$y = [(\bar{Q}^i x \bar{*} \bar{Q}^{i_1} x) \bar{*} (\bar{Q}^{i_2} x \bar{*} \bar{Q}^{i_3} x)] \bar{*} [(\bar{Q}^{i_4} x \bar{*} \bar{Q}^{i_5} x) \bar{*} \bar{Q}^j \bar{Q}^k x] \in H_*(D_2^4 S^1),$$

let $\sigma(y)$ denote the element

$$[\bar{Q}^i x \bar{*} \bar{Q}^{i_{\sigma(1)}} x) \bar{*} \bar{Q}^{i_{\sigma(2)}} x \bar{*} \bar{Q}^{i_{\sigma(3)}} x)] \bar{*} [(\bar{Q}^{i_{\sigma(4)}} x \bar{*} \bar{Q}^{i_{\sigma(5)}} x) \bar{*} \bar{Q}^j \bar{Q}^k x].$$

Now let $\alpha, \beta \in \Sigma_5$ be the permutations

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 5 & 2 & 3 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}.$$

Observe that $y+\alpha(y) \in \ker p_{3*}$ and $y+\beta(y) \in \ker p_{2*}$ where we recall that

$$p_1: D_2^4 S^1 \to D_2 D_2 D_4 S^1, \quad p_2: D_2^4 S^1 \to D_2 D_4 D_2 S^1 \quad \text{and} \quad p_3: D_2^4 S^1 \to D_4 D_2 D_2 S^1.$$

It is easily checked that the subgroup G generated by α and β has a presentation $\langle \alpha, \beta \mid \alpha^2 = \beta^2 = (\alpha\beta)^6 = 1 \rangle$ and thus can be identified with the dihedral group of order 12.

Example. Let

$$y = [(\bar{Q}^2 x \mathbin{\bar{*}} \bar{Q}^3 x) \mathbin{\bar{*}} (\bar{Q}^4 x \mathbin{\bar{*}} \bar{Q}^5 x)] \mathbin{\bar{*}} [\bar{Q}^6 x \mathbin{\bar{*}} \bar{Q}^7 x) \mathbin{\bar{*}} \bar{Q}^3 \bar{Q}^1 x]$$

and let $\bar{y} = \Sigma_{\sigma \in G} \sigma(y)$. Then $\bar{y} \in \ker p_{2*} \cap \ker p_{3*}$ by our observation above, and $\bar{y} \in \ker p_{1*}$ because $\bar{Q}^3 \bar{Q}^1 x$ is in the kernel of $H_*(D_2 D_2 S^1) \to H_*(D_4 S^1)$. $\bar{y}$ is not in $\operatorname{Im} T_1 \cap \operatorname{Im} T_2 \cap \operatorname{Im} T_3$.

Remarks. (1) Although (*) does not hold in general, it is true when restricted to the primitives in the coalgebra $H_*(D_2^k S^1)$ (or equivalently, the subspace of 'pure wreath product elements' in $H_*(D_2^k S^1)$). (2) We do not know whether or not Y_k is a stable wedge summand in $\Sigma^{k-1} D_2^k S^1$. If it is, it would follow that $\Sigma^{k-1} X_k$ is a summand in $\Sigma^\infty Y_k$.

REFERENCE

[1] N. J. KUHN. A Kahn–Priddy sequence and a conjecture of G. W. Whitehead. *Math. Proc. Cambridge Philos. Soc.* **92** (1982), 467–483.